

Chapter 1. Irrational Numbers

Ex 1.1

Answer 1A.

$$\frac{3}{5}$$

$$5 = 1 \times 5 = 2^0 \times 5^1$$

i.e., 5 can be expressed as $2^m \times 5^n$.

$\therefore \frac{3}{5}$ has terminating decimal representation.

Answer 1B.

$$\frac{5}{7}$$

$$7 = 1 \times 7$$

i.e., 7 cannot be expressed as $2^m \times 5^n$.

$\therefore \frac{5}{7}$ does not have terminating decimal representation.

Answer 1C.

$$\frac{25}{49}$$

$$49 = 7 \times 7$$

i.e. 49 cannot be expressed as $2^m \times 5^n$.

Hence, $\frac{25}{49}$ does not have terminating decimal representation.

Answer 1D.

$$\frac{37}{40}$$

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$$

i.e. 40 can be expressed as $2^m \times 5^n$.

Hence, $\frac{37}{40}$ has terminating decimal representation.

Answer 1E.

$$\frac{57}{64}$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 \times 5^0$$

i.e. 64 can be expressed as $2^m \times 5^n$.

Hence, $\frac{57}{64}$ has terminating decimal representation.

Answer 1F.

$$\frac{59}{75}$$

$$75 = 5 \times 5 \times 3 = 2^2 \times 3^1$$

i.e. 75 cannot be expressed as $2^m \times 5^n$.

Hence, $\frac{59}{75}$ does not have terminating decimal representation.

Answer 1G.

$$\frac{89}{125}$$

$$125 = 5 \times 5 \times 5 = 2^0 \times 5^3$$

i.e. 125 can be expressed as $2^m \times 5^n$.

Hence, $\frac{89}{125}$ has terminating decimal representation.

Answer 1H.

$$\frac{125}{213}$$

$$213 = 3 \times 71$$

i.e. 213 cannot be expressed as $2^m \times 5^n$.

Hence, $\frac{125}{213}$ does not have terminating decimal representation.

Answer 1.

$$\frac{147}{160}$$

$$160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^5 \times 5^1$$

i.e. 160 can be expressed as $2^m \times 5^n$.

Hence, $\frac{147}{160}$ has terminating decimal representation.

Answer 2A.

$$0.93 = \frac{93}{100}$$

Answer 2B.

$$4.56 = \frac{456}{100} = \frac{456 \div 4}{100 \div 4} = \frac{114}{25}$$

Answer 2C.

$$0.614 = \frac{614}{1000} = \frac{614 \div 2}{1000 \div 2} = \frac{307}{500}$$

Answer 2D.

$$21.025 = \frac{21025}{1000} = \frac{21025 \div 25}{1000 \div 25} = \frac{841}{40}$$

Answer 3.

(i) $\frac{3}{5}$

$$\frac{3}{5} = 0.6$$

(ii) $\frac{8}{11}$

$$\frac{8}{11} = 0.72727272... = 0.\overline{72}$$

(iii) $\frac{-2}{7}$

$$\frac{-2}{7} = -0.285714285714... = -0.\overline{285714}$$

(iv) $\frac{12}{21}$

$$\frac{12}{21} = 0.571428571428... = 0.\overline{571428}$$

(v) $\frac{13}{25}$

$$\frac{13}{25} = 0.52$$

(vi)

$$\frac{2}{3} = 0.6666..... = 0.\overline{6}$$

Answer 4A.

$$\text{Let } x = 0.\dot{7}$$

$$\text{Then, } x = 0.7777\ldots \quad \dots(1)$$

Here, the number of digits recurring is only 1, so we multiply both sides of the equation (1) by 10.

$$\therefore 10x = 10 \times 0.7777\ldots = 7.777\ldots \quad \dots(2)$$

On subtracting (1) from (2), we get

$$9x = 7$$

$$\therefore x = \frac{7}{9}$$

$$\therefore 0.\dot{7} = \frac{7}{9}$$

Answer 4B.

$$\text{Let } x = 0.\overline{35}$$

$$\text{Then, } x = 0.353535\ldots \quad \dots(1)$$

Here, the number of digits recurring is 2, so we multiply both sides of the equation (1) by 100.

$$\therefore 100x = 100 \times 0.353535\ldots = 35.3535\ldots \quad \dots(2)$$

On subtracting (1) from (2), we get

$$99x = 35$$

$$\therefore x = \frac{35}{99}$$

$$\therefore 0.\overline{35} = \frac{35}{99}$$

Answer 4C.

$$\text{Let } x = 0.\overline{89}$$

$$\text{Then, } x = 0.898989\ldots \quad \dots(1)$$

Here, the number of digits recurring is 2, so we multiply both sides of the equation (1) by 100.

$$\therefore 100x = 100 \times 0.898989\ldots = 89.8989\ldots \quad \dots(2)$$

On subtracting (1) from (2), we get

$$99x = 89$$

$$\therefore x = \frac{89}{99}$$

$$\therefore 0.\overline{89} = \frac{89}{99}$$

Answer 4D.

$$\text{Let } x = 0.\overline{057}$$

$$\text{Then, } x = 0.057057\dots \quad \dots(1)$$

Here, the number of digits recurring is 3, so we multiply both sides of the equation (1) by 1000.

$$\therefore 1000x = 1000 \times 0.057057\dots = 57.057\dots \quad \dots(2)$$

On subtracting (1) from (2), we get

$$999x = 57$$

$$\therefore x = \frac{57}{999} = \frac{19}{333}$$

$$\therefore 0.\overline{057} = \frac{19}{333}$$

Answer 4E.

$$\text{Let } x = 0.\overline{763}$$

$$\text{Then, } x = 0.763763\dots \quad \dots(1)$$

Here, the number of digits recurring is 3, so we multiply both sides of the equation (1) by 1000.

$$\therefore 1000x = 1000 \times 0.763763\dots = 763.763\dots \quad \dots(2)$$

On subtracting (1) from (2), we get

$$999x = 763$$

$$\therefore x = \frac{763}{999}$$

$$\therefore 0.\overline{763} = \frac{763}{999}$$

Answer 4F.

$$\text{Let } x = 2.\overline{67}$$

$$\text{Then, } x = 2.676767\dots \quad \dots(1)$$

Here, the number of digits recurring is 2, so we multiply both sides of the equation (1) by 100.

$$\therefore 100x = 100 \times 2.676767\dots = 267.6767\dots \quad \dots(2)$$

On subtracting (1) from (2), we get

$$99x = 265$$

$$\therefore x = \frac{265}{99}$$

$$\therefore 2.\overline{67} = \frac{265}{99}$$

Answer 4G.

Let $x = 4.\overline{6724} = 4.6724724...$

Here, only numbers 724 is being repeated, so first we need to remove 6 which proceeds 724.

We multiply by 10 so that only the recurring digits remain after decimal.

$$\therefore 10x = 46.724724.... \quad(1)$$

The number of digits recurring in equation (1) is 3, so we multiply both sides of the equation (1) by 1000.

$$\therefore 10000x = 1000 \times 46.724724.... = 46724.724..... \quad(2)$$

On subtracting (1) from (2), we get

$$9990x = 46678$$

$$\therefore x = \frac{46678}{9990} = \frac{23339}{4995}$$

$$\therefore 4.\overline{6724} = \frac{763}{999} = \frac{23339}{4995}$$

Answer 4H.

Let $x = 0.0\overline{17} = 0.01717..$

Here, only numbers 17 is being repeated, so first we need to remove 0 which proceeds 17.

We multiply by 10 so that only the recurring digits remain after decimal.

$$\therefore 10x = 0.1717.... \quad(1)$$

The number of digits recurring in equation (1) is 2, so we multiply both sides of the equation (1) by 100.

$$\therefore 1000x = 100 \times 0.1717.... = 17.1717..... \quad(2)$$

On subtracting (1) from (2), we get

$$990x = 17$$

$$\therefore x = \frac{17}{990}$$

$$\therefore 0.0\overline{17} = \frac{17}{990}$$

Answer 4I.

Let $x = 17.0\overline{27} = 17.027777..$

Here, only number 7 is being repeated, so first we need to remove 02 which proceeds 7.

We multiply by 100 so that only the recurring digits remain after decimal.

$$\therefore 100x = 1702.7777.... \quad(1)$$

The number of digits recurring in equation (1) is 1, so we multiply both sides of the equation (1) by 10.

$$\therefore 1000x = 10 \times 1702.7777.... = 17027.7774..... \quad(2)$$

On subtracting (1) from (2), we get

$$900x = 15325$$

$$\therefore x = \frac{15325}{900} = \frac{613}{36}$$

$$\therefore 17.0\overline{27} = \frac{613}{36}$$

Answer 5A.

A rational number lying between $\frac{2}{5}$ and $\frac{3}{4}$

$$= \frac{\frac{2}{5} + \frac{3}{4}}{2}$$

$$= \frac{\frac{8+15}{20}}{2}$$

$$= \frac{\frac{23}{20}}{2}$$

$$= \frac{23}{40}$$

Answer 5B.

A rational number lying between $\frac{3}{4}$ and $\frac{5}{7}$

$$= \frac{\frac{3}{4} + \frac{5}{7}}{2}$$

$$= \frac{\frac{21+20}{28}}{2}$$

$$= \frac{\frac{41}{28}}{2}$$

$$= \frac{41}{56}$$

Answer 5C.

A rational number lying between $\frac{4}{3}$ and $\frac{7}{5}$

$$= \frac{\frac{4}{3} + \frac{7}{5}}{2}$$

$$= \frac{\frac{20+21}{15}}{2}$$

$$= \frac{\frac{41}{15}}{2}$$

$$= \frac{41}{30}$$

Answer 5D.

A rational number lying between $\frac{5}{9}$ and $\frac{6}{7}$

$$\begin{aligned} &= \frac{\frac{5}{9} + \frac{6}{7}}{2} \\ &= \frac{\frac{35 + 54}{63}}{2} \\ &= \frac{\frac{89}{63}}{2} \\ &= \frac{89}{126} \end{aligned}$$

Answer 6A.

A rational number lying between 3 and 4

$$\begin{aligned} &= \frac{3 + 4}{2} \\ &= \frac{7}{2} \\ &= 3.5 \end{aligned}$$

Answer 6B.

A rational number lying between 7.6 and 7.7

$$\begin{aligned} &= \frac{7.6 + 7.7}{2} \\ &= \frac{15.3}{2} \\ &= 7.65 \end{aligned}$$

Answer 6C.

A rational number lying between 8 and 8.04

$$\begin{aligned} &= \frac{8 + 8.04}{2} \\ &= \frac{16.04}{2} \\ &= 8.02 \end{aligned}$$

Answer 6D.

A rational number lying between 101 and 102

$$= \frac{101 + 102}{2}$$

$$= \frac{203}{2}$$

$$= 101.5$$

Answer 7A.

A rational number lying between 0 and 1 = $\frac{0+1}{2} = \frac{1}{2}$

A rational number lying between 0 and $\frac{1}{2} = \frac{0+\frac{1}{2}}{2} = \frac{1}{4}$

A rational number lying between 0 and $\frac{1}{4} = \frac{0+\frac{1}{4}}{2} = \frac{1}{8}$

$$0 < \frac{1}{8} < \frac{1}{4} < \frac{1}{2} < 1$$

Hence, three rational numbers between 0 and 1 are

$$\frac{1}{8}, \frac{1}{4} \text{ and } \frac{1}{2}.$$

Answer 7B.

A rational number lying between 6 and 7 = $\frac{6+7}{2} = \frac{13}{2}$

A rational number lying between 6 and $\frac{13}{2} = \frac{6+\frac{13}{2}}{2} = \frac{\frac{25}{2}}{2} = \frac{25}{4}$

A rational number lying between $\frac{13}{2}$ and 7 = $\frac{\frac{13}{2}+7}{2} = \frac{\frac{27}{2}}{2} = \frac{27}{4}$

$$6 < \frac{25}{4} < \frac{13}{2} < \frac{27}{4} < 7$$

Hence, three rational numbers between 6 and 7 are

$$\frac{25}{4}, \frac{13}{2} \text{ and } \frac{27}{4}.$$

Answer 7C.

A rational number lying between -3 and $3 = \frac{-3+3}{2} = \frac{0}{2} = 0$

A rational number lying between -3 and $0 = \frac{-3+0}{2} = -\frac{3}{2}$

A rational number lying between 0 and $3 = \frac{0+3}{2} = \frac{3}{2}$

$$-3 < -\frac{3}{2} < 0 < \frac{3}{2} < 3$$

Hence, three rational numbers between -3 and 3 are

$$-\frac{3}{2}, 0 \text{ and } \frac{3}{2}.$$

Answer 7D.

A rational number lying between -5 and $-4 = \frac{-5+(-4)}{2} = -\frac{9}{2}$

A rational number lying between -5 and $-\frac{9}{2} = \frac{-5+\left(-\frac{9}{2}\right)}{2} = \frac{-\frac{19}{2}}{2} = -\frac{19}{4}$

A rational number lying between $-\frac{9}{2}$ and $-4 = \frac{-\frac{9}{2}+(-4)}{2} = \frac{-\frac{17}{2}}{2} = -\frac{17}{4}$

$$-5 < -\frac{19}{4} < -\frac{9}{2} < -\frac{17}{4} < -4$$

Hence, three rational numbers between -5 and -4 are

$$-\frac{19}{4}, -\frac{9}{2} \text{ and } -\frac{17}{4}.$$

Answer 8A.

Since, $\frac{2}{5} < \frac{2}{3}$

Let $a = \frac{2}{5}$, $b = \frac{2}{3}$ and $n = 5$

$$\therefore d = \frac{b-a}{n+1} = \frac{\frac{2}{3} - \frac{2}{5}}{5+1} = \frac{\frac{10-6}{15}}{6} = \frac{4}{90} = \frac{2}{45}$$

Hence, required rational numbers are:

$$a+d = \frac{2}{5} + \frac{2}{45} = \frac{18+2}{45} = \frac{20}{45} = \frac{4}{9}$$

$$a+2d = \frac{2}{5} + 2 \times \frac{2}{45} = \frac{2}{5} + \frac{4}{45} = \frac{18+4}{45} = \frac{22}{45}$$

$$a+3d = \frac{2}{5} + 3 \times \frac{2}{45} = \frac{2}{5} + \frac{2}{15} = \frac{6+2}{15} = \frac{8}{15}$$

$$a+4d = \frac{2}{5} + 4 \times \frac{2}{45} = \frac{2}{5} + \frac{8}{45} = \frac{18+8}{45} = \frac{26}{45}$$

$$a+5d = \frac{2}{5} + 5 \times \frac{2}{45} = \frac{2}{5} + \frac{2}{9} = \frac{18+10}{45} = \frac{28}{45}$$

Thus, five rational numbers between $\frac{2}{5}$ and $\frac{2}{3}$ are

$$\frac{4}{9}, \frac{22}{45}, \frac{8}{15}, \frac{26}{45} \text{ and } \frac{28}{45}.$$

Answer 8B.

Since, $-\frac{3}{4} < -\frac{2}{5}$

Let $a = -\frac{2}{5}$, $b = -\frac{3}{4}$ and $n = 5$

$$\therefore d = \frac{b-a}{n+1} = \frac{-\frac{3}{4} - \left(-\frac{2}{5}\right)}{5+1} = \frac{-\frac{3}{4} + \frac{2}{5}}{6} = \frac{\frac{-15+8}{20}}{6} = -\frac{7}{120}$$

Hence, required rational numbers are:

$$a+d = -\frac{2}{5} + \left(-\frac{7}{120}\right) = -\frac{2}{5} - \frac{7}{120} = \frac{-48-7}{120} = -\frac{55}{120} = -\frac{11}{24}$$

$$a+2d = \frac{2}{5} + 2 \times \left(-\frac{7}{120}\right) = \frac{2}{5} + \frac{4}{45} = \frac{18+4}{45} = \frac{22}{45}$$

$$a+3d = \frac{2}{5} + 3 \times \left(-\frac{7}{120}\right) = \frac{2}{5} + \frac{2}{15} = \frac{6+2}{15} = \frac{8}{15}$$

$$a+4d = \frac{2}{5} + 4 \times \left(-\frac{7}{120}\right) = \frac{2}{5} + \frac{8}{45} = \frac{18+8}{45} = \frac{26}{45}$$

$$a+5d = \frac{2}{5} + 5 \times \left(-\frac{7}{120}\right) = \frac{2}{5} + \frac{2}{9} = \frac{18+10}{45} = \frac{28}{45}$$

Thus, five rational numbers between $\frac{2}{5}$ and $\frac{2}{3}$ are

$$\frac{4}{9}, \frac{22}{45}, \frac{8}{15}, \frac{26}{45} \text{ and } \frac{28}{45}.$$

Answer 9A.

Given numbers: $\frac{6}{7}$, $\frac{9}{14}$ and $\frac{23}{28}$

The L.C.M. of 7, 14 and 28 is 28.

Thus, numbers are:

$$\frac{6}{7} = \frac{6 \times 4}{7 \times 4} = \frac{24}{28}; \quad \frac{9}{14} = \frac{9 \times 2}{14 \times 2} = \frac{18}{28} \text{ and } \frac{23}{28}.$$

Since $24 > 23 > 18$, we have $\frac{6}{7} > \frac{23}{28} > \frac{9}{14}$.

Hence, the greatest rational number is $\frac{6}{7}$ and

the smallest rational number is $\frac{9}{14}$.

Answer 9B.

Given numbers: $\frac{-2}{3}$, $\frac{-7}{9}$ and $\frac{-5}{6}$

The L.C.M. of 3, 9 and 6 is 18.

Thus, numbers are:

$$\frac{-2}{3} = \frac{-2 \times 6}{3 \times 6} = \frac{-12}{18}; \quad \frac{-7}{9} = \frac{-7 \times 2}{9 \times 2} = \frac{-14}{18}; \quad \frac{-5}{6} = \frac{-5 \times 3}{6 \times 3} = \frac{-15}{18}$$

Since $-12 > -14 > -15$, we have $\frac{-2}{3} > \frac{-7}{9} > \frac{-5}{6}$.

Hence, the greatest rational number is $\frac{-2}{3}$ and

the smallest rational number is $\frac{-5}{6}$.

Answer 10A.

Given numbers: $\frac{4}{5}$, $\frac{6}{7}$ and $\frac{7}{10}$

The L.C.M. of 5, 7 and 10 is 70.

Thus, numbers are:

$$\frac{4}{5} = \frac{4 \times 14}{5 \times 14} = \frac{56}{70}; \quad \frac{6}{7} = \frac{6 \times 10}{7 \times 10} = \frac{60}{70} \quad \text{and} \quad \frac{7}{10} = \frac{7 \times 7}{10 \times 7} = \frac{49}{70}.$$

Since $49 < 56 < 60$, we have $\frac{7}{10} < \frac{4}{5} < \frac{6}{7}$.

Answer 10B.

Given numbers: $\frac{-7}{12}$, $\frac{-3}{10}$ and $\frac{-2}{5}$

The L.C.M. of 12, 10 and 5 is 60.

Thus, numbers are:

$$\frac{-7}{12} = \frac{-7 \times 5}{12 \times 5} = \frac{-35}{60}; \quad \frac{-3}{10} = \frac{-3 \times 6}{10 \times 6} = \frac{-18}{60}; \quad \frac{-2}{5} = \frac{-2 \times 12}{5 \times 12} = \frac{-24}{60}$$

Since $-35 < -24 < -18$, we have $\frac{-7}{12} < \frac{-2}{5} < \frac{-3}{10}$.

Answer 10C.

Given numbers: $\frac{10}{9}$, $\frac{13}{12}$ and $\frac{19}{18}$

The L.C.M. of 9, 12 and 18 is 36.

Thus, numbers are:

$$\frac{10}{9} = \frac{10 \times 4}{9 \times 4} = \frac{40}{36}, \quad \frac{13}{12} = \frac{13 \times 3}{12 \times 3} = \frac{39}{36} \quad \text{and} \quad \frac{19}{18} = \frac{19 \times 2}{18 \times 2} = \frac{38}{36}.$$

Since $38 < 39 < 40$, we have $\frac{19}{18} < \frac{13}{12} < \frac{10}{9}$.

Answer 10D.

Given numbers: $\frac{7}{4}$, $\frac{-6}{5}$ and $\frac{-5}{2}$

The L.C.M. of 4, 5 and 2 is 20.

Thus, numbers are:

$$\frac{7}{4} = \frac{7 \times 5}{4 \times 5} = \frac{35}{20}, \quad \frac{-6}{5} = \frac{-6 \times 4}{5 \times 4} = \frac{-36}{20} \quad \text{and} \quad \frac{-5}{2} = \frac{-5 \times 10}{2 \times 10} = \frac{-50}{20}$$

Since $-50 < -36 < 35$, we have $\frac{-5}{2} < \frac{-6}{5} < \frac{7}{4}$.

Answer 11A.

Given numbers: $\frac{7}{13}$, $\frac{8}{15}$ and $\frac{3}{5}$

The L.C.M. of 13, 15 and 5 is 195.

Thus, numbers are:

$$\frac{7}{13} = \frac{7 \times 15}{13 \times 15} = \frac{105}{195}, \quad \frac{8}{15} = \frac{8 \times 13}{15 \times 13} = \frac{104}{195}, \quad \frac{3}{5} = \frac{3 \times 39}{5 \times 39} = \frac{117}{195}$$

Since $117 > 105 > 104$, we have $\frac{3}{5} > \frac{7}{13} > \frac{8}{15}$.

Answer 11B.

Given numbers: $\frac{4}{3}$, $\frac{-14}{5}$ and $\frac{17}{15}$

The L.C.M. of 3 and 5 is 15.

Thus, numbers are:

$$\frac{4}{3} = \frac{4 \times 5}{3 \times 5} = \frac{20}{15}, \quad \frac{-14}{5} = \frac{-14 \times 3}{5 \times 3} = \frac{-42}{15}, \quad \frac{17}{15}$$

Since $20 > 17 > -42$, we have $\frac{4}{3} > \frac{17}{15} > \frac{-14}{5}$.

Answer 11C.

Given numbers: $\frac{-7}{10}$, $\frac{-8}{15}$ and $\frac{-11}{30}$

The L.C.M. of 10, 15 and 30 is 30.

Thus, numbers are:

$$\frac{-7}{10} = \frac{-7 \times 3}{10 \times 3} = \frac{-21}{30}, \quad \frac{-8}{15} = \frac{-8 \times 2}{15 \times 2} = \frac{-16}{30}, \quad \frac{-11}{30}$$

Since $-11 > -16 > -21$, we have $\frac{-11}{30} > \frac{-8}{15} > \frac{-7}{10}$.

Answer 11D.

Given numbers: $\frac{-3}{8}$, $\frac{2}{5}$ and $\frac{-1}{3}$

The L.C.M. of 8, 5 and 3 is 120.

Thus, numbers are:

$$\frac{-3}{8} = \frac{-3 \times 15}{8 \times 15} = \frac{-45}{120}, \quad \frac{2}{5} = \frac{2 \times 24}{5 \times 24} = \frac{48}{120}, \quad \frac{-1}{3} = \frac{-1 \times 40}{3 \times 40} = \frac{-40}{120}$$

Since $48 > -40 > -45$, we have $\frac{2}{5} > \frac{-1}{3} > \frac{-3}{8}$.

Answer 12A.

$$\text{Let } x = 2.\overline{65} = 2.6555\dots$$

$$\Rightarrow 10x = 26.\overline{5} \quad \dots(i)$$

$$\Rightarrow 100x = 265.\overline{5} \quad \dots(ii)$$

Subtracting (i) from (ii),

$$90x = 239$$

$$\Rightarrow x = \frac{239}{90}$$

$$\text{Let } y = 1.\overline{25} \quad \dots(iii)$$

$$\Rightarrow 100y = 125.\overline{25} \quad \dots(iv)$$

Subtracting (iii) from (iv),

$$99y = 124$$

$$\Rightarrow y = \frac{124}{99}$$

$$\therefore 2.\overline{65} + 1.\overline{25} = x + y$$

$$\begin{aligned} &= \frac{239}{90} + \frac{124}{99} \\ &= \frac{239 \times 11 + 124 \times 10}{990} \\ &= \frac{2629 + 1240}{990} \\ &= \frac{3869}{990} \\ &= 3.\overline{908} \end{aligned}$$

Answer 12B.

$$\text{Let } x = 1.\overline{32} \dots (i)$$

$$\Rightarrow 100x = 132.\overline{32} \dots (ii)$$

Subtracting (i) from (ii),

$$99x = 131$$

$$\Rightarrow x = \frac{131}{99}$$

$$\text{Let } y = 0.9\overline{1}$$

$$\Rightarrow 10y = 9.\overline{1} \dots (iii)$$

$$\Rightarrow 100y = 91.\overline{1} \dots (iv)$$

Subtracting (iii) from (iv),

$$90y = 82$$

$$\Rightarrow y = \frac{82}{90} = \frac{41}{45}$$

$$\therefore 1.\overline{32} - 0.9\overline{1} = x - y$$

$$= \frac{131}{99} - \frac{41}{45}$$

$$= \frac{131 \times 5 - 41 \times 11}{495}$$

$$= \frac{655 - 451}{495}$$

$$= \frac{204}{495}$$

$$= 0.4\overline{12}$$

Answer 12C.

$$\text{Let } x = 2.\overline{12} \quad \dots(i)$$

$$\Rightarrow 100x = 212.\overline{12} \quad \dots(ii)$$

Subtracting (i) from (ii),

$$99x = 210$$

$$\Rightarrow x = \frac{210}{99} = \frac{70}{33}$$

$$\text{Let } y = 0.4\overline{5}$$

$$\Rightarrow 10y = 4.\overline{5} \quad \dots(iii)$$

$$\Rightarrow 100y = 45.\overline{5} \quad \dots(iv)$$

Subtracting (iii) from (iv),

$$90y = 41$$

$$\Rightarrow y = \frac{41}{90}$$

$$\therefore 2.\overline{12} - 0.4\overline{5} = x - y$$

$$\begin{aligned} &= \frac{70}{33} - \frac{41}{90} \\ &= \frac{70 \times 30 - 41 \times 11}{990} \\ &= \frac{2100 - 451}{990} \\ &= \frac{1649}{990} \\ &= 1.6\overline{65} \end{aligned}$$

Answer 12D.

$$\text{Let } x = 1.\overline{35}$$

$$\Rightarrow 10x = 13.\overline{5} \quad \dots(i)$$

$$\Rightarrow 100x = 135.\overline{5} \quad \dots(ii)$$

Subtracting (i) from (ii),

$$90x = 122$$

$$\Rightarrow x = \frac{122}{90} = \frac{61}{45}$$

$$\text{Let } y = 1.\overline{5} \quad \dots(iii)$$

$$\Rightarrow 10y = 15.\overline{5} \quad \dots(iv)$$

$$\Rightarrow 9y = 14$$

$$\Rightarrow y = \frac{14}{9}$$

$$\therefore 1.\overline{35} + 1.\overline{5} = x + y$$

$$= \frac{61}{45} + \frac{14}{9}$$

$$= \frac{61 \times 1 + 14 \times 5}{45}$$

$$= \frac{61 + 70}{45}$$

$$= \frac{131}{45}$$

$$= 2.\overline{91}$$

Ex 1.2**Answer 1A.**

$$\begin{aligned}(3 + \sqrt{3})^2 &= (3)^2 + (\sqrt{3})^2 + 2 \times 3 \times \sqrt{3} \\&= 9 + 3 + 6\sqrt{3} \\&= 12 + 6\sqrt{3}, \text{ which is irrational}\end{aligned}$$

Answer 1B.

$$\begin{aligned}(5 - \sqrt{5})^2 &= (5)^2 + (\sqrt{5})^2 - 2 \times 5 \times \sqrt{5} \\&= 25 + 5 - 10\sqrt{5} \\&= 30 - 10\sqrt{5}, \text{ which is irrational}\end{aligned}$$

Answer 1C.

$$\begin{aligned}(2 + \sqrt{2})(2 - \sqrt{2}) &= (2)^2 - (\sqrt{2})^2 \\&= 4 - 2 \\&= 2, \text{ which is rational}\end{aligned}$$

Answer 1D.

$$\left(\frac{\sqrt{5}}{3\sqrt{2}}\right)^2 = \frac{5}{9 \times 2} = \frac{5}{18}, \text{ which is rational}$$

Answer 2A.

$$(3\sqrt{2})^2 = 9 \times 2 = 18, \text{ which is rational}$$

Answer 2B.

$$\begin{aligned}
& (3 + \sqrt{2})^2 \\
&= (3)^2 + (\sqrt{2})^2 + 2 \times 3 \times \sqrt{2} \\
&= 9 + 2 + 6\sqrt{2} \\
&= 11 + 6\sqrt{2}, \text{ which is irrational}
\end{aligned}$$

Answer 2C.

$$\begin{aligned}
& \left(\frac{3\sqrt{2}}{2} \right)^2 \\
&= \frac{9 \times 2}{4} \\
&= \frac{9}{2}, \text{ which is rational}
\end{aligned}$$

Answer 2D.

$$\begin{aligned}
& (\sqrt{2} + \sqrt{3})^2 \\
&= (\sqrt{2})^2 + (\sqrt{3})^2 + 2 \times \sqrt{2} \times \sqrt{3} \\
&= 2 + 3 + 2\sqrt{6} \\
&= 5 + 2\sqrt{6}, \text{ which is irrational}
\end{aligned}$$

Answer 3.

2.23606...	
2	5.0000000000...
	-4
42	100
	- 84
443	1600
	- 1329
4466	27 100
	- 26796
447206	3040000
	- 2683236
	356764 ...

Clearly, $\sqrt{5} = 2.23606\dots$; which is an irrational number.
Hence, $\sqrt{5}$ is an irrational number.

Answer 4.

Let $\sqrt{7}$ be a rational number.

$$\therefore \sqrt{7} = \frac{a}{b}$$

$$\Rightarrow 7 = \frac{a^2}{b^2}$$

$$\Rightarrow a^2 = 7b^2$$

Since a^2 is divisible by 7, a is also divisible by 7.(I)

Let $a = 7c$

$$\Rightarrow a^2 = 49c^2$$

$$\Rightarrow 7b^2 = 49c^2$$

$$\Rightarrow b^2 = 7c^2$$

Since b^2 is divisible by 7, b is also divisible by 7.(II)

From (I) and (II), we get a and b both divisible by 7.

i.e., a and b have a common factor 7.

This contradicts our assumption that $\frac{a}{b}$ is rational.

i.e. a and b do not have any common factor other than unity (1).

$$\Rightarrow \frac{a}{b} \text{ is not rational}$$

$$\Rightarrow \sqrt{7} \text{ is not rational, i.e. } \sqrt{7} \text{ is irrational.}$$

Answer 5A.

$(\sqrt{3} + 5)$ and $(\sqrt{5} - 3)$ are irrational numbers whose sum is irrational.

Thus, we have

$$(\sqrt{3} + 5) + (\sqrt{5} - 3)$$

$$= \sqrt{3} + 5 + \sqrt{5} - 3$$

$$= \sqrt{3} + \sqrt{5} + 2, \text{ which is irrational.}$$

Answer 5B.

$(\sqrt{3} + 5)$ and $(4 - \sqrt{3})$ are two irrational numbers whose sum is rational.

Thus, we have

$$(\sqrt{3} + 5) + (4 - \sqrt{3})$$

$$= \sqrt{3} + 5 + 4 - \sqrt{3}$$

$$= 9, \text{ which is a rational number.}$$

Answer 5C.

$(\sqrt{3} + 2)$ and $(\sqrt{2} - 3)$ are irrational numbers whose difference is irrational.

Thus, we have

$$\begin{aligned} & (\sqrt{3} + 2) - (\sqrt{2} - 3) \\ &= \sqrt{3} + 2 - \sqrt{2} + 3 \\ &= \sqrt{3} - \sqrt{2} + 5, \text{ which is irrational.} \end{aligned}$$

Answer 5D.

$(\sqrt{5} - 3)$ and $(\sqrt{5} + 3)$ are irrational numbers whose difference is rational.

Thus, we have

$$\begin{aligned} & (\sqrt{5} - 3) - (\sqrt{5} + 3) \\ &= \sqrt{5} - 3 - \sqrt{5} - 3 \\ &= -6, \text{ which is a rational number.} \end{aligned}$$

Answer 5E.

Consider two irrational numbers $(5 + \sqrt{2})$ and $(\sqrt{5} - 2)$.

Thus, we have

$$\begin{aligned} & (5 + \sqrt{2})(\sqrt{5} - 2) \\ &= 5(\sqrt{5} - 2) + \sqrt{2}(\sqrt{5} - 2) \\ &= 5\sqrt{5} - 10 + \sqrt{10} - 2\sqrt{2}, \text{ which is irrational} \end{aligned}$$

Answer 5F.

$(\sqrt{3} + \sqrt{2})$ and $(\sqrt{3} - \sqrt{2})$ are irrational numbers whose product is rational.

Thus, we have

$$(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1, \text{ which is a rational number.}$$

Answer 6A.

$$\sqrt[4]{12} = 12^{\frac{1}{4}} \text{ has power } \frac{1}{4}$$

$$\sqrt[3]{15} = 15^{\frac{1}{3}} \text{ has power } \frac{1}{3}$$

Now, L.C.M. of 4 and 3 = 12

$$\sqrt[4]{12} = 12^{\frac{1}{4}} = 12^{\frac{3}{12}} = (12^3)^{\frac{1}{12}} = (1728)^{\frac{1}{12}}$$

$$\sqrt[3]{15} = 15^{\frac{1}{3}} = 15^{\frac{4}{12}} = (15^4)^{\frac{1}{12}} = (50625)^{\frac{1}{12}}$$

Since $1728 < 50625$, we have $(1728)^{\frac{1}{12}} < (50625)^{\frac{1}{12}}$.

Hence, $\sqrt[4]{12} < \sqrt[3]{15}$.

Answer 6B.

$$\sqrt[3]{48} = 48^{\frac{1}{3}} \text{ has power } \frac{1}{3}$$

$$\sqrt{36} = 6$$

Now, L.C.M. of 3 and 1 = 3

$$\sqrt[3]{48} = 48^{\frac{1}{3}}$$

$$\sqrt{36} = 6 = 6^{\frac{3}{3}} = (6^3)^{\frac{1}{3}} = 216^{\frac{1}{3}}$$

Since $48 < 216$, we have $48^{\frac{1}{3}} < 216^{\frac{1}{3}}$.

Hence, $\sqrt[3]{48} < \sqrt{36}$.

Answer 7A.

$$2\sqrt{5} = \sqrt{2^2 \times 5} = \sqrt{4 \times 5} = \sqrt{20}$$

$$\sqrt{3} = \sqrt{3}$$

$$5\sqrt{2} = \sqrt{5^2 \times 2} = \sqrt{25 \times 2} = \sqrt{50}$$

Since, $3 < 20 < 50$, we have $\sqrt{3} < \sqrt{20} < \sqrt{50}$.

Hence, $\sqrt{3} < 2\sqrt{5} < 5\sqrt{2}$.

Answer 7B.

$$\text{Since } 2\sqrt[3]{3} = \sqrt[3]{2^3 \times 3} = \sqrt[3]{8 \times 3} = \sqrt[3]{24}$$

$$4\sqrt[3]{3} = \sqrt[3]{4^3 \times 3} = \sqrt[3]{64 \times 3} = \sqrt[3]{192}$$

$$3\sqrt[3]{3} = \sqrt[3]{3^3 \times 3} = \sqrt[3]{27 \times 3} = \sqrt[3]{81}$$

$$\text{Since, } 24 < 81 < 192, \text{ we have } \sqrt[3]{24} < \sqrt[3]{81} < \sqrt[3]{192}.$$

$$\text{Hence, } 2\sqrt[3]{3} < 3\sqrt[3]{3} < 4\sqrt[3]{3}.$$

Answer 7C.

$$5\sqrt{7} = \sqrt{5^2 \times 7} = \sqrt{25 \times 7} = \sqrt{175}$$

$$7\sqrt{5} = \sqrt{7^2 \times 5} = \sqrt{49 \times 5} = \sqrt{245}$$

$$6\sqrt{2} = \sqrt{6^2 \times 2} = \sqrt{36 \times 2} = \sqrt{72}$$

$$\text{Since, } 72 < 175 < 245, \text{ we have } \sqrt{72} < \sqrt{175} < \sqrt{245}.$$

$$\text{Hence, } 6\sqrt{2} < 5\sqrt{7} < 7\sqrt{5}.$$

Answer 7D.

$$\text{Since } 7\sqrt[3]{5} = \sqrt[3]{7^3 \times 5} = \sqrt[3]{343 \times 5} = \sqrt[3]{1715}$$

$$6\sqrt[3]{4} = \sqrt[3]{6^3 \times 4} = \sqrt[3]{216 \times 4} = \sqrt[3]{864}$$

$$5\sqrt[3]{6} = \sqrt[3]{5^3 \times 6} = \sqrt[3]{125 \times 6} = \sqrt[3]{750}$$

$$\text{Since, } 750 < 864 < 1715, \text{ we have } \sqrt[3]{750} < \sqrt[3]{864} < \sqrt[3]{1715}.$$

$$\text{Hence, } 5\sqrt[3]{6} < 6\sqrt[3]{4} < 7\sqrt[3]{5}.$$

Answer 8A.

Since $\sqrt{2} = 2^{\frac{1}{2}}$ has power $\frac{1}{2}$,

$\sqrt[3]{5} = 5^{\frac{1}{3}}$ has power $\frac{1}{3}$

$\sqrt[4]{10} = 10^{\frac{1}{4}}$ has power $\frac{1}{4}$

Now, L.C.M. of 2, 3 and 4 = 12

$$\therefore \sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{6}{12}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$$

$$\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{4}{12}} = (5^4)^{\frac{1}{12}} = (625)^{\frac{1}{12}}$$

$$\sqrt[4]{10} = 10^{\frac{1}{4}} = 10^{\frac{3}{12}} = (10^3)^{\frac{1}{12}} = (1000)^{\frac{1}{12}}$$

Since, $1000 > 625 > 64$, we have $(1000)^{\frac{1}{12}} > (625)^{\frac{1}{12}} > (64)^{\frac{1}{12}}$.

Hence, $\sqrt[4]{10} > \sqrt[3]{5} > \sqrt{2}$.

Answer 8B.

Since $5\sqrt{3} = \sqrt{5^2 \times 3} = \sqrt{25 \times 3} = \sqrt{75}$

$\sqrt{15} = \sqrt{15}$

$3\sqrt{5} = \sqrt{3^2 \times 5} = \sqrt{9 \times 5} = \sqrt{45}$

Since, $75 > 45 > 15$, we have $\sqrt{75} > \sqrt{45} > \sqrt{15}$.

Hence, $5\sqrt{3} > 3\sqrt{5} > \sqrt{15}$.

Answer 8C.

Since $\sqrt{6} = 6^{\frac{1}{2}}$ has power $\frac{1}{2}$,

$\sqrt[3]{8} = 2$

$\sqrt[4]{3} = 3^{\frac{1}{4}}$ has power $\frac{1}{4}$

Now, L.C.M. of 2, 1 and 4 = 4

$$\therefore \sqrt{6} = 6^{\frac{1}{2}} = 6^{\frac{2}{4}} = (6^2)^{\frac{1}{4}} = (36)^{\frac{1}{4}}$$

$$\sqrt[3]{8} = 2 = 2^{\frac{4}{4}} = (2^4)^{\frac{1}{4}} = (16)^{\frac{1}{4}}$$

$$\sqrt[4]{3} = 3^{\frac{1}{4}} = (3^1)^{\frac{1}{4}} = (3)^{\frac{1}{4}}$$

Since, $36 > 16 > 3$, we have $(36)^{\frac{1}{4}} > (16)^{\frac{1}{4}} > (3)^{\frac{1}{4}}$.

Hence, $\sqrt{6} > \sqrt[3]{8} > \sqrt[4]{3}$.

Answer 9.

Since 3 and 4 are rational numbers and $3 \times 4 = 12$ is not a perfect square.

$\therefore$ One irrational number between 3 and 4 = $\sqrt{3 \times 4} = \sqrt{12}$

And, an irrational number between 3 and $\sqrt{12} = \sqrt{3 \times \sqrt{12}} = \sqrt{3\sqrt{12}}$

$\therefore$ Required irrational numbers between 3 and 4 are: $\sqrt{12}$ and $\sqrt{3\sqrt{12}}$

Answer 10.

We know that $2\sqrt{3} = \sqrt{4 \times 3} = \sqrt{12}$ and $3\sqrt{5} = \sqrt{9 \times 5} = \sqrt{45}$.

Thus, we have $\sqrt{12} < \sqrt{13} < \sqrt{14} < \sqrt{17} < \dots < \sqrt{43} < \sqrt{44} < \sqrt{45}$

So, any five irrational numbers between $2\sqrt{3}$ and $3\sqrt{5}$ are:

$\sqrt{13}, \sqrt{14}, \sqrt{23}, \sqrt{37}, \sqrt{41}$

Answer 11.

Since squares of $\sqrt{3}$ and $\sqrt{7}$ are 3 and 7 respectively.

Now, find two rational numbers between 3 and 7 such that each of them is a perfect square.

Let the numbers be 4 and 5.76,

where,

$$\sqrt{4} = 2$$

$$\sqrt{5.76} = 2.4$$

Hence, required rational numbers between $\sqrt{3}$ and $\sqrt{7}$ are 2 and 2.4.

Answer 12.

Since squares of $\sqrt{2}$ and $\sqrt{3}$ are 2 and 3 respectively.

Now, find four rational numbers between 2 and 3 such that each of them is a perfect square.

Let the numbers be 2.25, 2.4025, 2.56, 2.89,

where,

$$\sqrt{2.25} = 1.5$$

$$\sqrt{2.4025} = 1.55$$

$$\sqrt{2.56} = 1.6$$

$$\sqrt{2.89} = 1.7$$

Hence, required rational numbers between $\sqrt{2}$ and $\sqrt{3}$ are 1.5, 1.55, 1.6 and 1.7.

Answer 13A.

$$\sqrt{150} = \sqrt{25 \times 6} = 5\sqrt{6}, \text{ which is an irrational number.}$$

Hence, $\sqrt{150}$ is a surd.

Answer 13B.

$\sqrt[3]{4}$ is an irrational number.

Hence, $\sqrt[3]{4}$ is a surd.

Answer 13C.

$$\sqrt[3]{50} \cdot \sqrt[3]{20} = \sqrt[3]{50 \times 20} = \sqrt[3]{1000} = 10, \text{ which is a rational number.}$$

Hence, $\sqrt[3]{50} \cdot \sqrt[3]{20}$ is not a surd.

Answer 13D.

$$\sqrt[3]{-27} = -3, \text{ which is a rational number.}$$

Hence, $\sqrt[3]{-27}$ is not a surd.

Answer 13E.

$\sqrt{2 + \sqrt{3}}$ is an irrational number.

Hence, $\sqrt{2 + \sqrt{3}}$ is a surd

Answer 13F.

$$\sqrt[12]{8} \div \sqrt[6]{6}$$

$$= \frac{\sqrt[12]{8}}{\sqrt[6]{6}}$$

Numerator and Denominator, both are irrational numbers.

Hence, $\sqrt[12]{8} \div \sqrt[6]{6}$ is a surd.

Answer 14.

Let us find $\sqrt{5}$.

Draw a number line.

Mark a point O representing zero.

Take point A on numberline such that $OA = 2$

Construct $AB \perp OA$ such that $AB = 1$ unit.

$\therefore \triangle OAB$ is a right triangle.

In $\triangle OAB$, $(OB)^2 = (OA)^2 + (AB)^2$ (Pythagoras' Theorem)

$$\therefore (OB)^2 = 2^2 + 1^2$$

$$\therefore (OB)^2 = 5 \Rightarrow OB = \sqrt{5}$$

Now, let us find $\sqrt{6}$.

Construct $BC \perp OB$, such that $BC = 1$ unit.

$\therefore \triangle OBC$ is a right triangle.

In $\triangle OBC$, $OC^2 = OB^2 + BC^2$ (Pythagoras' Theorem)

$$\therefore OC^2 = (\sqrt{5})^2 + 1^2$$

$$\therefore OC^2 = 6 \Rightarrow OC = \sqrt{6}$$

Now, let us find $\sqrt{7}$.

Construct $CD \perp OC$, such that $CD = 1$ unit.

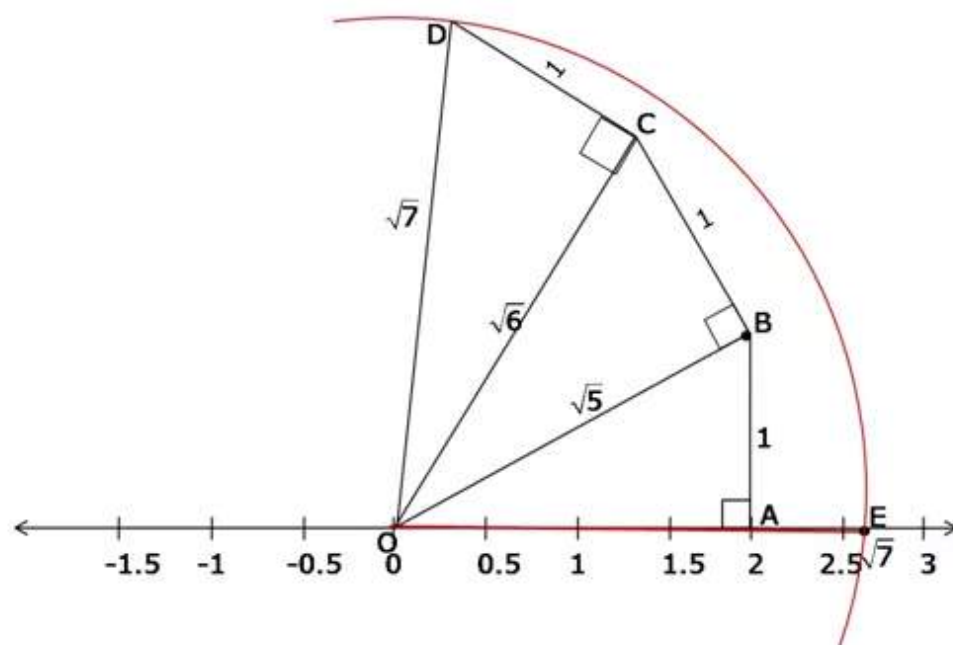
In $\triangle OCD$, $OD^2 = OC^2 + CD^2$ (Pythagoras' Theorem)

$$\therefore OD^2 = (\sqrt{6})^2 + 1^2$$

$$\therefore OD^2 = 7 \Rightarrow \sqrt{7}$$

Draw an arc of radius OD and centre O and let it intersect the number line at point E.

$\therefore \sqrt{7}$ is thus marked at point E on the number line.



Ex 1.3**Answer 1A.**

$$\begin{aligned}\frac{3\sqrt{2}}{\sqrt{5}} &= \frac{3\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{3\sqrt{2} \times \sqrt{5}}{(\sqrt{5})^2} \\ &= \frac{3\sqrt{10}}{5}\end{aligned}$$

Answer 1B.

$$\begin{aligned}\frac{1}{5+\sqrt{2}} &= \frac{1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} \\ &= \frac{5-\sqrt{2}}{(5)^2 - (\sqrt{2})^2} \\ &= \frac{5-\sqrt{2}}{25-2} \\ &= \frac{5-\sqrt{2}}{23}\end{aligned}$$

Answer 1C.

$$\begin{aligned}\frac{1}{\sqrt{3}+\sqrt{2}} &= \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \\ &= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{\sqrt{3}-\sqrt{2}}{3-2} \\ &= \frac{\sqrt{3}-\sqrt{2}}{1} \\ &= \sqrt{3}-\sqrt{2}\end{aligned}$$

Answer 1D.

$$\begin{aligned}& \frac{2}{3+\sqrt{7}} \\&= \frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} \\&= \frac{2(3-\sqrt{7})}{(3)^2 - (\sqrt{7})^2} \\&= \frac{2(3-\sqrt{7})}{9-7} \\&= \frac{2(3-\sqrt{7})}{2} \\&= 3-\sqrt{7}\end{aligned}$$

Answer 1E.

$$\begin{aligned}& \frac{5}{\sqrt{7}-\sqrt{2}} \\&= \frac{5}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}} \\&= \frac{5(\sqrt{7}+\sqrt{2})}{(\sqrt{7})^2 - (\sqrt{2})^2} \\&= \frac{5(\sqrt{7}+\sqrt{2})}{7-2} \\&= \frac{5(\sqrt{7}+\sqrt{2})}{5} \\&= \sqrt{7}+\sqrt{2}\end{aligned}$$

Answer 1F.

$$\begin{aligned}& \frac{42}{2\sqrt{3} + 3\sqrt{2}} \\&= \frac{42}{2\sqrt{3} + 3\sqrt{2}} \times \frac{2\sqrt{3} - 3\sqrt{2}}{2\sqrt{3} - 3\sqrt{2}} \\&= \frac{42(2\sqrt{3} - 3\sqrt{2})}{(2\sqrt{3})^2 - (3\sqrt{2})^2} \\&= \frac{84\sqrt{3} - 126\sqrt{2}}{12 - 18} \\&= \frac{84\sqrt{3} - 126\sqrt{2}}{-6} \\&= -14\sqrt{3} + 21\sqrt{2} \\&= 21\sqrt{2} - 14\sqrt{3} \\&= 7(3\sqrt{2} - 2\sqrt{3})\end{aligned}$$

Answer 1G.

$$\begin{aligned}& \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\&= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\&= \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - (1)^2} \\&= \frac{(\sqrt{3})^2 + 2 \times \sqrt{3} \times 1 + (1)^2}{3 - 1} \\&= \frac{3 + 2\sqrt{3} + 1}{2} \\&= \frac{4 + 2\sqrt{3}}{2} \\&= 2 + \sqrt{3}\end{aligned}$$

Answer 1H.

$$\begin{aligned} & \frac{\sqrt{5} - \sqrt{7}}{\sqrt{3}} \\ &= \frac{\sqrt{5} - \sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{5} \times \sqrt{3} - \sqrt{7} \times \sqrt{3}}{(\sqrt{3})^2} \\ &= \frac{\sqrt{15} - \sqrt{21}}{3} \end{aligned}$$

Answer 1I.

$$\begin{aligned} & \frac{3 - \sqrt{3}}{2 + \sqrt{2}} \\ &= \frac{3 - \sqrt{3}}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}} \\ &= \frac{3(2 - \sqrt{2}) - \sqrt{3}(2 - \sqrt{2})}{(2)^2 - (\sqrt{2})^2} \\ &= \frac{6 - 3\sqrt{2} - 2\sqrt{3} + \sqrt{6}}{4 - 2} \\ &= \frac{6 - 3\sqrt{2} - 2\sqrt{3} + \sqrt{6}}{2} \end{aligned}$$

Answer 2.

$$(i) \frac{5 + \sqrt{6}}{5 - \sqrt{6}}$$

$$\frac{5 + \sqrt{6}}{5 - \sqrt{6}}$$

$$\begin{aligned} &= \frac{5 + \sqrt{6}}{5 - \sqrt{6}} \times \frac{5 + \sqrt{6}}{5 + \sqrt{6}} \\ &= \frac{(5 + \sqrt{6})^2}{(5)^2 - (\sqrt{6})^2} = \frac{25 + 6 + 10\sqrt{6}}{25 - 6} \\ &= \frac{31 + 10\sqrt{6}}{19} \end{aligned}$$

$$(ii) \frac{4 + \sqrt{8}}{4 - \sqrt{8}}$$

$$\frac{4 + \sqrt{8}}{4 - \sqrt{8}}$$

$$= \frac{4 + \sqrt{8}}{4 - \sqrt{8}} \times \frac{4 + \sqrt{8}}{4 + \sqrt{8}}$$

$$= \frac{(4 + \sqrt{8})^2}{(4)^2 - (\sqrt{8})^2} = \frac{16 + 8 + 8\sqrt{8}}{16 - 8}$$

$$= \frac{24 + 8\sqrt{8}}{8} = 3 + \sqrt{8}$$

$$(iii) \frac{\sqrt{15} + 3}{\sqrt{15} - 3}$$

$$\frac{\sqrt{15} + 3}{\sqrt{15} - 3}$$

$$= \frac{\sqrt{15} + 3}{\sqrt{15} - 3} \times \frac{\sqrt{15} + 3}{\sqrt{15} + 3}$$

$$= \frac{(\sqrt{15} + 3)^2}{(\sqrt{15})^2 - (3)^2} = \frac{15 + 9 + 6\sqrt{15}}{15 - 9}$$

$$= \frac{24 + 6\sqrt{15}}{6} = 4 + \sqrt{15}$$

$$(iv) \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}}$$

$$\frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}}$$

$$= \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}}$$

$$= \frac{(\sqrt{7} - \sqrt{5})^2}{(\sqrt{7})^2 - (\sqrt{5})^2} = \frac{7 + 5 - 2\sqrt{35}}{7 - 5} = \frac{12 - 2\sqrt{35}}{2}$$

$$= 6 - \sqrt{35}$$

$$(v) \frac{3\sqrt{5} + \sqrt{7}}{3\sqrt{5} - \sqrt{7}}$$

$$\frac{3\sqrt{5} + \sqrt{7}}{3\sqrt{5} - \sqrt{7}}$$

$$= \frac{3\sqrt{5} + \sqrt{7}}{3\sqrt{5} - \sqrt{7}} \times \frac{3\sqrt{5} + \sqrt{7}}{3\sqrt{5} + \sqrt{7}}$$

$$= \frac{(3\sqrt{5} + \sqrt{7})^2}{(3\sqrt{5})^2 - (\sqrt{7})^2} = \frac{45 + 7 + 6\sqrt{35}}{45 - 7}$$

$$= \frac{52 + 6\sqrt{35}}{38} = \frac{26 + 3\sqrt{35}}{19}$$

$$(vi) \frac{2\sqrt{3} - \sqrt{6}}{2\sqrt{3} + \sqrt{6}}$$

$$\frac{2\sqrt{3} - \sqrt{6}}{2\sqrt{3} + \sqrt{6}}$$

$$= \frac{2\sqrt{3} - \sqrt{6}}{2\sqrt{3} + \sqrt{6}} \times \frac{2\sqrt{3} - \sqrt{6}}{2\sqrt{3} - \sqrt{6}}$$

$$= \frac{(2\sqrt{3} - \sqrt{6})^2}{(2\sqrt{3})^2 - (\sqrt{6})^2} = \frac{12 + 6 - 4\sqrt{18}}{12 - 6}$$

$$= \frac{18 - 4\sqrt{18}}{6} = \frac{9 - 2\sqrt{18}}{3} = \frac{9 - 6\sqrt{2}}{3} = 3 - 2\sqrt{2}$$

$$(vii) \frac{5\sqrt{3} - \sqrt{15}}{5\sqrt{3} + \sqrt{15}}$$

$$\frac{5\sqrt{3} - \sqrt{15}}{5\sqrt{3} + \sqrt{15}}$$

$$= \frac{5\sqrt{3} - \sqrt{15}}{5\sqrt{3} + \sqrt{15}} \times \frac{5\sqrt{3} - \sqrt{15}}{5\sqrt{3} - \sqrt{15}}$$

$$= \frac{(5\sqrt{3} - \sqrt{15})^2}{(5\sqrt{3})^2 - (\sqrt{15})^2} = \frac{75 + 15 - 10\sqrt{45}}{75 - 15}$$

$$= \frac{90 - 10\sqrt{45}}{60} = \frac{9 - \sqrt{45}}{6} = \frac{9 - 3\sqrt{5}}{6}$$

$$= \frac{3 - \sqrt{5}}{2}$$

$$(viii) \frac{2\sqrt{6} - \sqrt{5}}{3\sqrt{5} - 2\sqrt{6}}$$

$$\frac{2\sqrt{6} - \sqrt{5}}{3\sqrt{5} - 2\sqrt{6}}$$

$$= \frac{2\sqrt{6} - \sqrt{5}}{3\sqrt{5} - 2\sqrt{6}} \times \frac{3\sqrt{5} + 2\sqrt{6}}{3\sqrt{5} + 2\sqrt{6}}$$

$$= \frac{6\sqrt{30} + 24 - 15 - 2\sqrt{30}}{(3\sqrt{5})^2 - (2\sqrt{6})^2}$$

$$= \frac{6\sqrt{30} + 9 - 2\sqrt{30}}{45 - 24} = \frac{4\sqrt{30} + 9}{21}$$

$$(ix) \frac{7\sqrt{3} - 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$$

$$\frac{7\sqrt{3} - 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$$

$$= \frac{7\sqrt{3} - 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} \times \frac{\sqrt{48} - \sqrt{18}}{\sqrt{48} - \sqrt{18}}$$

$$= \frac{7\sqrt{144} - 7\sqrt{54} - 5\sqrt{96} + 5\sqrt{36}}{(\sqrt{48})^2 - (\sqrt{18})^2}$$

$$= \frac{84 - 21\sqrt{6} - 20\sqrt{6} + 30}{48 - 18}$$

$$= \frac{114 - 41\sqrt{6}}{30}$$

$$(x) \frac{\sqrt{12} + \sqrt{18}}{\sqrt{75} - \sqrt{50}}$$

$$\frac{\sqrt{12} + \sqrt{18}}{\sqrt{75} - \sqrt{50}}$$

$$= \frac{\sqrt{12} + \sqrt{18}}{\sqrt{75} - \sqrt{50}} \times \frac{\sqrt{75} + \sqrt{50}}{\sqrt{75} + \sqrt{50}}$$

$$= \frac{(2\sqrt{3} + 3\sqrt{2})(5\sqrt{3} + 5\sqrt{2})}{(\sqrt{75})^2 - (\sqrt{50})^2}$$

$$= \frac{30 + 10\sqrt{6} + 15\sqrt{6} + 30}{75 - 50}$$

$$= \frac{60 + 25\sqrt{6}}{25} = \frac{12 + 5\sqrt{6}}{5}$$

Answer 3.

$$(i) \frac{3}{5-\sqrt{3}} + \frac{2}{5+\sqrt{3}}$$

$$\frac{3}{5-\sqrt{3}} + \frac{2}{5+\sqrt{3}}$$

$$= \frac{3(5+\sqrt{3})+2(5-\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})}$$

$$= \frac{15+3\sqrt{3}+10-2\sqrt{3}}{(5)^2-(\sqrt{3})^2}$$

$$= \frac{25+\sqrt{3}}{25-3} = \frac{25+\sqrt{3}}{22}$$

$$(ii) \frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$= \frac{(4+\sqrt{5})^2 + (4-\sqrt{5})^2}{(4-\sqrt{5})(4+\sqrt{5})}$$

$$= \frac{16+5+8\sqrt{5}+16+5-8\sqrt{5}}{16-5}$$

$$= \frac{42}{11}$$

$$(iii) \frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$$

$$\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$$

$$= \frac{(\sqrt{5}-2)^2 - (\sqrt{5}+2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)}$$

$$= \frac{5+4-4\sqrt{5}-5-4-4\sqrt{5}}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{-8\sqrt{5}}{5-4} = -8\sqrt{5}$$

$$\begin{aligned}
 \text{(iv)} \quad & \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}} - \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} \\
 & \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}} - \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} \\
 & = \frac{(\sqrt{7}-\sqrt{3})^2 - (\sqrt{7}+\sqrt{3})^2}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} \\
 & = \frac{7+3-2\sqrt{21}-7-3-2\sqrt{21}}{(\sqrt{7})^2 - (\sqrt{3})^2} \\
 & = \frac{-4\sqrt{21}}{7-3} = -\sqrt{21}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\
 & \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\
 & = \frac{(\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}-\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\
 & = \frac{5+3+\sqrt{15}+5+3-\sqrt{15}}{5-3} \\
 & = \frac{16}{2} = 8
 \end{aligned}$$

Answer 4.

$$\begin{aligned}
 \text{(i)} \quad & \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \\
 & \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}
 \end{aligned}$$

Rationalizing the denominator of each term, we have

$$\begin{aligned}
 & = \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} + \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} \\
 & = \frac{\sqrt{12}-\sqrt{18}}{2-3} + \frac{3\sqrt{12}-3\sqrt{6}}{6-3} - \frac{4\sqrt{18}-4\sqrt{6}}{6-2} \\
 & = \frac{\sqrt{12}-\sqrt{18}}{-1} + \frac{3\sqrt{12}-3\sqrt{6}}{3} - \frac{4\sqrt{18}-4\sqrt{6}}{4} \\
 & = \sqrt{18}-\sqrt{12} + \sqrt{12}-\sqrt{6} - \sqrt{18} + \sqrt{6} \\
 & = 0
 \end{aligned}$$

$$(ii) \frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2}$$

$$\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2}$$

Rationalizing the denominator of each term, we have

$$\begin{aligned} &= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} + \frac{2\sqrt{3}(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)} \\ &= \frac{3\sqrt{12}+3\sqrt{6}}{6-3} - \frac{4\sqrt{18}+4\sqrt{6}}{6-2} + \frac{2\sqrt{18}-4\sqrt{3}}{6-4} \\ &= \frac{3\sqrt{12}+3\sqrt{6}}{3} - \frac{4\sqrt{18}+4\sqrt{6}}{4} + \frac{2\sqrt{18}-4\sqrt{3}}{2} \\ &= \sqrt{12}+\sqrt{6}-\sqrt{18}-\sqrt{6}+\sqrt{18}-2\sqrt{3} \\ &= \sqrt{12}-2\sqrt{3} \\ &= 2\sqrt{3}-2\sqrt{3} \\ &= 0 \end{aligned}$$

$$(iii) \frac{6}{2\sqrt{3}-\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$$

$$\frac{6}{2\sqrt{3}-\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$$

Rationalizing the denominator of each term, we have

$$\begin{aligned} &= \frac{6(2\sqrt{3}+\sqrt{6})}{(2\sqrt{3}-\sqrt{6})(2\sqrt{3}+\sqrt{6})} + \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} \\ &= \frac{12\sqrt{3}+6\sqrt{6}}{12-6} + \frac{\sqrt{18}-\sqrt{12}}{3-2} - \frac{4\sqrt{18}+4\sqrt{6}}{6-2} \\ &= \frac{12\sqrt{3}+6\sqrt{6}}{6} + \frac{\sqrt{18}-\sqrt{12}}{1} - \frac{4\sqrt{18}+4\sqrt{6}}{4} \\ &= 2\sqrt{3}+\sqrt{6}+\sqrt{18}-\sqrt{12}-\sqrt{18}-\sqrt{6} \\ &= 2\sqrt{3}-\sqrt{12}=2\sqrt{3}-2\sqrt{3} \\ &= 0 \end{aligned}$$

$$(iv) \frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$$

$$\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$$

Rationalizing the denominator of each term, we have

$$\begin{aligned} &= \frac{7\sqrt{3}(\sqrt{10} - \sqrt{3})}{(\sqrt{10} + \sqrt{3})(\sqrt{10} - \sqrt{3})} - \frac{2\sqrt{5}(\sqrt{6} - \sqrt{5})}{(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{5})} - \frac{3\sqrt{2}(\sqrt{15} - 3\sqrt{2})}{(\sqrt{15} + 3\sqrt{2})(\sqrt{15} - 3\sqrt{2})} \\ &= \frac{7\sqrt{30} - 21}{10 - 3} - \frac{2\sqrt{30} - 10}{6 - 5} - \frac{3\sqrt{30} - 18}{15 - 18} \\ &= \frac{7\sqrt{30} - 21}{7} - \frac{2\sqrt{30} - 10}{1} - \frac{3\sqrt{30} - 18}{-3} \\ &= \frac{7\sqrt{30} - 21}{7} - \frac{2\sqrt{30} - 10}{1} + \frac{3\sqrt{30} - 18}{3} \\ &= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6 \\ &= -1 \end{aligned}$$

$$(v) \frac{4\sqrt{3}}{(2 - \sqrt{2})} - \frac{30}{(4\sqrt{3} - 3\sqrt{2})} - \frac{3\sqrt{2}}{(3 + 2\sqrt{3})}$$

$$\frac{4\sqrt{3}}{(2 - \sqrt{2})} - \frac{30}{(4\sqrt{3} - 3\sqrt{2})} - \frac{3\sqrt{2}}{(3 + 2\sqrt{3})}$$

Rationalizing the denominator of each term, we have

$$\begin{aligned} &= \frac{4\sqrt{3}(2 + \sqrt{2})}{(2 - \sqrt{2})(2 + \sqrt{2})} - \frac{30(4\sqrt{3} + 3\sqrt{2})}{(4\sqrt{3} - 3\sqrt{2})(4\sqrt{3} + 3\sqrt{2})} - \frac{3\sqrt{2}(3 - 2\sqrt{3})}{(3 + 2\sqrt{3})(3 - 2\sqrt{3})} \\ &= \frac{8\sqrt{3} + 4\sqrt{6}}{4 - 2} - \frac{120\sqrt{3} + 90\sqrt{2}}{48 - 18} - \frac{9\sqrt{2} - 6\sqrt{6}}{9 - 12} \\ &= \frac{8\sqrt{3} + 4\sqrt{6}}{2} - \frac{120\sqrt{3} + 90\sqrt{2}}{30} - \frac{9\sqrt{2} - 6\sqrt{6}}{-3} \\ &= \frac{8\sqrt{3} + 4\sqrt{6}}{2} - \frac{120\sqrt{3} + 90\sqrt{2}}{30} + \frac{9\sqrt{2} - 6\sqrt{6}}{3} \\ &= 4\sqrt{3} + 2\sqrt{6} - 4\sqrt{3} - 3\sqrt{2} + 3\sqrt{2} - 2\sqrt{6} \\ &= 0 \end{aligned}$$

Answer 5.

$$\begin{aligned}& \frac{\sqrt{2.5} - \sqrt{0.75}}{\sqrt{2.5} + \sqrt{0.75}} \\&= \frac{\sqrt{2.5} - \sqrt{0.75}}{\sqrt{2.5} + \sqrt{0.75}} \times \frac{\sqrt{2.5} - \sqrt{0.75}}{\sqrt{2.5} - \sqrt{0.75}} \\&= \frac{(\sqrt{2.5} - \sqrt{0.75})^2}{(\sqrt{2.5})^2 - (\sqrt{0.75})^2} \\&= \frac{2.5 - 2 \times \sqrt{2.5} \times \sqrt{0.75} + 0.75}{2.5 - 0.75} \\&= \frac{3.25 - 2 \times \sqrt{0.25 \times 10} \times \sqrt{0.25 \times 3}}{1.75} \\&= \frac{3.25 - 2 \times 0.25\sqrt{30}}{1.75} \\&= \frac{3.25 - 0.5\sqrt{30}}{1.75} \\&= \frac{3.25}{1.75} - \frac{0.5}{1.75}\sqrt{30} \\&= \frac{325}{175} - \frac{50}{175}\sqrt{30} \\&= \frac{13}{7} - \frac{2}{7}\sqrt{30} \\&= \frac{13}{7} + \left(-\frac{2}{7}\right)\sqrt{30} \\&= p + q\sqrt{30}\end{aligned}$$

Hence, $p = \frac{13}{7}$ and $q = -\frac{2}{7}$.

Answer 6A.

$$\begin{aligned}& \frac{\sqrt{3}-1}{\sqrt{3}+1} \\&= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \\&= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - (1)^2} \\&= \frac{3 - 2 \times \sqrt{3} \times 1 + 1}{3 - 1} \\&= \frac{4 - 2\sqrt{3}}{2} \\&= 2 - \sqrt{3} \\&= 2 + (-1)\sqrt{3} \\&= a + b\sqrt{3}\end{aligned}$$

Hence, $a = 2$ and $b = -1$.

Answer 6B.

$$\begin{aligned}& \frac{3+\sqrt{7}}{3-\sqrt{7}} \\&= \frac{3+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} \\&= \frac{(3+\sqrt{7})^2}{(3)^2 - (\sqrt{7})^2} \\&= \frac{9 + 6\sqrt{7} + 7}{9 - 7} \\&= \frac{16 + 6\sqrt{7}}{2} \\&= 8 + 3\sqrt{7} \\&= a + b\sqrt{7}\end{aligned}$$

Hence, $a = 8$ and $b = 3$.

Answer 6C.

$$\begin{aligned}& \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \\&= \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} \\&= \frac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2} \\&= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48} \\&= \frac{11-6\sqrt{3}}{1} \\&= 11+(-6)\sqrt{3} \\&= a+b\sqrt{3}\end{aligned}$$

Hence, $a = 11$ and $b = -6$

Answer 6D.

$$\begin{aligned}& \frac{1}{\sqrt{5}-\sqrt{3}} \\&= \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\&= \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} \\&= \frac{\sqrt{5}+\sqrt{3}}{5-3} \\&= \frac{\sqrt{5}+\sqrt{3}}{2} \\&= \frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{3} \\&= \frac{1}{2}\sqrt{5}-\left(-\frac{1}{2}\right)\sqrt{3} \\&= a\sqrt{5}-b\sqrt{3}\end{aligned}$$

Hence, $a = \frac{1}{2}$ and $b = -\frac{1}{2}$.

Answer 6E.

$$\begin{aligned}
& \frac{\sqrt{3}-2}{\sqrt{3}+2} \\
&= \frac{\sqrt{3}-2}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} \\
&= \frac{\sqrt{3}(\sqrt{3}-2)-2(\sqrt{3}-2)}{(\sqrt{3})^2-(2)^2} \\
&= \frac{3-2\sqrt{3}-2\sqrt{3}+4}{3-4} \\
&= \frac{7-4\sqrt{3}}{-1} \\
&= -(7-4\sqrt{3}) \\
&= -7+4\sqrt{3} \\
&= 4\sqrt{3}-7 \\
&= 4\sqrt{3}+(-7) \\
&= a\sqrt{3}+b
\end{aligned}$$

Hence, $a = 4$ and $b = -7$.

Answer 6F.

$$\begin{aligned}
& \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \\
&= \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \times \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}} \\
&= \frac{(\sqrt{11}-\sqrt{7})^2}{(\sqrt{11})^2-(\sqrt{7})^2} \\
&= \frac{(\sqrt{11})^2+(\sqrt{7})^2-2\times\sqrt{11}\times\sqrt{7}}{11-7} \\
&= \frac{11+7-2\sqrt{77}}{4} \\
&= \frac{18-2\sqrt{77}}{4} \\
&= \frac{18}{4}-\frac{2}{4}\sqrt{77} \\
&= \frac{9}{2}-\frac{1}{2}\sqrt{77} \\
&= a-b\sqrt{77}
\end{aligned}$$

Hence, $a = \frac{9}{2}$ and $b = \frac{1}{2}$.

Answer 6G.

$$\begin{aligned}
& \frac{7\sqrt{3} - 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}} \\
&= \frac{7\sqrt{3} - 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}} \times \frac{4\sqrt{3} - 3\sqrt{2}}{4\sqrt{3} - 3\sqrt{2}} \\
&= \frac{7\sqrt{3}(4\sqrt{3} - 3\sqrt{2}) - 5\sqrt{2}(4\sqrt{3} - 3\sqrt{2})}{(4\sqrt{3})^2 - (3\sqrt{2})^2} \\
&= \frac{84 - 21\sqrt{6} - 20\sqrt{6} + 30}{48 - 18} \\
&= \frac{110 - 41\sqrt{6}}{30} \\
&= \frac{110}{30} - \frac{41\sqrt{6}}{30} \\
&= \frac{11}{3} - \frac{41}{30}\sqrt{6} \\
&= a - b\sqrt{6} \\
&\text{Hence, } a = \frac{11}{3} \text{ and } b = \frac{41}{30}.
\end{aligned}$$

Answer 6H.

$$\begin{aligned}
& \frac{\sqrt{2} + \sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} \\
&= \frac{\sqrt{2} + \sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} \times \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} \\
&= \frac{(\sqrt{2} + \sqrt{3})(3\sqrt{2} + 2\sqrt{3})}{(3\sqrt{2})^2 - (2\sqrt{3})^2} \\
&= \frac{\sqrt{2}(3\sqrt{2} + 2\sqrt{3}) + \sqrt{3}(3\sqrt{2} + 2\sqrt{3})}{(9 \times 2) - (4 \times 3)} \\
&= \frac{(3 \times 2 + 2\sqrt{6}) + (3\sqrt{6} + 2 \times 3)}{18 - 12} \\
&= \frac{6 + 2\sqrt{6} + 3\sqrt{6} + 6}{6} \\
&= \frac{12 + 5\sqrt{6}}{6} \\
&= 2 + \frac{5\sqrt{6}}{6} \\
&= 2 - \left(-\frac{5}{6}\right)\sqrt{6} \\
&= a - b\sqrt{6} \\
&\text{Hence, } a = 2 \text{ and } b = -\frac{5}{6}.
\end{aligned}$$

Answer 6I.

$$\begin{aligned}& \frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} \\&= \frac{7+\sqrt{5}}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}} \\&= \frac{(7+\sqrt{5})^2}{7^2-(\sqrt{5})^2} - \frac{(7-\sqrt{5})^2}{7^2-(\sqrt{5})^2} \\&= \frac{7^2+2 \times 7 \times \sqrt{5}+(\sqrt{5})^2}{49-5} - \frac{7^2-2 \times 7 \times \sqrt{5}+(\sqrt{5})^2}{49-5} \\&= \frac{49+14\sqrt{5}+5}{44} - \frac{49-14\sqrt{5}+5}{44} \\&= \frac{54+14\sqrt{5}}{44} - \frac{54-14\sqrt{5}}{44}\end{aligned}$$

$$\begin{aligned}&= \frac{2(27+7\sqrt{5})}{44} - \frac{2(22-7\sqrt{5})}{44} \\&= \frac{27+7\sqrt{5}}{22} - \frac{22-7\sqrt{5}}{22} \\&= \frac{27}{22} + \frac{7\sqrt{5}}{22} - \frac{22}{22} + \frac{7\sqrt{5}}{22} \\&= \frac{14\sqrt{5}}{22} \\&= \frac{7\sqrt{5}}{11} \\&= 0 + \frac{7\sqrt{5}}{11} \\&= a + b\sqrt{5}\end{aligned}$$

Hence, $a = 0$ and $b = \frac{7}{11}$.

Answer 6J.

$$\begin{aligned}
& \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} \\
&= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\
&= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1} + \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2-1} \\
&= \frac{(\sqrt{3})^2-2 \times \sqrt{3} \times 1+1^2}{3-1} + \frac{(\sqrt{3})^2+2 \times \sqrt{3} \times 1+1^2}{3-1} \\
&= \frac{3-2\sqrt{3}+1}{2} + \frac{3+2\sqrt{3}+1}{2} \\
&= \frac{4-2\sqrt{3}}{2} + \frac{4+2\sqrt{3}}{2} \\
&= \frac{2(2-\sqrt{3})}{2} + \frac{2(2+\sqrt{3})}{2} \\
&= 2-\sqrt{3}+2+\sqrt{3} \\
&= 4+0
\end{aligned}$$

Hence, $a = 4$ and $b = 0$

Answer 7.

(i) $\sqrt{x} + \frac{1}{\sqrt{x}}$

Squaring Both sides we get

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2 \quad \text{--- (1)}$$

we will first find out $x + \frac{1}{x}$

$$\begin{aligned}
x + \frac{1}{x} &= (7 + 4\sqrt{3}) + \frac{1}{(7 + 4\sqrt{3})} \\
&= \frac{(7 + 4\sqrt{3})^2 + 1}{(7 + 4\sqrt{3})} \\
&= \frac{49 + 48 + 56\sqrt{3} + 1}{(7 + 4\sqrt{3})} \\
&= \frac{98 + 56\sqrt{3}}{(7 + 4\sqrt{3})} = \frac{14(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})} = 14
\end{aligned}$$

substituting in (1)

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2 = 14 + 2 = 16$$

$$\therefore \sqrt{x} + \frac{1}{\sqrt{x}} = 4$$

$$(ii) x^2 + \frac{1}{x^2}$$

$$\left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2 \quad \text{---(1)}$$

we will first find out $x + \frac{1}{x}$

$$\begin{aligned} x + \frac{1}{x} &= (7 + 4\sqrt{3}) + \frac{1}{(7 + 4\sqrt{3})} \\ &= \frac{(7 + 4\sqrt{3})^2 + 1}{(7 + 4\sqrt{3})} \\ &= \frac{49 + 48 + 56\sqrt{3} + 1}{(7 + 4\sqrt{3})} \\ &= \frac{98 + 56\sqrt{3}}{(7 + 4\sqrt{3})} = \frac{14(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})} = 14 \end{aligned}$$

substituting in (1)

$$\left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2 = 196 - 2 = 194$$

$$\therefore \left(x^2 + \frac{1}{x^2}\right) = 194$$

$$(iii) x^3 + \frac{1}{x^3}$$

$$\left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) \quad \text{---(1)}$$

we will first find out $x + \frac{1}{x}$

$$\begin{aligned} x + \frac{1}{x} &= (7 + 4\sqrt{3}) + \frac{1}{(7 + 4\sqrt{3})} \\ &= \frac{(7 + 4\sqrt{3})^2 + 1}{(7 + 4\sqrt{3})} \\ &= \frac{49 + 48 + 56\sqrt{3} + 1}{(7 + 4\sqrt{3})} \\ &= \frac{98 + 56\sqrt{3}}{(7 + 4\sqrt{3})} = \frac{14(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})} = 14 \end{aligned}$$

substituting in (1)

$$\begin{aligned} \left(x^3 + \frac{1}{x^3}\right) &= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = (14)^3 - 3 \times 14 = 2744 - 42 \\ &= 2702 \end{aligned}$$

$$\therefore \left(x^3 + \frac{1}{x^3}\right) = 2702$$

(iv)

$$x = 7 + 4\sqrt{3}$$

$$\therefore \frac{1}{x} = \frac{1}{7 + 4\sqrt{3}}$$

$$= \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}}$$

$$= \frac{7 - 4\sqrt{3}}{7^2 - (4\sqrt{3})^2}$$

$$= \frac{7 - 4\sqrt{3}}{49 - 48}$$

$$= \frac{7 - 4\sqrt{3}}{1}$$

$$= 7 - 4\sqrt{3}$$

$$\therefore x + \frac{1}{x} = (7 + 4\sqrt{3}) + (7 - 4\sqrt{3}) = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14$$

$$\text{Hence, } \left(x + \frac{1}{x}\right)^2 = (14)^2 = 196$$

Answer 8.

(i) $\frac{1}{x}$

$$\frac{1}{x} = \frac{1}{(4 - \sqrt{15})}$$

$$= \frac{1}{(4 - \sqrt{15})} \times \frac{(4 + \sqrt{15})}{(4 + \sqrt{15})}$$

$$= \frac{(4 + \sqrt{15})}{16 - 15} = (4 + \sqrt{15})$$

(ii) $x + \frac{1}{x}$

$$x + \frac{1}{x} = (4 - \sqrt{15}) + \frac{1}{(4 - \sqrt{15})}$$

$$= \frac{(4 - \sqrt{15})^2 + 1}{(4 - \sqrt{15})}$$

$$= \frac{16 + 15 - 8\sqrt{15} + 1}{(4 - \sqrt{15})}$$

$$= \frac{8(4 - \sqrt{15})}{(4 - \sqrt{15})} = 8$$

$$(iii) x^2 + \frac{1}{x^2}$$

$$\left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2 \text{ ----- (1)}$$

we will first find the value of $x + \frac{1}{x}$

$$x + \frac{1}{x} = (4 - \sqrt{15}) + \frac{1}{(4 - \sqrt{15})}$$

$$= \frac{(4 - \sqrt{15})^2 + 1}{(4 - \sqrt{15})}$$

$$= \frac{16 + 15 - 8\sqrt{15} + 1}{(4 - \sqrt{15})}$$

$$= \frac{8(4 - \sqrt{15})}{(4 - \sqrt{15})} = 8$$

substituting the values in (1)

$$\left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2 = 8^2 - 2 = 64 - 2 = 62$$

$$\left(x^2 + \frac{1}{x^2}\right) = 62$$

$$(iv) x^3 + \frac{1}{x^3}$$

$$\left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) \text{ ----- (1)}$$

we will first find the value of $x + \frac{1}{x}$

$$x + \frac{1}{x} = (4 - \sqrt{15}) + \frac{1}{(4 - \sqrt{15})}$$

$$= \frac{(4 - \sqrt{15})^2 + 1}{(4 - \sqrt{15})}$$

$$= \frac{16 + 15 - 8\sqrt{15} + 1}{(4 - \sqrt{15})}$$

$$= \frac{8(4 - \sqrt{15})}{(4 - \sqrt{15})} = 8$$

substituting the values in (1)

$$\left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 8^3 - 24 = 488$$

$$\left(x^3 + \frac{1}{x^3}\right) = 488$$

(v)

$$x = 4 - \sqrt{15}$$

$$\therefore \frac{1}{x} = \frac{1}{4 - \sqrt{15}}$$

$$= \frac{1}{4 - \sqrt{15}} \times \frac{4 + \sqrt{15}}{4 + \sqrt{15}}$$

$$= \frac{4 + \sqrt{15}}{4^2 - (\sqrt{15})^2}$$

$$= \frac{4 + \sqrt{15}}{16 - 15}$$

$$= \frac{4 + \sqrt{15}}{1}$$

$$= 4 + \sqrt{15}$$

$$\therefore x + \frac{1}{x} = (4 - \sqrt{15}) + (4 + \sqrt{15}) = 4 - \sqrt{15} + 4 + \sqrt{15} = 8$$

$$\text{Hence, } \left(x + \frac{1}{x}\right)^2 = (8)^2 = 64$$

Answer 9.

$$x = \frac{(2 + \sqrt{5})}{(2 - \sqrt{5})}$$

$$= \frac{(2 + \sqrt{5})}{(2 - \sqrt{5})} \times \frac{(2 + \sqrt{5})}{(2 + \sqrt{5})}$$

$$= \frac{(2 + \sqrt{5})^2}{4 - 5} = -(4 + 5 + 4\sqrt{5})$$

$$= -9 - 4\sqrt{5}$$

$$y = \frac{(2 - \sqrt{5})}{(2 + \sqrt{5})}$$

$$= \frac{(2 - \sqrt{5})}{(2 + \sqrt{5})} \times \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$$

$$= \frac{(2 - \sqrt{5})^2}{4 - 5} = -(4 + 5 - 4\sqrt{5})$$

$$= -9 + 4\sqrt{5}$$

$$\therefore x^2 - y^2 = (x + y)(x - y)$$

$$= (-9 - 4\sqrt{5} - 9 + 4\sqrt{5})(-9 - 4\sqrt{5} + 9 - 4\sqrt{5})$$

$$= (-18)(-8\sqrt{5}) = 144\sqrt{5}$$

Answer 10.

(i) $x^2 + y^2$

$$x^2 + y^2 = (x + y)^2 - 2xy \text{ --- (1)}$$

$$\begin{aligned} \therefore (x + y) &= \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} + \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \\ &= \frac{(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2}{3 - 1} \\ &= \frac{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}{2} \\ &= \frac{8}{2} = 4 \end{aligned}$$

$$\begin{aligned} \text{and } xy &= \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \\ &= 1 \end{aligned}$$

substituting in (1), we get

$$x^2 + y^2 = (x + y)^2 - 2xy = 16 - 2 = 14$$

(ii) $x^3 + y^3$

$$x^3 + y^3 = (x + y)^3 - 3xy(x + y) \text{ --- (1)}$$

$$\begin{aligned} \therefore (x + y) &= \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} + \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \\ &= \frac{(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2}{3 - 1} \\ &= \frac{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}{2} \\ &= \frac{8}{2} = 4 \end{aligned}$$

$$\begin{aligned} \text{and } xy &= \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \\ &= \frac{3 - 1}{3 - 1} = 1 \end{aligned}$$

substituting in (1), we get

$$\begin{aligned} x^3 + y^3 &= (x + y)^3 - 3xy(x + y) \\ &= 64 - 3 \times 4 = 64 - 12 = 52 \end{aligned}$$

$$(iii) \ x^2 - y^2 + xy$$

$$x^2 - y^2 + xy = (x + y)(x - y) + xy \text{ --- (1)}$$

$$\begin{aligned} \therefore (x + y) &= \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} + \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \\ &= \frac{(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2}{3 - 1} \\ &= \frac{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}{2} \\ &= \frac{8}{2} = 4 \end{aligned}$$

$$\begin{aligned} (x - y) &= \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} - \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \\ &= \frac{(\sqrt{3} + 1)^2 - (\sqrt{3} - 1)^2}{3 - 1} \\ &= \frac{3 + 1 + 2\sqrt{3} - 3 - 1 + 2\sqrt{3}}{2} \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{and } xy &= \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \\ &= \frac{3 - 1}{3 - 1} = 1 \end{aligned}$$

substituting in (1), we get

$$\begin{aligned} x^2 - y^2 + xy &= (x + y)(x - y) + xy \\ &= 4 \times 2\sqrt{3} + 1 = 8\sqrt{3} + 1 \end{aligned}$$

Answer 11.

(i) $x^2 + y^2$

$$(x^2 + y^2) = (x + y)^2 - 2xy \text{ -----(1)}$$

$$\begin{aligned} \text{Now, } x + y &= \frac{1}{(3 - 2\sqrt{2})} + \frac{1}{(3 + 2\sqrt{2})} \\ &= \frac{(3 + 2\sqrt{2}) + (3 - 2\sqrt{2})}{(3 - 2\sqrt{2})(3 + 2\sqrt{2})} \\ &= \frac{6}{9 - 8} = 6 \end{aligned}$$

$$\begin{aligned} \text{and } xy &= \frac{1}{(3 - 2\sqrt{2})} \times \frac{1}{(3 + 2\sqrt{2})} \\ &= \frac{1}{9 - 8} = 1 \end{aligned}$$

sustituting the values in (1), we get

$$(x^2 + y^2) = (x + y)^2 - 2xy = 36 - 2 = 34$$

$$(x^2 + y^2) = 34$$

(ii) $x^3 + y^3$

$$(x^3 + y^3) = (x + y)^3 - 3xy(x + y) \text{ -----(1)}$$

$$\begin{aligned} \text{Now, } x + y &= \frac{1}{(3 - 2\sqrt{2})} + \frac{1}{(3 + 2\sqrt{2})} \\ &= \frac{(3 + 2\sqrt{2}) + (3 - 2\sqrt{2})}{(3 - 2\sqrt{2})(3 + 2\sqrt{2})} \\ &= \frac{6}{9 - 8} = 6 \end{aligned}$$

$$\begin{aligned} \text{and } xy &= \frac{1}{(3 - 2\sqrt{2})} \times \frac{1}{(3 + 2\sqrt{2})} \\ &= \frac{1}{9 - 8} = 1 \end{aligned}$$

sustituting the values in (1), we get

$$\begin{aligned} (x^3 + y^3) &= (x + y)^3 - 3xy(x + y) \\ &= 216 - 3 \times 6 = 198 \end{aligned}$$