

Chapter 4. Expansions

Ex 4.1

Answer 1.

$$(i) \quad (a + 4)(a + 7) = a^2 + 4a + 7a + 28 = a^2 + 11a + 28$$

(Using identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$$(ii) \quad (m + 8)(m - 7) = m^2 + 8m - 7m - 56 = m^2 + m - 56$$

(Using identity: $(x + a)(x - b) = x^2 + (a - b)x - ab$)

$$(iii) \quad (x - 5)(x - 4) = x^2 - 5x - 4x + 20 = x^2 - 9x + 20$$

(Using identity: $(x - a)(x - b) = x^2 - (a + b)x + ab$)

$$(iv) \quad (3x + 4)(2x - 1) = 6x^2 - 3x + 8x - 4 = 6x^2 + 5x - 4$$

(Using identity: $(x + a)(x - b) = x^2 + (a - b)x - ab$)

(v)

$$(2x - 5)(2x + 5)(2x - 3)$$

$$= (4x^2 - 25)(2x - 3) = 8x^3 - 12x^2 - 50x + 75$$

(Using identity: $(x - a)(x + b) = x^2 - (a - b)x - ab$)

Answer 2.

a. Using $(x + y)^2 = x^2 + 2xy + y^2$, we get

$$\begin{aligned}(a + 3b)^2 &= a^2 + 2(a)(3b) + (3b)^2 \\ &= a^2 + 6ab + 9b^2\end{aligned}$$

$$\begin{aligned}b. (2p - 3q)^2 &= (2p)^2 - 2(2p)(3q) + (3q)^2 \\ &= 4p^2 - 12pq + 9q^2\end{aligned}$$

$$\begin{aligned}c. \left(2a + \frac{1}{2a}\right)^2 &= (2a)^2 + 2(2a)\left(\frac{1}{2a}\right) + \left(\frac{1}{2a}\right)^2 \\ &= 4a^2 + 2 + \frac{1}{4a^2}\end{aligned}$$

d. Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$$\begin{aligned}(x - 3y - 2z)^2 &= x^2 + (3y)^2 + (2z)^2 + 2(x)(-3y) + 2(-3y)(-2z) + 2(x)(-2z) \\ &= x^2 + 9y^2 + 4z^2 - 6xy + 12yz - 4xz\end{aligned}$$

Answer 3.

a. Using $(x + y)^2 = x^2 + 2xy + y^2$, we get

$$\begin{aligned}(9m - 2n)^2 &= (9m)^2 + 2(9m)(-2n) + (-2n)^2 \\ &= 81m^2 - 36mn + 4n^2\end{aligned}$$

$$\begin{aligned}\text{b. } (3p - 4q)^2 &= (3p)^2 - 2(3p)(4q) + (4q)^2 \\ &= 9p^2 - 12pq + 16q^2\end{aligned}$$

$$\begin{aligned}\text{c. } \left(\frac{7x}{9y} - \frac{9y}{7x}\right)^2 &= \left(\frac{7x}{9y}\right)^2 + 2\left(\frac{7x}{9y}\right)\left(\frac{9y}{7x}\right) + \left(\frac{9y}{7x}\right)^2 \\ &= \frac{49x^2}{81y^2} + 2 + \frac{81y^2}{49x^2}\end{aligned}$$

d. Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$$\begin{aligned}(2a + 3b - 4c)^2 &= (2a)^2 + (3b)^2 + (4c)^2 + 2(2a)(3b) + 2(3b)(-4c) + 2(2a)(-4c) \\ &= 4a^2 + 9b^2 + 16c^2 + 12ab - 24bc - 8ac\end{aligned}$$

Answer 4.

$$\text{(i) } (5x - 9)(5x + 9) = (5x)^2 - (9)^2 = 25x^2 - 81$$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

$$\text{(ii) } (2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2$$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

$$\begin{aligned}\text{(iii) } (a + b - c)(a - b + c) &= (a + b - c)[a - (b - c)] \\ &= (a)^2 - (b - c)^2\end{aligned}$$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

$$= a^2 - (b^2 + c^2 - 2bc)$$

$$= a^2 - b^2 - c^2 + 2bc$$

$$\text{(iv) } (x + y - 3)(x + y + 3) = (x + y)^2 - (3)^2$$

$$= x^2 + y^2 + 2xy - 9$$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

$$\begin{aligned}
 \text{(v)} \quad (1 + a)(1 - a)(1 + a^2) &= [(1)^2 - (a)^2](1 + a^2) \\
 &= (1 - a^2)(1 + a^2) \\
 \text{(Using identity: } (a+b)(a-b) &= a^2 - b^2) \\
 &= (1)^2 - (a^2)^2 \\
 &= 1 - a^4
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad \left(a + \frac{2}{a} - 1\right)\left(a - \frac{2}{a} - 1\right) &= (a - 1)^2 - \left(\frac{2}{a}\right)^2 \\
 &= a^2 + 1 - 2a - \frac{4}{a^2} \\
 \text{(Using identity: } (a+b)(a-b) &= a^2 - b^2)
 \end{aligned}$$

Answer 5.

a. Using $(x + y)^2 = x^2 + 2xy + y^2$, we get

$$\begin{aligned}
 (95)^2 &= (100 - 5)^2 \\
 &= (100)^2 - 2(100)(5) + (5)^2 \\
 &= 10000 - 1000 + 25 \\
 &= 9025
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } (103)^2 &= (100 + 3)^2 \\
 &= (100)^2 + 2(100)(3) + (3)^2 \\
 &= 10000 + 600 + 9 \\
 &= 10609
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } (999)^2 &= (1000 - 1)^2 \\
 &= (1000)^2 - 2(1000)(1) + (1)^2 \\
 &= 1000000 - 2000 + 1 \\
 &= 998001
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } (1005)^2 &= (1000 + 5)^2 \\
 &= (1000)^2 + 2(1000)(5) + (5)^2 \\
 &= 1000000 + 10000 + 25 \\
 &= 1010025
 \end{aligned}$$

Answer 6.

$$\begin{aligned}\text{a. } 399 \times 401 &= (400 - 1) \times (400 + 1) \\ &= (400)^2 - (1)^2 \\ &= 160000 - 1 \\ &= 159999\end{aligned}$$

$$\begin{aligned}\text{b. } 999 \times 1001 &= (1000 - 1) \times (1000 + 1) \\ &= (1000)^2 - (1)^2 \\ &= 1000000 - 1 \\ &= 999999\end{aligned}$$

$$\begin{aligned}\text{c. } 4.9 \times 5.1 &= (5 - 0.1) \times (5 + 0.1) \\ &= (5)^2 - (0.1)^2 \\ &= 25 - 0.01 \\ &= 24.99\end{aligned}$$

$$\begin{aligned}\text{d. } 15.9 \times 16.1 &= (16 - 0.1) \times (16 + 0.1) \\ &= (16)^2 - (0.1)^2 \\ &= 256 - 0.01 \\ &= 255.99\end{aligned}$$

Answer 7.

$$a - b = 10, ab = 11$$

We know that:

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$\Rightarrow (10)^2 = a^2 + b^2 - 2 \times 11$$

$$\Rightarrow 100 = a^2 + b^2 - 22$$

$$\Rightarrow a^2 + b^2 = 100 + 22 = 122$$

Using $(a + b)^2 = a^2 + b^2 + 2ab$, we get

$$(a + b)^2 = 122 + 2(11) = 122 + 22 = 144$$

$$\Rightarrow (a + b) = \sqrt{144} = \pm 12$$

Answer 8.

$$x + y = 9, xy = 20$$

(i) We know $(a + b)^2 = a^2 + 2ab + b^2$

$$\therefore (x + y)^2 = 81 \quad x^2 + y^2 + 2xy$$

$$\Rightarrow x^2 + y^2 = 81 - 2(120) = 41$$

We also know $(a - b)^2 = a^2 - 2ab + b^2$

$$\Rightarrow (x - y)^2 = x^2 - 2xy + y^2$$

$$\Rightarrow (x - y)^2 = 41 - 2(20) = 1$$

$$\Rightarrow x - y = \pm 1$$

(ii) We know $(x - y)(x + y) = x^2 - y^2$

$$\Rightarrow x^2 - y^2 = (\pm 1)(9) = \pm 9$$

Answer 9.

(i) $\left(a + \frac{1}{a}\right)^2 = (a^2) + 2(a)\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

$$\Rightarrow 36 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 34$$

$$\begin{aligned}\left(a - \frac{1}{a}\right)^2 &= (a^2) - 2(a)\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2 \\ &= a^2 + \frac{1}{a^2} - 2 \\ &= 34 - 2 = 32\end{aligned}$$

$$\Rightarrow a - \frac{1}{a} = \pm \sqrt{32} = \pm 4\sqrt{2}$$

(ii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$$= (6)(\pm 4\sqrt{2}) = \pm 24\sqrt{2}$$

Answer 10.

$$a - \frac{1}{a} = 10$$

$$(i) \left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2(a) \left(\frac{1}{a}\right)$$

$$\Rightarrow (10)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 102$$

$$\begin{aligned} \text{Now, } \left(a + \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} + 2(a) \left(\frac{1}{a}\right) \\ &= 102 + 2 = 104 \end{aligned}$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{104} = \pm 2\sqrt{26}$$

$$(ii) a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right) \left(a - \frac{1}{a}\right)$$

$$\begin{aligned} &= (\pm 2\sqrt{26}) (10) \\ &= \pm 20\sqrt{26} \end{aligned}$$

Answer 11.

$$(i) \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2(x) \left(\frac{1}{x}\right)$$

$$\Rightarrow (3)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

(ii) squaring both sides of the equation $\left(x^2 + \frac{1}{x^2}\right) = 7$, we get:

$$x^4 + \frac{1}{x^4} + 2 = 49$$

$$x^4 + \frac{1}{x^4} = 47$$

Answer 12.

$$(i) (p + q)^2 = (8)^2$$

$$p^2 + q^2 + 2pq = 64 \quad \dots(i)$$

$$(p - q)^2 = (4)^2$$

$$p^2 + q^2 - 2pq = 16$$

$$p^2 + q^2 = 16 + 2pq \quad \dots(ii)$$

Using (ii) in (i), we get:

$$16 + 2pq + 2pq = 64$$

$$\Rightarrow 4pq = 64 - 16 = 48$$

$$\Rightarrow pq = 12$$

(ii) Putting $pq = 12$ in (i) we get:

$$p^2 + q^2 = 64 - 2(12) = 64 - 24 = 40$$

Answer 13.

Given $m - n = 0.9$ and $mn = 0.36$

$$a. (m - n)^2 = m^2 - 2mn + n^2$$

$$\Rightarrow (0.9)^2 = m^2 - 2mn + n^2$$

$$\Rightarrow 0.81 = m^2 + n^2 - 2(0.36)$$

$$\Rightarrow 0.81 = m^2 + n^2 - 0.72$$

$$\Rightarrow m^2 + n^2 = 1.53$$

$$\text{So, } (m + n)^2 = m^2 + 2mn + n^2$$

$$\Rightarrow (m + n)^2 = m^2 + n^2 + 2mn$$

$$\Rightarrow (m + n)^2 = 1.53 + 2(0.36)$$

$$\Rightarrow (m + n)^2 = 2.25$$

$$\Rightarrow m + n = \pm 1.5$$

$$b. m^2 - n^2 = (m + n)(m - n)$$

$$= (\pm 1.5)(0.9)$$

$$= \pm 1.35$$

Answer 14.

$$(i) (x + y)^2 = (1)^2$$

$$\Rightarrow x^2 + y^2 + 2xy = 1$$

$$\Rightarrow x^2 + y^2 = 1 - 2(-12) = 1 + 24 = 25$$

$$\text{Now, } (x - y)^2 = x^2 + y^2 - 2xy$$

$$= 25 - 2(-12)$$

$$= 25 + 24$$

$$= 49$$

$$\Rightarrow x - y = \pm 7$$

$$(ii) x^2 - y^2 = (x + y)(x - y)$$

$$= (1)(\pm 7) = \pm 7$$

Answer 15.

(i) Dividing the given equation by a , we get:

$$\frac{a^2}{a} - \frac{7a}{a} + \frac{1}{a} = 0, a - 7 + \frac{1}{a} = 0$$

$$\Rightarrow a + \frac{1}{a} = 7$$

$$(ii) a + \frac{1}{a} = 7$$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 49$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 49 - 2 = 47$$

Answer 16.

(i) Dividing the given equation by a we get

$$a - 3 - \frac{1}{a} = 0$$

$$\Rightarrow a - \frac{1}{a} = 3$$

(ii) $a - \frac{1}{a} = 3$

Squaring both sides, we get

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2 = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 11$$

Now,

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 = 11 + 2 = 13$$

$$\Rightarrow a + \frac{1}{a} = \pm\sqrt{13}$$

$$\begin{aligned} \text{(iii)} \quad a^2 - \frac{1}{a^2} &= \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) \\ &= (\pm\sqrt{13})(3) \\ &= \pm 3\sqrt{13} \end{aligned}$$

Answer 17.

Given $2x + 3y = 10$ and $xy = 5$

$$4x^2 + 9y^2 = (2x)^2 + (3y)^2$$

$$= (2x + 3y)^2 - 2(2x)(3y)$$

$$\dots\dots[\because (a+b)^2 = a^2 + b^2 + 2ab, \text{ so, } a^2 + b^2 = (a+b)^2 - 2ab]$$

$$= (10)^2 - 12(5)$$

$$= 100 - 60$$

$$= 40$$

Answer 18.

$$\begin{aligned}
(x + y + z)^2 &= (12)^2 \\
\Rightarrow x^2 + y^2 + z^2 + 2xy + 2yz + 2zx &= 144 \\
\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) &= 144 \\
\Rightarrow x^2 + y^2 + z^2 + 2(27) &= 144 \\
\Rightarrow x^2 + y^2 + z^2 &= 144 - 54 = 90
\end{aligned}$$

Answer 19.

$$\begin{aligned}
(a + b + c)^2 &= (9)^2 \\
a^2 + b^2 + c^2 + 2ab + 2bc + 2ca &= 81 \\
\Rightarrow 41 + 2(ab + bc + ca) &= 81 \\
\Rightarrow 2(ab + bc + ca) &= 81 - 41 = 40 \\
\Rightarrow ab + bc + ca &= 20
\end{aligned}$$

Answer 20.

$$\begin{aligned}
(p + q + r)^2 &= p^2 + q^2 + r^2 + 2pq + 2qr + 2pr \\
&= 8^2 + 2(18) \\
&= 64 + 36 \\
&= 100 \\
\Rightarrow p + q + r &= \sqrt{100} = \pm 10
\end{aligned}$$

Answer 21.

$$\begin{aligned}
&\text{Given } x + y + z = p \text{ and } xy + yz + zx = q \\
(x + y + z)^2 &= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx \\
\Rightarrow x^2 + y^2 + z^2 &= (x + y + z)^2 - 2xy + 2yz + 2zx \\
\Rightarrow x^2 + y^2 + z^2 &= (x + y + z)^2 - 2(xy + yz + zx) \\
\Rightarrow x^2 + y^2 + z^2 &= (p)^2 - 2(q) \\
\Rightarrow x^2 + y^2 + z^2 &= p^2 - 2q
\end{aligned}$$

Ex 4.2

Answer 1.

(i) Using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$$\begin{aligned}(2a - 5b)^3 &= (2a)^3 - (5b)^3 - 3(2a)(5b)(2a - 5b) \\ &= 8a^3 - 125b^3 - 30ab(2a - 5b) \\ &= 8a^3 - 125b^3 - 60a^2b + 150ab^2\end{aligned}$$

(ii) Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\begin{aligned}(4x + 7y)^3 &= (4x)^3 + (7y)^3 + 3(4x)(7y)(4x + 7y) \\ &= 64x^3 + 343y^3 + 84xy(4x + 7y) \\ &= 64x^3 + 343y^3 + 336x^2y + 588xy^2\end{aligned}$$

(iii) $\left(3a + \frac{1}{3a}\right)^3 = (3a)^3 + \left(\frac{1}{3a}\right)^3 + 3(3a)\left(\frac{1}{3a}\right)\left(3a + \frac{1}{3a}\right)$

$$= 27a^3 + \frac{1}{27a^3} + 9a + \frac{1}{a}$$

(iv) $\left(4p - \frac{1}{p}\right)^3 = (4p)^3 - \left(\frac{1}{p}\right)^3 - 3(4p)\left(\frac{1}{p}\right)\left(4p - \frac{1}{p}\right)$

$$= 64p^3 - \frac{1}{p^3} - 48p + \frac{12}{p}$$

(v) $\left(\frac{2m}{3n} + \frac{3n}{2m}\right)^3 = \left(\frac{2m}{3n}\right)^3 + \left(\frac{3n}{2m}\right)^3 + 3\left(\frac{2m}{3n}\right)\left(\frac{3n}{2m}\right)\left(\frac{2m}{3n} + \frac{3n}{2m}\right)$

$$= \frac{8m^3}{27n^3} + \frac{27n^3}{8m^3} + \frac{2m}{n} + \frac{9n}{2m}$$

(vi)

Using $(a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2a + 3c^2a + 6abc$

$$\begin{aligned}\left(a - \frac{1}{a} + b\right)^3 &= a^3 + \left(-\frac{1}{a}\right)^3 + b^3 + 3a^2\left(-\frac{1}{a}\right) + 3a^2b + 3\left(-\frac{1}{a}\right)^2b + 3\left(-\frac{1}{a}\right)a^2 + 3b^2a + 3b^2\left(-\frac{1}{a}\right) + 6a\left(-\frac{1}{a}\right)b \\ &= a^3 - \frac{1}{a^3} + b^3 - 3a + 3a^2b + \frac{3b}{a^2} + \frac{3}{a} + 3b^2a - \frac{3b^2}{a} - 6b\end{aligned}$$

Answer 2.

$$5x + \frac{1}{5x} = 7$$

Using $\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$, we get :

$$\begin{aligned}\left(5x + \frac{1}{5x}\right)^3 &= (5x)^3 + \left(\frac{1}{5x}\right)^3 + 3\left(5x + \frac{1}{5x}\right) \\ \Rightarrow 343 &= 125x^2 + \frac{1}{125x^3} + 3(7) \\ \Rightarrow 125x^3 + \frac{1}{125x^3} &= 343 - 21 = 322\end{aligned}$$

Answer 3.

$$3x - \frac{1}{3x} = 9$$

Using $\left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)$, we get :

$$\begin{aligned}\left(3x - \frac{1}{3x}\right)^3 &= (3x)^3 - \left(\frac{1}{3x}\right)^3 - 3\left(3x - \frac{1}{3x}\right) \\ \Rightarrow 729 &= 27x^3 - \frac{1}{27x^3} - 3(9) \\ \Rightarrow 27x^3 - \frac{1}{27x^3} &= 729 + 27 = 756\end{aligned}$$

Answer 4.

$$x + \frac{1}{x} = 5 \quad \dots(1)$$

Squaring both sides of (1),

$$\begin{aligned}\left(x + \frac{1}{x}\right)^2 &= (5)^2 \\ \Rightarrow x^2 + \frac{1}{x^2} + 2 &= 25\end{aligned}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 25 - 2 = 23 \quad \dots(2)$$

Cubing both sides of (1),

$$\begin{aligned}\left(x + \frac{1}{x}\right)^3 &= 5^3 \\ x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) &= 125 \\ \Rightarrow x^3 + \frac{1}{x^3} + 3(5) &= 125 \\ \Rightarrow x^3 + \frac{1}{x^3} &= 125 - 15 = 110\end{aligned}$$

Squaring both sides of (2),

$$\begin{aligned}\left(x^2 + \frac{1}{x^2}\right)^2 &= (23)^2 \\ \Rightarrow x^4 + \frac{1}{x^4} + 2 &= 529 \\ \Rightarrow x^4 + \frac{1}{x^4} &= 529 - 2 = 527\end{aligned}$$

Answer 5.

$$a - \frac{1}{a} = 7 \quad \dots(1)$$

Squaring both sides of (1),

$$\begin{aligned}\left(a - \frac{1}{a}\right)^2 &= (7)^2 \\ \Rightarrow a^2 + \frac{1}{a^2} - 2 &= 49 \\ \Rightarrow a^2 + \frac{1}{a^2} &= 49 + 2 = 51\end{aligned}$$

$$\begin{aligned}\text{Now, } \left(a + \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} + 2 \\ &= 51 + 2 = 53\end{aligned}$$

$$\Rightarrow a + \frac{1}{a} = \pm \sqrt{53}$$

$$\text{Now } a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) = (\pm\sqrt{53})(7) = \pm 7\sqrt{53}$$

Answer 6A.

Using $(a+b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}\left(a + \frac{1}{a}\right)^2 &= a^2 + 2a\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2 \\ \Rightarrow \left(a + \frac{1}{a}\right)^2 &= a^2 + 2 + \frac{1}{a^2} \\ \Rightarrow \left(a + \frac{1}{a}\right)^2 &= a^2 + \frac{1}{a^2} + 2 \\ \Rightarrow \left(a + \frac{1}{a}\right)^2 &= 14 + 2 \\ \Rightarrow \left(a + \frac{1}{a}\right)^2 &= 16 \\ \Rightarrow a + \frac{1}{a} &= \pm 4\end{aligned}$$

Answer 6B.

$$\text{Using } (a+b)^2 = a^2 + 2ab + b^2$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2a\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 14 + 2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 16$$

$$\Rightarrow a + \frac{1}{a} = \pm 4$$

$$\begin{aligned} a^3 + \frac{1}{a^3} &= \left(a + \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right) \dots \dots [\text{Using } a^3 + b^3 = (a+b)(a^2 + b^2 - ab)] \\ &= (\pm 4)(14 - 1) \\ &= (\pm 4)(13) \\ &= \pm 52 \end{aligned}$$

Answer 7.

$$m^2 + \frac{1}{m^2} = 51$$

We know that

$$\left(m - \frac{1}{m}\right)^2 = m^2 + \frac{1}{m^2} - 2$$

$$\Rightarrow \left(m - \frac{1}{m}\right)^2 = 51 - 2$$

$$\Rightarrow \left(m - \frac{1}{m}\right)^2 = 49 = 7^2$$

$$\Rightarrow m - \frac{1}{m} = 7$$

$$\Rightarrow \left(m - \frac{1}{m}\right)^3 = 7^3$$

$$\Rightarrow m^3 - \frac{1}{m^3} - 3\left(m - \frac{1}{m}\right) = 343$$

$$\Rightarrow m^3 - \frac{1}{m^3} - 3 \times 7 = 343$$

$$\Rightarrow m^3 - \frac{1}{m^3} = 343 + 21 = 364$$

Answer 8.

$$9a^2 + \frac{1}{9a^2} = 23$$

$$\text{Using } \left(3a + \frac{1}{3a}\right)^2 = (3a)^2 + \left(\frac{1}{3a}\right)^2 + 2(3a)\left(\frac{1}{3a}\right)$$

$$\Rightarrow \left(3a + \frac{1}{3a}\right)^2 = 9a^2 + \frac{1}{9a^2} + 2$$
$$= 23 + 2 = 25$$

$$\Rightarrow 3a + \frac{1}{3a} = 5$$

Cubing both sides, we get:

$$(3a)^3 + \left(\frac{1}{3a}\right)^3 + 3(3a)\left(\frac{1}{3a}\right)\left(3a + \frac{1}{3a}\right) = (5)^3$$

$$\Rightarrow 27a^3 + \frac{1}{27a^3} + 3(5) = 125$$

$$\Rightarrow 27a^3 + \frac{1}{27a^3} = 125 - 15 = 110$$

Answer 9.

$$x^2 + \frac{1}{x^2} = 18$$

$$\text{(i) Using } \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 18 - 2 = 16$$

$$\Rightarrow x - \frac{1}{x} = 4$$

$$\text{(ii) } \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$\Rightarrow 64 = x^3 - \frac{1}{x^3} - 3(4)$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 64 + 12 = 76$$

Answer 10.

$$(i) \left(p + \frac{1}{p}\right)^2 = p^2 + \frac{1}{p^2} + 2$$

$$\Rightarrow 36 = p^2 + \frac{1}{p^2} + 2$$

$$\Rightarrow p^2 + \frac{1}{p^2} = 36 - 2 = 34$$

$$(ii) \left(p^2 + \frac{1}{p^2}\right)^2 = p^4 + \frac{1}{p^4} + 2$$

$$\Rightarrow (34)^2 = p^4 + \frac{1}{p^4} + 2$$

$$\Rightarrow p^4 + \frac{1}{p^4} = 1158 - 2 = 1154$$

$$(iii) \left(p + \frac{1}{p}\right)^3 = p^3 + \frac{1}{p^3} + 3\left(p + \frac{1}{p}\right)$$

$$\Rightarrow 216 = p^3 + \frac{1}{p^3} + 3(6)$$

$$\Rightarrow p^3 + \frac{1}{p^3} = 216 - 18 = 198$$

Answer 11.

$$(i) \left(r - \frac{1}{r}\right)^2 = r^2 + \frac{1}{r^2} - 2$$

$$\Rightarrow (4)^2 = r^2 + \frac{1}{r^2} - 2$$

$$\Rightarrow r^2 + \frac{1}{r^2} = 16 + 2 = 18$$

$$(ii) \left(r^2 + \frac{1}{r^2}\right)^2 = r^4 + \frac{1}{r^4} + 2$$

$$\Rightarrow (18)^2 = r^4 + \frac{1}{r^4} + 2$$

$$\Rightarrow r^4 + \frac{1}{r^4} = 324 - 2 = 322$$

$$(iii) \left(r - \frac{1}{r}\right)^3 = r^3 - \frac{1}{r^3} - 3\left(r - \frac{1}{r}\right)$$

$$\Rightarrow (4)^3 = r^3 - \frac{1}{r^3} - 3(4)$$

$$\Rightarrow r^3 - \frac{1}{r^3} = 64 + 12 = 76$$

Answer 12.

$$a + \frac{1}{a} = 2$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow (2)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 4 - 2 = 2$$

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow (2)^3 = a^3 + \frac{1}{a^3} + 3(2)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 8 - 6 = 2$$

$$\left(a^2 + \frac{1}{a^2}\right)^2 = a^4 + \frac{1}{a^4} + 2$$

$$\Rightarrow (2)^2 = a^4 + \frac{1}{a^4} + 2$$

$$\Rightarrow a^4 + \frac{1}{a^4} = 4 - 2 = 2$$

$$\text{Thus, } a^2 + \frac{1}{a^2} = a^3 + \frac{1}{a^3} = a^4 + \frac{1}{a^4}$$

Answer 13.

$$x + \frac{1}{x} = p, \quad x - \frac{1}{x} = q$$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow p^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = p^2 - 2 \quad \dots(1)$$

$$\text{Also, } \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow q^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = q^2 + 2 \quad \dots(2)$$

Equating the value of $x^2 + \frac{1}{x^2}$ from and (2), we get:

$$p^2 - 2 = q^2 + 2$$

$$\Rightarrow p^2 - q^2 = 4$$

Answer 14.

$$a + \frac{1}{a} = p$$

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow p^3 = a^3 + \frac{1}{a^3} + 3(p)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = p^3 - 3p = p(p^2 - 3)$$

Answer 15.

$$\left(a + \frac{1}{a}\right)^2 = 3$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{3}$$

$$\text{Now, } \left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow (\sqrt{3})^3 = a^3 + \frac{1}{a^3} + 3(\sqrt{3})$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

Answer 16.

$$a + b + c = 0 \quad \dots(i)$$

$$\Rightarrow (a + b) + c = 0$$

cubing both sides

$$\Rightarrow (a + b)^3 + c^3 + 3(a + b)(c)(a + b + c) = 0$$

$$\Rightarrow a^3 + b^3 + 3ab(a + b) + c^3 + 0 = 0$$

$$\Rightarrow a^3 + b^3 + c^3 + 3ab(a + b) = 0 \quad \dots(2)$$

Using (i), we get, $a + b = -c$ From (2),

$$a^3 + b^3 + c^3 + 3ab(-c) = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

Answer 17.

$$a + 2b + c = 0 \quad \dots(i)$$

$$\Rightarrow (a + 2b) + c = 0$$

$$\Rightarrow (a + 2b)^3 + c^3 + 3(a + 2b) c (a + 2b + c) = 0$$

$$\Rightarrow a^3 + 8b^3 + 6ab(a + 2b) + c^3 + 0 = 0$$

$$\Rightarrow a^3 + 8b^3 + c^3 + 6ab(a + 2b) = 0 \quad \dots(2)$$

Using (1), we get $a + 2b = -c$

From (2),

$$a^3 + 8b^3 + 6ab(-c) = 0$$

$$\Rightarrow a^3 + 8b^3 + c^3 = 6abc$$

Answer 18.

$$x^3 + y^3 = 9, \quad x + y = 3$$

$$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\Rightarrow (3)^3 = 9 + 3xy(3)$$

$$\Rightarrow 27 = 9 + 9xy$$

$$\Rightarrow 9xy = 27 - 9 = 18$$

$$\Rightarrow xy = 2$$

Answer 19.

$$\text{Using } (a + b)^2 = a^2 + 2ab + b^2$$

$$a^2 + b^2 = (a + b)^2 - 2ab$$

$$\Rightarrow a^2 + b^2 = (5)^2 - 2(2)$$

$$\Rightarrow a^2 + b^2 = (5)^2 - 2(2)$$

$$\Rightarrow a^2 + b^2 = 25 - 4$$

$$\Rightarrow a^2 + b^2 = 21$$

$$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$= (5)(21 - 2)$$

$$= (5)(19)$$

$$= 95$$

Answer 20.

$$p - q = -1, \quad pq = -12$$

$$\begin{aligned}(p - q)^3 &= p^3 - q^3 - 3pq(p - q) \\ \Rightarrow (-1)^3 &= p^3 - q^3 - 3(-12)(-1) \\ \Rightarrow p^3 - q^3 &= -14 + 36 = 22\end{aligned}$$

Answer 21.

$$m - n = -2, \quad m^3 - n^3 = -26$$

$$\begin{aligned}(m - n)^3 &= m^3 - n^3 - 3mn(m - n) \\ \Rightarrow (-2)^3 &= -26 - 3mn(-2) \\ \Rightarrow 6mn &= -8 + 26 = 18 \\ \Rightarrow mn &= 3\end{aligned}$$

Answer 22.

$$2a - 3b = 10$$

$$\begin{aligned}(2a - 3b)^3 &= (2a)^3 - (3b)^3 - 2(2a)(3b)(2a - 3b) \\ \Rightarrow 1000 &= 8a^3 - 27b^3 - 12(16)(10) \\ \Rightarrow 8a^3 - 27b^3 &= 1000 + 1920 = 2920\end{aligned}$$

Answer 23.

$$\text{Given } x + 2y = 5$$

$$(x + 2y)^3 = 5^3$$

$$\Rightarrow (x)^3 + (2y)^3 + 3(x)(2y)(x + 2y) = 5^3 \dots \left[\text{Using } (a + b)^3 = (a)^3 + (b)^3 + 3ab(a + b) \right]$$

$$\Rightarrow (x)^3 + (2y)^3 + 6xy(x + 2y) = 125$$

$$\Rightarrow (x)^3 + (2y)^3 + 6xy(5) = 125$$

$$\Rightarrow x^3 + 8y^3 + 30xy = 125$$

Answer 24A.

$$\begin{aligned}& (4x + 5y)^2 + (4x - 5y)^2 \\&= (4x)^2 + (5y)^2 + 2(4x)(5y) + (4x)^2 + (5y)^2 - 2(4x)(5y) \\&= 16x^2 + 25y^2 + 40xy + 16x^2 + 25y^2 - 40xy \\&= 32x^2 + 50y^2\end{aligned}$$

Answer 24B.

$$\begin{aligned}& (7a + 5b)^2 - (7a - 5b)^2 \\&= (7a)^2 + (5b)^2 + 2(7a)(5b) - [(7a)^2 + (5b)^2 - 2(7a)(5b)] \\&= 49a^2 + 25b^2 + 70ab - [49a^2 + 25b^2 - 70ab] \\&= 70a + 70ab \\&= 140ab\end{aligned}$$

Answer 24C.

$$\begin{aligned}& (a + b)^3 + (a - b)^3 \\&= a^3 + b^3 + 3ab(a + b) + a^3 - 3ab(a - b) - b^3 \\&= a^3 + b^3 + 3a^2b + 3ab^2 + a^3 - 3a^2b + 3ab^2 - b^3 \\&= 2a^3 + 6ab^2\end{aligned}$$

Answer 24D.

$$\begin{aligned}& \left(a - \frac{1}{a}\right)^2 + \left(a + \frac{1}{a}\right)^2 \\&= (a)^2 + \left(\frac{1}{a}\right)^2 - 2(a)\left(\frac{1}{a}\right) + (a)^2 + \left(\frac{1}{a}\right)^2 + 2(a)\left(\frac{1}{a}\right) \\&= a^2 + \frac{1}{a^2} - 2 + a^2 + \frac{1}{a^2} + 2 \\&= 2a^2 + \frac{2}{a^2}\end{aligned}$$

Answer 24E.

$$\begin{aligned}
& (x + y - z)^2 + (x - y + z)^2 \\
&= x^2 + y^2 + z^2 + 2(x)(y) + 2(y)(-z) + 2(x)(-z) + x^2 + y^2 + z^2 + 2(x)(-y) + 2(-y)(z) + 2(x)(z) \\
&= x^2 + y^2 + z^2 + 2xy - 2yz - 2xz + x^2 + y^2 + z^2 - 2xy - 2yz + 2xz \\
&= 2x^2 + 2y^2 + 2z^2 - 4yz
\end{aligned}$$

Answer 24F.

$$\begin{aligned}
& \left(a + \frac{1}{a}\right)^3 - \left(a - \frac{1}{a}\right)^3 \\
&= (a)^3 + \left(\frac{1}{a}\right)^3 + 3(a)\left(\frac{1}{a}\right)\left(a + \frac{1}{a}\right) - \left[(a)^3 - \left(\frac{1}{a}\right)^3 - 3(a)\left(\frac{1}{a}\right)\left(a - \frac{1}{a}\right)\right] \\
&= a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) - \left[a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)\right] \\
&= a^3 + \frac{1}{a^3} + 3a + \frac{3}{a} - a^3 + \frac{1}{a^3} + 3a - \frac{3}{a} \\
&= \frac{2}{a^3} + 6a
\end{aligned}$$

Answer 24G.

$$\begin{aligned}
& (2x + y)(4x^2 - 2xy + y^2) \\
&= 2x(4x^2 - 2xy + y^2) + y(4x^2 - 2xy + y^2) \\
&= 8x^3 - 4x^2y + 2xy^2 + 4x^2y - 2xy^2 + y^3 \\
&= 8x^3 + y^3
\end{aligned}$$

Answer 24H.

$$\begin{aligned}
& \left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) \\
&= x\left(x^2 + 1 + \frac{1}{x^2}\right) - \frac{1}{x}\left(x^2 + 1 + \frac{1}{x^2}\right) \\
&= x^3 + x + \frac{1}{x} - x - \frac{1}{x} - \frac{1}{x^3} \\
&= x^3 - \frac{1}{x^3}
\end{aligned}$$

Answer 24I.

$$\begin{aligned}
& (x + 2y + 3z)(x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3zx) \\
&= x(x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3zx) + 2y(x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3zx) \\
&\quad + 3z(x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3zx) \\
&= x^3 + 4xy^2 + 9xz^2 - 2x^2y - 6xyz - 3zx^2 + 2x^2y + 8y^3 + 18yz^2 - 4xy^2 - 12y^2z - 6xyz \\
&\quad + 3x^2z + 12y^2z + 27z^3 - 6xyz - 18yz^2 - 9xz^2 \\
&= x^3 + 8y^3 + 27z^3 - 18xyz
\end{aligned}$$

Answer 24J.

$$\begin{aligned}
& (1 + x)(1 - x)(1 - x + x^2)(1 + x + x^2) \\
&= (1 + x)(1 - x)(x^2 + 1 - x)(x^2 + 1 + x) \\
&= (1^2 - x^2)\left[(x^2 + 1)^2 - x^2\right] \quad \dots \text{(Using } a^2 - b^2 = (a + b)(a - b)\text{)} \\
&= (1 - x^2)[x^4 + 2x^2 + 1 - x^2] \\
&= (1 - x^2)(x^4 + x^2 + 1) \\
&= 1(x^4 + x^2 + 1) - x^2(x^4 + x^2 + 1) \\
&= x^4 + x^2 + 1 - x^6 - x^4 - x^2 \\
&= 1 - x^6
\end{aligned}$$

Answer 24K.

$$\begin{aligned}
& (3a + 2b - c)(9a^2 + 4b^2 + c^2 - 6ab + 2bc + 3ca) \\
&= 3a(9a^2 + 4b^2 + c^2 - 6ab + 2bc + 3ca) + 2b(9a^2 + 4b^2 + c^2 - 6ab + 2bc + 3ca) \\
&\quad - c(9a^2 + 4b^2 + c^2 - 6ab + 2bc + 3ca) \\
&= 27a^3 + 12ab^2 + 3ac^2 - 18a^2b + 6abc + 9a^2c + 18a^2b + 8b^3 + 2bc^2 - 12ab^2 + 4b^2c + 6abc \\
&\quad - 9a^2c - 4b^2c - c^3 + 6abc - 2bc^2 - 3ac^2 \\
&= 27a^3 + 8b^3 - c^3 + 18abc
\end{aligned}$$

Answer 24L.

$$\begin{aligned}
& (3x + 5y + 2z)(3x - 5y + 2z) \\
&= (3x + 2z + 5y)(3x + 2z - 5y) \\
&= (3x + 2z)^2 - (5y)^2 \\
&= 9x^2 + 2(3x)(2z) + 4z^2 - 25y^2 \\
&= 9x^2 - 25y^2 + 4z^2 + 12xz
\end{aligned}$$

Answer 24M.

$$\begin{aligned}
& (2x - 4y + 7)(2x + 4y + 7) \\
&= (2x + 7 - 4y)(2x + 7 + 4y) \\
&= (2x + 7)^2 - (4y)^2 \\
&= 4x^2 + 2(2x)(7) + 7^2 - 16y^2 \\
&= 4x^2 - 16y^2 + 28x + 49
\end{aligned}$$

Answer 24N.

$$\begin{aligned}
& (3a - 7b + 3)(3a - 7b + 5) \\
&= 3a(3a - 7b + 5) - 7b(3a - 7b + 5) + 3(3a - 7b + 5) \\
&= 9a^2 - 21ab + 15a - 21ab + 49b^2 - 35b + 9a - 21b + 15 \\
&= 9a^2 - 42ab + 24a + 49b^2 - 56b + 15
\end{aligned}$$

Answer 24O.

$$\begin{aligned}
& (4m - 5n - 8)(4m - 5n + 5) \\
&= 4m(4m - 5n + 5) - 5n(4m - 5n + 5) - 8(4m - 5n + 5) \\
&= 16m^2 - 20mn + 20m - 20mn + 25n^2 - 25n - 32m + 40n - 40 \\
&= 16m^2 + 25n^2 - 40mn - 12m + 15n - 40
\end{aligned}$$

Answer 25.

$$\begin{aligned}
& \text{(i) } (3.29)^3 + (6.71)^3 \\
&= (3.29 + 6.71)^3 - 3(3.29)(6.71)(3.029 + 6.71) \\
&= (10)^3 - 3(3.29)(6.71)(10) \\
&= 1000 - 30(5 - 1.71)(5 + 1.71) \\
&= 1000 - 30(5)^2 - (1.71)^2 \\
&= 1000 - 30(25 - 2.9241) \\
&= 1000 - 30 \times 22.0759 \\
&= 1000 - 662.277 \\
&= 337.723
\end{aligned}$$

$$\text{(ii) } (5.45)^3 + (3.55)^3$$

$$\begin{aligned}
&= (5.45 + 3.55)^3 - 3(5.45)(3.55)(5.45 + 3.55) \\
&= (9)^3 - 3(4 + 1.45)(4 - 1.45)(9) \\
&= 81 - 3(16 - (1.45)^2)(9) \\
&= 81 - 27(16 - 2.1025) \\
&= 81 - 27 \times 13.8975 \\
&= 81 - 522.3825 = 206.6175
\end{aligned}$$

$$(iii) (8.12)^3 - (3.12)^3$$

$$= (8.12 - 3.12)^3 + 3(8.12)(3.12)(8.12 - 3.12)$$

$$= 5^3 + 3(8.12)(3.12) \times 5$$

$$= 125 + 15 \times (8.12)(3.12)$$

$$= 125 + 15 \times 25.3344$$

$$= 125 + 380.016 = 505.016$$

$$(iv) 7.16 \times 7.16 + 2.16 \times 7.16 + 2.16 \times 2.16$$

$$= (7.16)^2 + (2.16)(7.16) + (2.16)^2$$

$$= (7.16)^2 + (2.16)(7.16) + (2.16)^2 + (2.16)(7.16) - (2.16)(7.16)$$

$$= (7.16)^2 + 2(2.16)(7.16) + (2.16)^2 - (2.16)(7.16)$$

$$= (7.16 + 2.16)^2 - (2.16)(7.16)$$

$$= (9.32)^2 - 15.4656$$

$$= 86.8624 - 15.4656 = 71.3968$$

$$(v) 1.81 \times 1.81 - 1.81 \times 2.19 + 2.19 \times 2.19$$

$$\text{Sol: } 1.81 \times 1.81 - 1.81 \times 2.19 + 2.19 \times 2.19$$

$$= (1.81)^2 - (1.81 \times 2.19) + (2.19)^2$$

$$= (1.81)^2 - (1.81 \times 2.19) + (2.19)^2 - (1.81 \times 2.19) + (1.81 \times 2.19)$$

$$= (1.81)^2 - 2(1.81 \times 2.19) + (2.19)^2 + (1.81 \times 2.19)$$

$$= (1.81 - 2.19)^2 + (2.00 - 0.19)(2.00 + 0.19)$$

$$= (0.38)^2 + (4 - (0.19)^2)$$

$$= 0.1444 + (4 - 0.0361)$$

$$= 0.1444 + 3.9639 = 4.1083$$