

Chapter 5. Factorisation

Ex 5.1

Answer 1A.

$$4x^2y^3 - 6x^3y^2 - 12xy^2$$

Here, the common factor is $2xy^2$.

Dividing throughout by $2xy^2$, we get

$$\frac{4x^2y^3}{2xy^2} - \frac{6x^3y^2}{2xy^2} - \frac{12xy^2}{2xy^2}$$

$$= 2xy - 3x^2 - 6$$

$$\therefore 4x^2y^3 - 6x^3y^2 - 12xy^2 = 2xy^2(2xy - 3x^2 - 6)$$

Answer 1B.

$$5a(x^2 - y^2) + 35b(x^2 - y^2)$$

Here, the common factor is $5(x^2 - y^2)$.

Dividing throughout by $5(x^2 - y^2)$, we get

$$\frac{5a(x^2 - y^2)}{5(x^2 - y^2)} + \frac{35b(x^2 - y^2)}{5(x^2 - y^2)}$$

$$= a + 7b$$

$$\therefore 5a(x^2 - y^2) + 35b(x^2 - y^2) = 5(x^2 - y^2)(a + 7b)$$

Answer 1C.

$$2x^5y + 8x^3y^2 - 12x^2y^3$$

Here, the common factor is $2x^2y$.

Dividing throughout by $2x^2y$, we get

$$\frac{2x^5y}{2x^2y} + \frac{8x^3y^2}{2x^2y} - \frac{12x^2y^3}{2x^2y}$$

$$= x^3 + 4xy - 6y^2$$

$$\therefore 2x^5y + 8x^3y^2 - 12x^2y^3 = 2x^2y(x^3 + 4xy - 6y^2)$$

Answer 1D.

$$12a^3 + 15a^2b - 21ab^2$$

Here, the common factor is $3a$.

Dividing throughout by $3a$, we get

$$\frac{12a^3}{3a} + \frac{15a^2b}{3a} - \frac{21ab^2}{3a}$$

$$= 4a^2 + 5ab - 7b^2$$

$$\therefore 12a^3 + 15a^2b - 21ab^2 = 3a(4a^2 + 5ab - 7b^2)$$

Answer 1E.

$$24m^4n^6 + 56m^6n^4 - 72m^2n^2$$

Here, the common factor is $8m^2n^2$.

Dividing throughout by $8m^2n^2$, we get

$$\frac{24m^4n^6}{8m^2n^2} + \frac{56m^6n^4}{8m^2n^2} - \frac{72m^2n^2}{8m^2n^2}$$

$$= 3m^2n^4 + 7m^4n^2 - 9$$

$$\therefore 24m^4n^6 + 56m^6n^4 - 72m^2n^2 = 8m^2n^2(3m^2n^4 + 7m^4n^2 - 9)$$

Answer 1F.

$$(a-b)^2 - 2(a-b)$$

Here, the common factor is $(a-b)$.

Dividing throughout by $(a-b)$, we get

$$\frac{(a-b)^2}{(a-b)} - \frac{2(a-b)}{(a-b)}$$

$$= a - b - 2$$

$$\therefore (a-b)^2 - 2(a-b) = (a-b)(a-b-2)$$

Answer 1G.

$$2a(p^2 + q^2) + 4b(p^2 + q^2)$$

Here, the common factor is $2(p^2 + q^2)$.

Dividing throughout by $2(p^2 + q^2)$, we get

$$\frac{2a(p^2 + q^2)}{2(p^2 + q^2)} + \frac{4b(p^2 + q^2)}{2(p^2 + q^2)}$$

$$= a + 2b$$

$$\therefore 2a(p^2 + q^2) + 4b(p^2 + q^2) = 2(p^2 + q^2)(a + 2b)$$

Answer 1H.

$$81(p + q)^2 - 9p - 9q$$

$$= 81(p + q)^2 - 9(p + q)$$

Here, the common factor is $9(p + q)$

Dividing throughout by $9(p + q)$, we get

$$\frac{81(p + q)^2}{9(p + q)} - \frac{9(p + q)}{9(p + q)}$$

$$= 9(p + q) - 1$$

$$\therefore 81(p + q)^2 - 9p - 9q = 9(p + q)[9(p + q) - 1]$$

Answer 1I.

$$(mx + ny)^2 + (nx - my)^2$$

$$= m^2x^2 + n^2y^2 + 2mnxy + n^2x^2 + m^2y^2 - 2mnxy$$

$$= m^2x^2 + n^2y^2 + n^2x^2 + m^2y^2$$

$$= m^2x^2 + n^2x^2 + m^2y^2 + n^2y^2$$

$$= x^2(m^2 + n^2) + y^2(m^2 + n^2)$$

Here, the common factor is $(m^2 + n^2)$.

Dividing throughout by $(m^2 + n^2)$, we get

$$\frac{x^2(m^2 + n^2)}{(m^2 + n^2)} + \frac{y^2(m^2 + n^2)}{(m^2 + n^2)}$$

$$= x^2 + y^2$$

$$\therefore (mx + ny)^2 + (nx - my)^2 = (m^2 + n^2)(x^2 + y^2)$$

Answer 1J.

$$36(x + y)^3 - 54(x + y)^2$$

Here, the common factor is $18(x + y)^2$.

Dividing throughout by $18(x + y)^2$, we get

$$\frac{36(x + y)^3}{18(x + y)^2} - \frac{54(x + y)^2}{18(x + y)^2}$$

$$= 2(x + y) - 3$$

$$\therefore 36(x + y)^3 - 54(x + y)^2 = 18(x + y)^2[2(x + y) - 3]$$

Answer 1K.

$$\begin{aligned}
& p(p^2 + q^2 - r^2) + q(r^2 - q^2 - p^2) - r(p^2 + q^2 - r^2) \\
&= p(p^2 + q^2 - r^2) - q(r^2 + q^2 + p^2) - r(p^2 + q^2 - r^2) \\
&= p(p^2 + q^2 - r^2) - q(p^2 + q^2 - r^2) - r(p^2 + q^2 - r^2)
\end{aligned}$$

Here, the common factor is $(p^2 + q^2 - r^2)$.

Dividing throughout by $(p^2 + q^2 - r^2)$, we get

$$\begin{aligned}
& \frac{p(p^2 + q^2 - r^2)}{(p^2 + q^2 - r^2)} - \frac{q(p^2 + q^2 - r^2)}{(p^2 + q^2 - r^2)} - \frac{r(p^2 + q^2 - r^2)}{(p^2 + q^2 - r^2)} \\
&= p - q - r
\end{aligned}$$

$$\therefore p(p^2 + q^2 - r^2) + q(r^2 - q^2 - p^2) - r(p^2 + q^2 - r^2) = (p^2 + q^2 - r^2)(p - q - r)$$

Answer 2A.

$$\begin{aligned}
& 15xy - 9x - 25y + 15 \\
&= (15xy - 9x) - (25y + 15) \\
&= 3x(5y - 3) - 5(5y - 3) \\
&= (5y - 3)(3x - 5)
\end{aligned}$$

Answer 2B.

$$\begin{aligned}
& 15x^2 + 7y - 3x - 35xy \\
&= 15x^2 - 3x - 35xy + 7y \\
&= (15x^2 - 3x) - (35xy - 7y) \\
&= 3x(5x - 1) - 7y(5x - 1) \\
&= (5x - 1)(3x - 7y)
\end{aligned}$$

Answer 2C.

$$\begin{aligned}
& 9 + 3xy + x^2y + 3x \\
&= 9 + 3xy + 3x + x^2y \\
&= (9 + 3xy) + (3x + x^2y) \\
&= 3(3 + xy) + y(3 + xy) \\
&= (3 + xy)(3 + x)
\end{aligned}$$

Answer 2D.

$$\begin{aligned}
& 8(2a + b)^2 - 8a - 4b \\
&= 8(2a + b)^2 - (8a + 4b) \\
&= 8(2a + b)^2 - 4(2a + b) \\
&= 4(2a + b)[2(2a + b) - 1] \\
&= 4(2a + b)[4a + 2b - 1]
\end{aligned}$$

Answer 2E.

$$\begin{aligned} & x(x - 4) - x + 4 \\ &= x(x - 4) - 1(x - 4) \\ &= (x - 4)(x - 1) \end{aligned}$$

Answer 2F.

$$\begin{aligned} & 2m^3 - 5n^2 - 5m^2n + 2mn \\ &= 2m^3 + 2mn - 5m^2n - 5n^2 \\ &= (2m^3 + 2mn) - (5m^2n + 5n^2) \\ &= 2m(m^2 + n) - 5n(m^2 + n) \\ &= (m^2 + n)(2m - 5n) \end{aligned}$$

Answer 2H.

$$\begin{aligned} & 9x^3 + 6x^2y^2 - 4y^3 - 6xy \\ &= 9x^3 + 6x^2y^2 - 6xy - 4y^3 \\ &= (9x^3 + 6x^2y^2) - (6xy + 4y^3) \\ &= 3x^2(3x + 2y^2) - 2y(3x + 2y^2) \\ &= (3x + 2y^2)(3x^2 - 2y) \end{aligned}$$

Answer 2I.

$$\begin{aligned} & 3ax^2 - 5bx^2 + 9az^2 + 6ay^2 - 10by^2 - 15bz^2 \\ &= 3ax^2 + 6ay^2 + 9az^2 - 5bx^2 - 10by^2 - 15bz^2 \\ &= (3ax^2 + 6ay^2 + 9az^2) - (5bx^2 + 10by^2 + 15bz^2) \\ &= 3a(x^2 + 2y^2 + 3z^2) - 5b(x^2 + 2y^2 + 3z^2) \\ &= (x^2 + 2y^2 + 3z^2)(3a - 5b) \end{aligned}$$

Answer 2J.

$$\begin{aligned} & 8x^3 - 24x^2y + 54xy^2 - 162y^3 \\ &= (8x^3 - 24x^2y) + (54xy^2 - 162y^3) \\ &= 8x^2(x - 3y) + 54y^2(x - 3y) \\ &= (x - 3y)(8x^2 + 54y^2) \end{aligned}$$

Answer 2K.

$$\begin{aligned} & 2a + b + 3c - d + (2a + b)^3 + (2a + b)^2(3c - d) \\ &= (2a + b + 3c - d) + [(2a + b)^3 + (2a + b)^2(3c - d)] \\ &= 1(2a + b + 3c - d) + (2a + b)^2(2a + b + 3c - d) \\ &= (2a + b + 3c - d)[1 + (2a + b)^2] \end{aligned}$$

Answer 2L.

$$\begin{aligned} & xy(a^2 + 1) + a(x^2 + y^2) \\ &= a^2xy + xy + ax^2 + ay^2 \\ &= (a^2xy + ax^2) + (ay^2 + xy) \\ &= ax(ay + x) + y(ay + x) \\ &= (ay + x)(ax + y) \end{aligned}$$

Answer 2M.

$$\begin{aligned} & p^2x^2 + (px^2 + 1)x + p \\ &= p^2x^2 + px^3 + x + p \\ &= (p^2x^2 + px^3) + (p + x) \\ &= px^2(p + x) + 1(p + x) \\ &= (p + x)(px^2 + 1) \end{aligned}$$

Answer 2N.

$$\begin{aligned} & x^2 - (p + q)x + pq \\ &= x^2 - px - qx + pq \\ &= (x^2 - px) - (qx - pq) \\ &= x(x - p) - q(x - p) \\ &= (x - p)(x - q) \end{aligned}$$

Answer 2O.

$$\begin{aligned}
& p^2 + \frac{1}{p^2} - 2 - 5p + \frac{5}{p} \\
&= \left(p^2 + \frac{1}{p^2} - 2 \right) - \left(5p - \frac{5}{p} \right) \\
&= \left((p)^2 + \left(\frac{1}{p} \right)^2 - 2 \times p \times \frac{1}{p} \right) - \left(5p - \frac{5}{p} \right) \\
&= \left(p - \frac{1}{p} \right)^2 - 5 \left(p - \frac{1}{p} \right) \\
&= \left(p - \frac{1}{p} \right) \left(p - \frac{1}{p} - 5 \right)
\end{aligned}$$

Answer 2P.

$$\begin{aligned}
& x + y + m(x + y) \\
&= (x + y) + m(x + y) \\
&= (x + y)(1 + m)
\end{aligned}$$

Answer 2Q.

$$\begin{aligned}
& \frac{1}{25x^2} + 16x^2 + \frac{8}{5} - 12x - \frac{3}{5x} \\
&= \left(\frac{1}{25x^2} + 16x^2 + \frac{8}{5} \right) - \left(12x + \frac{3}{5x} \right) \\
&= \left(\left(\frac{1}{5x} \right)^2 + (4x)^2 + 2 \times \frac{1}{5x} \times 4x \right) - \left(12x + \frac{3}{5x} \right) \\
&= \left(\frac{1}{5x} + 4x \right)^2 - 3 \left(4x + \frac{1}{5x} \right) \\
&= \left(\frac{1}{5x} + 4x \right)^2 - 3 \left(\frac{1}{5x} + 4x \right) \\
&= \left(\frac{1}{5x} + 4x \right) \left(\frac{1}{5x} + 4x - 3 \right)
\end{aligned}$$

Answer 2R.

$$\begin{aligned}
& 2p(a^2 - 2b^2) - 14p + (a^2 - 2b^2)^2 - 7(a^2 - 2b^2) \\
&= 2p(a^2 - 2b^2) + (a^2 - 2b^2)^2 - 14p - 7(a^2 - 2b^2) \\
&= [2p(a^2 - 2b^2) + (a^2 - 2b^2)^2] - [14p + 7(a^2 - 2b^2)] \\
&= (a^2 - 2b^2)(2p + a^2 - 2b^2) - 7(2p + a^2 - 2b^2) \\
&= (2p + a^2 - 2b^2)(a^2 - 2b^2 - 7)
\end{aligned}$$

Ex 5.2

Answer 1Q.

$$\begin{aligned}x^2 + 6x + 8 \\&= x^2 + 4x + 2x + 8 \\&= x(x + 4) + 2(x + 4) \\&= (x + 4)(x + 2)\end{aligned}$$

Answer 1B.

$$\begin{aligned}x^2 - 11x + 24 \\&= x^2 - 8x - 3x + 24 \\&= x(x - 8) - 3(x - 8) \\&= (x - 8)(x - 3)\end{aligned}$$

Answer 1C.

$$\begin{aligned}x^2 + 5x - 6 \\&= x^2 + 6x - x - 6 \\&= x(x + 6) - 1(x + 6) \\&= (x + 6)(x - 1)\end{aligned}$$

Answer 1D.

$$\begin{aligned}p^2 - 12p - 64 \\&= p^2 - 16p + 4p - 64 \\&= p(p - 16) + 4(p - 16) \\&= (p - 16)(p + 4)\end{aligned}$$

Answer 1E.

$$\begin{aligned}y^2 - 2y - 24 \\&= y^2 - 6y + 4y - 24 \\&= y(y - 6) + 4(y - 6) \\&= (y - 6)(y + 4)\end{aligned}$$

Answer 1F.

$$\begin{aligned} & 3x^2 + 19x - 14 \\ &= 3x^2 + 21x - 2x - 14 \\ &= 3x(x + 7) - 2(x + 7) \\ &= (x + 7)(3x - 2) \end{aligned}$$

Answer 1G.

$$\begin{aligned} & 15a^2 - 14a - 16 \\ &= 15a^2 - 24a + 10a - 16 \\ &= 3a(5a - 8) + 2(5a - 8) \\ &= (5a - 8)(3a + 2) \end{aligned}$$

Answer 1H.

$$\begin{aligned} & 12 + x - 6x^2 \\ &= 12 + 9x - 8x - 6x^2 \\ &= 3(4 + 3x) - 2x(4 + 3x) \\ &= (4 + 3x)(3 - 2x) \end{aligned}$$

Answer 1I.

$$\begin{aligned} & 7x^2 + 40x - 12 \\ &= 7x^2 + 42x - 2x - 12 \\ &= 7x(x + 6) - 2(x + 6) \\ &= (x + 6)(7x - 2) \end{aligned}$$

Answer 2A.

$$\begin{aligned} & 5x^2 - 17xy + 6y^2 \\ &= 5x^2 - 15xy - 2xy + 6y^2 \\ &= 5x(x - 3y) - 2y(x - 3y) \\ &= (x - 3y)(5x - 2y) \end{aligned}$$

Answer 2B.

$$\begin{aligned} & 9x^2 - 22xy + 8y^2 \\ &= 9x^2 - 18xy - 4xy + 8y^2 \\ &= 9x(x - 2y) - 4y(x - 2y) \\ &= (x - 2y)(9x - 4y) \end{aligned}$$

Answer 2C.

$$\begin{aligned} & 2x^3 + 5x^2y - 12xy^2 \\ &= 2x^3 + 8x^2y - 3x^2y - 12xy^2 \\ &= 2x^2(x + 4y) - 3xy(x + 4y) \\ &= (x + 4y)(2x^2 - 3xy) \\ &= (x + 4y)x(2x - 3y) \\ &= x(x + 4y)(2x - 3y) \end{aligned}$$

Answer 2D.

$$\begin{aligned} & x^2y^2 + 15xy - 16 \\ &= x^2y^2 + 16xy - xy - 16 \\ &= xy(xy + 16) - 1(xy + 16) \\ &= (xy + 16)(xy - 1) \end{aligned}$$

Answer 2E.

$$\begin{aligned} & (2p + q)^2 - 10p - 5q - 6 \\ &= (2p + q)^2 - (10p - 5q) - 6 \\ &= (2p + q)^2 - 5(2p + q) - 6 \\ &= (2p + q)^2 - 6(2p + q) + (2p + q) - 6 \\ &= (2p + q)(2p + q - 6) + 1(2p + q - 6) \\ &= (2p + q - 6)(2p + q + 1) \end{aligned}$$

Answer 2F.

$$\begin{aligned} & y^2 + 3y + 2 + by + 2b \\ &= y^2 + y + 2y + 2 + by + 2b \\ &= y^2 + y + by + 2y + 2 + 2b \\ &= y(y + 1 + b) + 2(y + 1 + b) \\ &= (y + 1 + b)(y + 2) \end{aligned}$$

Answer 2G.

$$\begin{aligned} & x^3y^3 - 8x^2y^2 + 15xy \\ &= x^3y^3 - 3x^2y^2 - 5x^2y^2 + 15xy \\ &= x^2y^2(xy - 3) - 5xy(xy - 3) \\ &= (xy - 3)(x^2y^2 - 5xy) \\ &= (xy - 3)xy(xy - 5) \\ &= xy(xy - 3)(xy - 5) \end{aligned}$$

Answer 2H.

$$\begin{aligned} & 6\sqrt{3}x^2 - 19x + 5\sqrt{3} \\ &= 6\sqrt{3}x^2 - 10x - 9x + 5\sqrt{3} \\ &= 2x(3\sqrt{3}x - 5) - \sqrt{3}(3\sqrt{3}x - 5) \\ &= (3\sqrt{3}x - 5)(2x - \sqrt{3}) \end{aligned}$$

Answer 2I.

$$\begin{aligned} & 2\sqrt{5}x^2 - 7x - 3\sqrt{5} \\ &= 2\sqrt{5}x^2 - 10x + 3x - 3\sqrt{5} \\ &= 2\sqrt{5}x(x - \sqrt{5}) + 3(x - \sqrt{5}) \\ &= (x - \sqrt{5})(2\sqrt{5}x + 3) \end{aligned}$$

Answer 3A.

$$\begin{aligned} & 5(3x + y)^2 + 6(3x + y) - 8 \\ &= 5(3x + y)^2 + 10(3x + y) - 4(3x + y) - 8 \\ &= 5(3x + y)(3x + y + 2) - 4(3x + y + 2) \\ &= (3x + y + 2)[5(3x + y) - 4] \end{aligned}$$

Answer 3B.

$$\begin{aligned} & 5 - 4(a - b) - 12(a - b)^2 \\ &= 5 - 10(a - b) + 6(a - b) - 12(a - b)^2 \\ &= 5[1 - 2(a - b)] + 6(a - b)[1 - 2(a - b)] \\ &= [5 + 6(a - b)][1 - 2(a - b)] \\ &= (5 + 6a - 6b)(1 - 2a + 2b) \end{aligned}$$

Answer 3C.

$$\begin{aligned}(3a - 2b)^2 + 3(3a - 2b) - 10 \\&= (3a - 2b)^2 + 5(3a - 2b) - 2(3a - 2b) - 10 \\&= (3a - 2b)(3a - 2b + 5) - 2(3a - 2b + 5) \\&= (3a - 2b + 5)((3a - 2b - 2))\end{aligned}$$

Answer 3D.

$$\begin{aligned}(a^2 - 2a)^2 - 23(a^2 - 2a) + 120 \\&= (a^2 - 2a)^2 - 15(a^2 - 2a) - 8(a^2 - 2a) + 120 \\&= (a^2 - 2a)(a^2 - 2a - 15) - 8(a^2 - 2a - 15) \\&= (a^2 - 2a - 15)(a^2 - 2a - 8) \\&= (a^2 - 5a + 3a - 15)(a^2 - 4a + 2a - 8) \\&= [a(a - 5) + 3(a - 5)][a(a - 4) + 2(a - 4)] \\&= [(a - 5)(a + 3)][(a - 4)(a + 2)] \\&= (a - 5)(a + 3)(a - 4)(a + 2) \\&= (a + 2)(a + 3)(a - 4)(a - 5)\end{aligned}$$

Answer 3E.

$$\begin{aligned}(x + 4)^2 - 5xy - 20y - 6y^2 \\&= (x + 4)^2 - 5y(x + 4) - 6y^2 \\&= (x + 4)^2 - 6y(x + 4) + y(x + 4) - 6y^2 \\&= (x + 4)(x + 4 - 6y) + y(x + 4 - 6y) \\&= (x + 4 - 6y)(x + 4 + y) \\&= (x - 6y + 4)(x + y + 4)\end{aligned}$$

Answer 3F.

$$\begin{aligned}7(x - 2)^2 - 13(x - 2) - 2 \\&= 7(x - 2)^2 - 14(x - 2) + (x - 2) - 2 \\&= 7(x - 2)(x - 2 - 2) + 1(x - 2 - 2) \\&= 7(x - 2)(x - 4) + 1(x - 4) \\&= (x - 4)[7(x - 2) + 1] \\&= (x - 4)(7x - 14 + 1) \\&= (x - 4)(7x - 13)\end{aligned}$$

Answer 3G.

$$\begin{aligned}
& 12 - (y + y^2)(8 - y - y^2) \\
&= 12 - a(8 - a) \text{ [Taking } y + y^2 = a\text{]} \\
&= 12 - 8a + a^2 \\
&= 12 - 6a - 2a + a^2 \\
&= 6(2 - a) - a(2 - a) \\
&= (2 - a)(6 - a) \\
&= [2 - (y + y^2)][6 - (y + y^2)] \\
&= (2 - y - y^2)(6 - y - y^2) \\
&= (2 - 2y + y - y^2)(6 - 3y + 2y - y^2) \\
&= [2(1 - y) + y(1 - y)][3(2 - y) + y(2 - y)] \\
&= [(1 - y)(2 + y)][(2 - y)(3 + y)] \\
&= (1 - y)(2 + y)(2 - y)(3 + y) \\
&= (y - 1)(y + 2)(y - 2)(y + 3)
\end{aligned}$$

Answer 3H.

$$\begin{aligned}
& (p^2 + p)^2 - 8(p^2 + p) + 12 \\
&= (p^2 + p)^2 - 6(p^2 + p) - 2(p^2 + p) + 12 \\
&= (p^2 + p)(p^2 + p - 6) - 2(p^2 + p - 6) \\
&= (p^2 + p - 6)(p^2 + p - 2) \\
&= (p^2 + 3p - 2p - 6)(p^2 + 2p - p - 2) \\
&= [p(p + 3) - 2(p + 3)][p(p + 2) - 1(p + 2)] \\
&= [(p + 3)(p - 2)][(p + 2)(p - 1)] \\
&= (p + 3)(p - 2)(p + 2)(p - 1)
\end{aligned}$$

Answer 4A.

$$\begin{aligned}
& (y^2 - 3y)(y^2 - 3y + 7) + 10 \\
&= a(a + 7) + 10 \text{ [taking } (y^2 - 3y) = a\text{]} \\
&= a^2 + 7a + 10 \\
&= a^2 + 5a + 2a + 10 \\
&= a(a + 5) + 2(a + 5) \\
&= (a + 5)(a + 2) \\
&= (y^2 - 3y + 5)(y^2 - 3y + 2) \\
&= (y^2 - 3y + 5)(y^2 - 2y - y + 2) \\
&= (y^2 - 3y + 5)[y(y - 2) - 1(y - 2)] \\
&= (y^2 - 3y + 5)[(y - 2)(y - 1)] \\
&= (y - 1)(y - 2)(y^2 - 3y + 5)
\end{aligned}$$

Answer 4B.

$$\begin{aligned} & (t^2 - t)(4t^2 - 4t - 5) - 6 \\ &= (t^2 - t)[4(t^2 - t) - 5] - 6 \\ &= a[4a - 5] - 6 \text{ [Taking } (t^2 - t) = a] \\ &= 4a^2 - 5a - 6 \\ &= 4a^2 - 8a + 3a - 6 \\ &= 4a(a - 2) + 3(a - 2) \\ &= (a - 2)(4a + 3) \\ &= (t^2 - t - 2)[4(t^2 - t) + 3] \\ &= (t^2 - 2t + t - 2)(4t^2 - 4t + 3) \\ &= [t(t - 2) + 1(t - 2)](4t^2 - 4t + 3) \\ &= [(t - 2)(t + 1)](4t^2 - 4t + 3) \\ &= (t + 1)(t - 2)(4t^2 - 4t + 3) \end{aligned}$$

Answer 4C.

$$\begin{aligned} & 12(2x - 3y)^2 - 1(2x - 3y) - 1 \\ &= 12a^2 - a - 1 \text{ [Taking } (2x - 3y) = a] \\ &= 12a^2 - 4a + 3a - 1 \\ &= 4a(3a - 1) + 1(3a - 1) \\ &= (3a - 1)(4a + 1) \\ &= [3(2x - 3y) - 1][4(2x - 3y) + 1] \\ &= (6x - 9y - 1)(8x - 12y + 1) \end{aligned}$$

Answer 4D.

$$\begin{aligned} & 6 - 5x + 5y + (x - y)^2 \\ &= 6 - 5(x - y) + (x - y)^2 \\ &= 6 - 3(x - y) - 2(x - y) + (x - y)^2 \\ &= 3[2 - (x - y)] - (x - y)[2 - (x - y)] \\ &= 3(2 - x + y) - (x - y)(2 - x + y) \\ &= (2 - x + y)(3 - x + y) \end{aligned}$$

Answer 4E.

$$\begin{aligned} & 2x^2 + \frac{x}{6} - 1 \\ &= \frac{1}{6}(12x^2 + x - 6) \\ &= \frac{1}{6}(12x^2 + 9x - 8x - 6) \\ &= \frac{1}{6}[3x(4x + 3) - 2(4x + 3)] \\ &= \frac{1}{6}[(4x + 3)(3x - 2)] \\ &= \frac{1}{6}(4x + 3)(3x - 2) \end{aligned}$$

Answer 4F.

$$\begin{aligned} & p^4 + 23p^2q^2 + 90q^4 \\ &= p^4 + 18p^2q^2 + 5p^2q^2 + 90q^4 \\ &= p^2(p^2 + 18q^2) + 5q^2(p^2 + 18q^2) \\ &= (p^2 + 18q^2)(p^2 + 5q^2) \end{aligned}$$

Answer 4G.

$$\begin{aligned} & 2a^3 + 5a^2b - 12ab^2 \\ &= 2a^3 + 8a^2b - 3a^2b - 12ab^2 \\ &= 2a^2(a + 4b) - 3ab(a + 4b) \\ &= (a + 4b)(2a^2 - 3ab) \\ &= (a + 4b)a(2a - 3b) \\ &= a(a + 4b)(2a - 3b) \end{aligned}$$

Ex 5.3

Answer 1A.

$$\begin{aligned}x^2 - 16 \\&= x^2 - 4^2 \\&= (x - 4)(x + 4)\end{aligned}$$

Answer 1B.

$$\begin{aligned}64x^2 - 121y^2 \\&= (8x)^2 - (11y)^2 \\&= (8x - 11y)(8x + 11y)\end{aligned}$$

Answer 1C.

$$\begin{aligned}441 - 81y^2 \\&= (21)^2 - (9y)^2 \\&= (21 - 9y)(21 + 9y) \\&= 3(7 - 3y)3(7 + 3y) \\&= 9(7 - 3y)(7 + 3y)\end{aligned}$$

Answer 1D.

$$\begin{aligned}x^6 - 196 \\&= (x^3)^2 - (14)^2 \\&= (x^3 - 14)(x^3 + 14)\end{aligned}$$

Answer 1E.

$$\begin{aligned}625 - b^2 \\&= (25)^2 - (b)^2 \\&= (25 - b)(25 + b)\end{aligned}$$

Answer 1F.

$$\begin{aligned} & m^2 - \frac{1}{9}n^2 \\ &= m^2 - \left(\frac{1}{3}n\right)^2 \\ &= \left(m - \frac{1}{3}n\right)\left(m + \frac{1}{3}n\right) \end{aligned}$$

Answer 1G.

$$\begin{aligned} & 8xy^2 - 18x^3 \\ &= 2x(4y^2 - 9x^2) \\ &= 2x[(2y)^2 - (3x)^2] \\ &= 2x[(2y - 3x)(2y + 3x)] \\ &= 2x(2y - 3x)(2y + 3x) \end{aligned}$$

Answer 1H.

$$\begin{aligned} & 16a^4 - 81b^4 \\ &= (4a^2)^2 - (9b^2)^2 \\ &= (4a^2 - 9b^2)(4a^2 + 9b^2) \\ &= [(2a)^2 - (3b)^2](4a^2 + 9b^2) \\ &= [(2a - 3b)(2a + 3b)](4a^2 + 9b^2) \\ &= (2a - 3b)(2a + 3b)(4a^2 + 9b^2) \end{aligned}$$

Answer 1I.

$$\begin{aligned} & a(a - 1) - b(b - 1) \\ &= a^2 - a - b^2 + b \\ &= a^2 - b^2 - a + b \\ &= (a^2 - b^2) - (a - b) \\ &= (a - b)(a + b) - (a - b) \\ &= (a - b)(a + b - 1) \end{aligned}$$

Answer 1J.

$$\begin{aligned} & (x + y)^2 - 1 \\ &= (x + y)^2 - (1)^2 \\ &= (x + y + 1)(x + y - 1) \end{aligned}$$

Answer 1K.

$$\begin{aligned} & x^2 + y^2 - z^2 - 2xy \\ &= x^2 + y^2 - 2xy - z^2 \\ &= (x^2 + y^2 - 2xy) - z^2 \\ &= (x - y)^2 - (z)^2 \\ &= (x - y - z)(x - y + z) \end{aligned}$$

Answer 1L.

$$\begin{aligned} & (x - 2y)^2 - z^2 \\ &= (x - 2y)^2 - (z)^2 \\ &= (x - 2y - z)(x - 2y + z) \end{aligned}$$

Answer 2A.

$$\begin{aligned} & 9(a - b)^2 - (a + b)^2 \\ &= [3(a - b)]^2 - (a + b)^2 \\ &= [3(a - b) - (a + b)][3(a - b) + (a + b)] \\ &= (3a - 3b - a - b)(3a - 3b + a + b) \\ &= (2a - 4b)(4a - 2b) \\ &= 2(a - 2b)2(2a - b) \\ &= 4(a - 2b)(2a - b) \end{aligned}$$

Answer 2B.

$$\begin{aligned} & 25(x - y)^2 - 49(c - d)^2 \\ &= [5(x - y)]^2 - [7(c - d)]^2 \\ &= [5(x - y) - 7(c - d)][5(x - y) + 7(c - d)] \\ &= (5x - 5y - 7c + 7d)(5x - 5y + 7c - 7d) \end{aligned}$$

Answer 2C.

$$\begin{aligned} & (2a - b)^2 - 9(3c - d)^2 \\ &= (2a - b)^2 - [3(3c - d)]^2 \\ &= [(2a - b) - 3(3c - d)][(2a - b) + 3(3c - d)] \\ &= (2a - b - 9c + 3d)(2a - b + 9c - 3d) \end{aligned}$$

Answer 2D.

$$\begin{aligned}
& b^2 - 2bc + c^2 - a^2 \\
&= (b^2 - 2bc + c^2) - a^2 \\
&= (b - c)^2 - (a)^2 \\
&= (b - c - a)(b - c + a)
\end{aligned}$$

Answer 2E.

$$\begin{aligned}
& x^2 + \frac{1}{x^2} - 2 \\
&= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} \\
&= \left(x - \frac{1}{x}\right)^2 \\
&= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x}\right)
\end{aligned}$$

Answer 2F.

$$\begin{aligned}
& (x^2 + y^2 - z^2)^2 - 4x^2y^2 \\
&= (x^2 + y^2 - z^2)^2 - (2xy)^2 \\
&= (x^2 + y^2 - z^2 - 2xy)(x^2 + y^2 - z^2 + 2xy) \\
&= [(x^2 + y^2 - 2xy) - z^2][(x^2 + y^2 + 2xy) - z^2] \\
&= [(x - y)^2 - z^2][(x + y)^2 - z^2] \\
&= [(x - y - z)(x - y + z)][(x + y - z)(x + y + z)] \\
&= (x - y - z)(x - y + z)(x + y - z)(x + y + z)
\end{aligned}$$

Answer 2G.

$$\begin{aligned}
& a^2 + b^2 - c^2 - d^2 + 2ab - 2cd \\
&= (a^2 + b^2 + 2ab) - (c^2 + d^2 + 2cd) \\
&= (a + b)^2 - (c + d)^2 \\
&= (a + b + c + d)(a + b - c - d)
\end{aligned}$$

Answer 2H.

$$\begin{aligned}
& 4xy - x^2 - 4y^2 + z^2 \\
&= z^2 - x^2 - 4y^2 + 4xy \\
&= z^2 - (x^2 + 4y^2 - 4xy) \\
&= z^2 - (x - 2y)^2 \\
&= [z - (x - 2y)][z + (x - 2y)] \\
&= (z - x + 2y)(z + x - 2y)
\end{aligned}$$

Answer 2I.

$$\begin{aligned}
& 4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2 \\
&= (4x^2 - 12ax + 9a^2) - (y^2 + z^2 + 2yz) \\
&= (2x - 3a)^2 - (y + z)^2 \\
&= [(2x - 3a) + (y + z)][(2x - 3a) - (y + z)] \\
&= (2x - 3a + y + z)(2x - 3a - y - z)
\end{aligned}$$

Answer 2J.

$$\begin{aligned}
& (x + y)^3 - x - y \\
&= (x + y)(x + y)^2 - (x + y) \\
&= (x + y)[(x + y)^2 - 1] \\
&= (x + y)[(x + y + 1)(x + y - 1)] \\
&= (x + y)(x + y + 1)(x + y - 1)
\end{aligned}$$

Answer 2K.

$$\begin{aligned}
& y^4 + y^2 + 1 \\
&= y^4 + 2y^2 + 1 - y^2 \\
&= (y^2 + 1)^2 - y^2 \\
&= (y^2 + 1 + y)(y^2 + 1 - y)
\end{aligned}$$

Answer 2L.

$$\begin{aligned}
& (a^2 - b^2)(c^2 - d^2) - 4abcd \\
&= a^2c^2 - a^2d^2 - b^2c^2 + b^2d^2 - 4abcd \\
&= a^2c^2 + b^2d^2 - 2abcd - a^2d^2 - b^2c^2 - 2abcd \\
&= (a^2c^2 + b^2d^2 - 2abcd) - (a^2d^2 + b^2c^2 + 2abcd) \\
&= (ac - bd)^2 - (ad + bc)^2 \\
&= [(ac - bd) + (ad + bc)][(ac - bd) - (ad + bc)] \\
&= (ac - bd + ad + bc)(ac - bd - ad - bc)
\end{aligned}$$

Answer 3A.

$$\begin{aligned}
& (x^2 - 2x + 3)(x^2 + 2x + 3) \\
&= [x^2 + 3 - 2x][x^2 + 3 + 2x] \\
&= [(x^2 + 3) - 2x][(x^2 + 3) + 2x] \\
&= (x^2 + 3)^2 - (2x)^2 \\
&= (x^2 + 3)^2 - 4x^2
\end{aligned}$$

Answer 3B.

$$\begin{aligned}& (x^2 - 2x + 3)(x^2 - 2x - 3) \\&= [(x^2 - 2x) + 3][(x^2 - 2x) - 3] \\&= (x^2 - 2x)^2 - (3)^2 \\&= (x^2 - 2x)^2 - 9\end{aligned}$$

Answer 3C.

$$\begin{aligned}& (x^2 + 2x - 3)(x^2 - 2x + 3) \\&= [x^2 + (2x - 3)][x^2 - (2x - 3)] \\&= x^2 - (2x - 3)^2 \\&= x^4 - (2x - 3)^2\end{aligned}$$

Answer 4A.

$$\begin{aligned}& y^2 + \frac{1}{4y^2} + 1 - 6y - \frac{3}{y} \\&= \left(y^2 + \frac{1}{4y^2} + 1\right) - \left(6y + \frac{3}{y}\right) \\&= \left(y + \frac{1}{2y}\right)^2 - 6\left(y + \frac{1}{2y}\right) \\&= \left(y + \frac{1}{2y}\right)\left(y + \frac{1}{2y} - 6\right)\end{aligned}$$

Answer 4B.

$$\begin{aligned}& 4a^2 + \frac{1}{4a^2} - 2 - 6a + \frac{3}{2a} \\&= \left(4a^2 + \frac{1}{4a^2} - 2\right) - \left(6a - \frac{3}{2a}\right) \\&= \left(2a - \frac{1}{2a}\right)^2 - 3\left(2a - \frac{1}{2a}\right) \\&= \left(2a - \frac{1}{2a}\right)\left(2a - \frac{1}{2a} - 3\right)\end{aligned}$$

Answer 4C.

$$\begin{aligned} & x^4 + y^4 - 6x^2y^2 \\ &= (x^2)^2 + (y^2)^2 - 2x^2y^2 - 4x^2y^2 \\ &= \left[(x^2)^2 + (y^2)^2 - 2x^2y^2 \right] - (4x^2y^2) \\ &= (x^2 - y^2)^2 - (2xy)^2 \\ &= (x^2 - y^2 - 2xy)(x^2 - y^2 + 2xy) \end{aligned}$$

Answer 4D.

$$\begin{aligned} & 4x^4 + 25y^4 + 19x^2y^2 \\ &= 4x^4 + 25y^4 + 20x^2y^2 - x^2y^2 \\ &= (2x^2)^2 + (5y^2)^2 + 2 \times (2x^2) \times (5y^2) - x^2y^2 \\ &= \left[(2x^2)^2 + (5y^2)^2 + 2 \times (2x^2) \times (5y^2) \right] - x^2y^2 \\ &= [2x^2 + 5y^2]^2 - (xy)^2 \\ &= (2x^2 + 5y^2 - xy)(2x^2 + 5y^2 + xy) \end{aligned}$$

Answer 4E.

$$\begin{aligned} & p^2 + \frac{1}{p^2} - 3 \\ &= p^2 + \frac{1}{p^2} - 2 - 1 \\ &= \left(p^2 + \frac{1}{p^2} - 2 \times p \times \frac{1}{p} \right) - 1 \\ &= \left(p - \frac{1}{p} \right)^2 - 1^2 \\ &= \left(p - \frac{1}{p} + 1 \right) \left(p - \frac{1}{p} - 1 \right) \end{aligned}$$

Answer 4F.

$$\begin{aligned} & 5x^2 - y^2 - 4xy + 3x - 3y \\ &= x^2 + 4x^2 - y^2 - 4xy + 3x - 3y \\ &= (x^2 - y^2) + (4x^2 - 4xy) + (3x - 3y) \\ &= (x - y)(x + y) + 4x(x - y) + 3(x - y) \\ &= (x - y)[(x + y) + 4x + 3] \\ &= (x - y)(x + y + 4x + 3) \\ &= (x - y)(5x + y + 3) \end{aligned}$$