

Chapter 10. Logarithms

Ex 10.1

Answer 1.

$$\begin{aligned} \text{(i)} \quad 3^3 &= 27 \\ \Rightarrow \log_3 27 &= 3 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 5^4 &= 625 \\ \Rightarrow \log_5 625 &= 4 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad 9^0 &= 1 \\ \Rightarrow \log_9 1 &= 0 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \frac{1}{8} &= 2^{-3} \\ \Rightarrow \log_2 \frac{1}{8} &= -3 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad 11^2 &= 121 \\ \Rightarrow \log_{11} 121 &= 2 \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad 3^{-2} &= \frac{1}{9} \\ \Rightarrow \log_3 \frac{1}{9} &= -2 \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad 10^{-4} &= 0.0001 \\ \Rightarrow \log_{10} 0.0001 &= -4 \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad 7^0 &= 1 \\ \Rightarrow \log_7 1 &= 0 \end{aligned}$$

$$\begin{aligned} \text{(ix)} \quad \left(\frac{1}{3}\right)^4 &= \frac{1}{81} \\ \Rightarrow \log_{\frac{1}{3}} \frac{1}{81} &= 4 \end{aligned}$$

$$\begin{aligned} \text{(x)} \quad 9^{-4} &= \frac{1}{6561} \\ \Rightarrow \log_9 \frac{1}{6561} &= -4 \end{aligned}$$

Answer 2.

$$\begin{aligned} \text{(i)} \quad \log_2 128 &= 7 \\ \Rightarrow 128 &= 2^7 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \log_3 81 &= 4 \\ \Rightarrow 81 &= 3^4 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \log_{10} 0.001 &= -3 \\ \Rightarrow 0.001 &= 10^{-3} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \log_2 \frac{1}{32} &= -5 \\ \Rightarrow \frac{1}{32} &= 2^{-5} \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad \log_b a &= c \\ \Rightarrow a &= b^c \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad \log_2 \frac{1}{2} &= -1 \\ \Rightarrow \frac{1}{2} &= 2^{-1} \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad \log_5 a &= 3 \\ \Rightarrow a &= 5^3 \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad \log_{\sqrt{3}} 27 &= 6 \\ \Rightarrow 27 &= (\sqrt{3})^6 \end{aligned}$$

$$\begin{aligned} \text{(ix)} \quad \log_{25} \sqrt{5} &= \frac{1}{4} \\ \Rightarrow \sqrt{5} &= 25^{\frac{1}{4}} \end{aligned}$$

$$\begin{aligned} \text{(x)} \quad q &= \log_a p \\ \Rightarrow p &= a^q \end{aligned}$$

$$\begin{aligned} \text{(xi)} \quad \log_{\sqrt{6}} (6\sqrt{6}) &= 3 \\ \Rightarrow 6\sqrt{6} &= (\sqrt{6})^3 \end{aligned}$$

$$\begin{aligned} \text{(xii)} \quad -2 &= \log_2 0.25 \\ \Rightarrow 2^{-2} &= 0.25 \end{aligned}$$

Answer 3.

$$(i) \log_x 49 = 2$$

$$\Rightarrow x^2 = 49$$

$$\Rightarrow x^2 = 7^2$$

$$\therefore x = 7$$

$$(ii) \log_x 125 = 3$$

$$\Rightarrow x^3 = 125$$

$$\Rightarrow x^3 = 5^3$$

$$\therefore x = 5$$

$$(iii) \log_x 243 = 5$$

$$\Rightarrow x^5 = 243$$

$$\Rightarrow x^5 = 3^5$$

$$\therefore x = 3$$

$$(iv) \log_8 x = \frac{2}{3}$$

$$\Rightarrow x = 8^{\frac{2}{3}}$$

$$\Rightarrow x^3 = 8^2 = 64 = 4^3$$

$$\therefore x = 4$$

$$(v) \log_7 x = 3$$

$$\Rightarrow x = 7^3$$

$$\Rightarrow x = 343$$

$$(vi) \log_4 x = -4$$

$$\Rightarrow x = 4^{-4}$$

$$\Rightarrow x = \frac{1}{256}$$

$$(vii) \log_2 0.5 = x$$

$$\Rightarrow 2^x = 0.5 = \frac{1}{2}$$

$$\Rightarrow 2^x = 2^{-1}$$

$$\therefore x = -1$$

$$(viii) \log_3 243 = x$$

$$\Rightarrow 243 = 3^x$$

$$\Rightarrow 3^5 = 3^x$$

$$\therefore x = 5$$

$$(ix) \log_{10} 0.0001 = x$$

$$\Rightarrow 0.0001 = 10^x$$

$$\Rightarrow 10^x = 10^{-4}$$

$$\therefore x = -4$$

$$(x) \log_4 0.0625 = x$$

$$\Rightarrow 0.0625 = 4^x$$

$$\Rightarrow 4^x = 4^{-2}$$

$$\therefore x = -2$$

Answer 4.

(i) $\log_{10} 1000$

Let $\log_{10} 1000 = x$

$$\Rightarrow 10^x = 1000$$

$$\Rightarrow 10^x = 10^3$$

$$\therefore x = 3$$

(ii) $\log_3 81$

Let $\log_3 81 = x$

$$\Rightarrow 3^x = 81$$

$$\Rightarrow 3^x = 3^4$$

$$\therefore x = 4$$

(iii) $\log_5 3125$

Let $\log_5 3125 = x$

$$\Rightarrow 5^x = 3125$$

$$\Rightarrow 5^x = 5^5$$

$$\therefore x = 5$$

(iv) $\log_2 128$

Let $\log_2 128 = x$

$$\Rightarrow 2^x = 128$$

$$\Rightarrow 2^x = 2^7$$

$$\therefore x = 7$$

(v) $\log_{\frac{1}{5}} 125$

Let $\log_{\frac{1}{5}} 125 = x$

$$\Rightarrow \left(\frac{1}{5}\right)^x = 125$$

$$\Rightarrow 5^{-x} = 5^3$$

$$\therefore -x = 3$$

$$\Rightarrow x = -3$$

(vi) $\log_{10} 0.0001$

Let $\log_{10} 0.0001 = x$

$$\Rightarrow 0.0001 = 10^x$$

$$\Rightarrow 10^x = 10^{-4}$$

$$\therefore x = -4$$

(vii) $\log_5 125$

Let $\log_5 125 = x$

$$\Rightarrow 125 = 5^x$$

$$\Rightarrow 5^x = 5^3$$

$$\therefore x = 3$$

(viii) $\log_8 2$

Let $\log_8 2 = x$

$$\Rightarrow 2 = 8^x$$

$$\Rightarrow (2^3)^x = 2$$

$$\Rightarrow 2^{3x} = 2^1$$

$$\therefore 3x = 1 \Rightarrow x = \frac{1}{3}$$

(ix) $\log_{\frac{1}{2}} 16$

Let $\log_{\frac{1}{2}} 16 = x$

$$\Rightarrow 16 = \left(\frac{1}{2}\right)^x$$

$$\Rightarrow 2^{-x} = 2^4$$

$$\therefore -x = 4 \Rightarrow x = -4$$

(x) $\log_{0.01} 10$

Let $\log_{0.01} 10 = x$

$$\Rightarrow (0.01)^x = 10$$

$$\Rightarrow (10^{-2})^x = 10^1$$

$$\Rightarrow 10^{-2x} = 10^1$$

$$\therefore -2x = 1 \Rightarrow x = -\frac{1}{2}$$

(xi) $\log_3 81$

Let $\log_3 81 = x$

$$\Rightarrow 3^x = 81$$

$$\Rightarrow 3^x = 3^4$$

$$\therefore x = 4$$

(xii) $\log_5 \frac{1}{25}$

Let $\log_5 \frac{1}{25} = x$

$$\Rightarrow 5^x = \frac{1}{25}$$

$$\Rightarrow 5^x = 5^{-2}$$

$$\therefore x = -2$$

(xiii) $\log_2 8$

Let $\log_2 8 = x$

$$\Rightarrow 2^x = 8$$

$$\Rightarrow 2^x = 2^3$$

$$\therefore x = 3$$

$$(xiv) \log_a a^3$$

$$\text{Let } \log_a a^3 = x$$

$$\Rightarrow a^x = a^3$$

$$\therefore x = 3$$

$$(xv) \log_{0.1} 10$$

$$\text{Let } \log_{0.1} 10 = x$$

$$\Rightarrow 0.1^x = 10$$

$$\Rightarrow (10^{-1})^x = 10^1$$

$$\therefore -x = 1 \Rightarrow x = -1$$

$$(xvi) \log_{\sqrt{3}} (3\sqrt{3})$$

$$\text{Let } \log_{\sqrt{3}} (3\sqrt{3}) = x$$

$$\Rightarrow (\sqrt{3})^x = 3\sqrt{3}$$

$$\Rightarrow 3^{\frac{x}{2}} = 3^{1+\frac{1}{2}} = 3^{\frac{3}{2}}$$

$$\therefore \frac{x}{2} = \frac{3}{2} \Rightarrow x = 3$$

Answer 5.

$$(i) 10^{2a}$$

$$\log_{10} x = a$$

$$\Rightarrow x = 10^a$$

$$\therefore 10^{2a} = (10^a)^2 = x^2$$

$$(ii) 10^{a+3}$$

$$\log_{10} x = a$$

$$\Rightarrow x = 10^a$$

$$\therefore 10^{a+3} = 10^a \cdot 10^3 = x \cdot 1000 = 1000x$$

$$(iii) 10^{-a}$$

$$\log_{10} x = a$$

$$\Rightarrow x = 10^a$$

$$\therefore 10^{-a} = x^{-1} = 1/x$$

$$(iv) 10^{2a-3}$$

$$\log_{10} x = a$$

$$\Rightarrow x = 10^a$$

$$\therefore 10^{2a-3} = 10^{2a} \cdot 10^{-3} = (10^a)^2 10^{-3} = \frac{x^2}{1000}$$

Answer 6.

(i) 10^{n-1}

$\log_{10} m = n$

$\Rightarrow m = 10^n$

$\therefore 10^{n-1} = 10^n \cdot 10^{-1} = \frac{m}{10}$

(ii) 10^{2n+1}

$\log_{10} m = n$

$\Rightarrow m = 10^n$

$\therefore 10^{2n+1} = 10^{2n} \cdot 10^1 = (10^n)^2 \cdot 10 = 10 m^2$

(iii) 10^{-3n}

$\log_{10} m = n$

$\Rightarrow m = 10^n$

$\therefore 10^{-3n} = (10^n)^{-3} = (m)^{-3} = \frac{1}{m^3}$

Answer 7.

(i) 10^p

$\log_{10} x = p$

$\Rightarrow x = 10^p$

(ii) 10^{p+1}

$\log_{10} x = p$

$\Rightarrow x = 10^p$

$\therefore 10^{p+1} = 10^p \cdot 10^1 = 10x$

(iii) 10^{2p-3}

$\log_{10} x = p$

$\Rightarrow x = 10^p$

$\therefore 10^{2p-3} = 10^{2p} \cdot 10^{-3} = (10^p)^2 \cdot 10^{-3} = \frac{x^2}{1000}$

(iv) 10^{2-p}

$\log_{10} x = p$

$\Rightarrow x = 10^p$

$\therefore 10^{2-p} = 10^2 \cdot 10^{-p} = 100 \cdot x^{-1} = \frac{100}{x}$

Answer 8.

$\log_{10} x = a$

$\Rightarrow x = 10^a$

$\log_{10} y = b$

$\Rightarrow y = 10^b$

$\log_{10} z = 2a - 3b$

$\Rightarrow z = 10^{2a-3b}$

$\therefore z = 10^{2a-3b} = (10^a)^2 \cdot (10^b)^{-3} = (x)^2 (y)^{-3} = \frac{x^2}{y^3}$

Answer 9.(i) 10^{2x-3} in terms of a

$$\log_{10} a = x$$

$$\Rightarrow a = 10^x$$

$$10^{2x-3} = (10^x)^2 \cdot 10^{-3} = \frac{a^2}{1000}$$

(ii) 10^{3y-1} in terms of b

$$\log_{10} b = y$$

$$\Rightarrow b = 10^y$$

$$10^{3y-1} = (10^y)^3 \cdot 10^{-1} = \frac{b^3}{10}$$

(iii) 10^{x-y+z} in terms of a, b and c.

$$\log_{10} a = x$$

$$\Rightarrow a = 10^x$$

$$\log_{10} b = y$$

$$\Rightarrow b = 10^y$$

$$\log_{10} c = z$$

$$\Rightarrow c = 10^z$$

$$10^{x-y+z} = 10^x \cdot 10^{-y} \cdot 10^z = a \cdot b^{-1} \cdot c = \frac{ac}{b}$$

Answer 10.(i) If $\log_{10} 100 = 2$, then $10^2 = 100$

The statement is TRUE.

(ii) If $\log_{10} p = q$, then $10^p = q$

The statement is FALSE.

$$\log_{10} p = q \text{ implies } 10^q = p$$

(iii) If $4^3 = 64$, then $\log_3 64 = 4$

The statement is FALSE

$$4^3 = 64 \text{ implies } \log_4 64 = 3$$

(iv) If $x^y = z$, then $y = \log_x z$

The statement is TRUE.

(v) If $\log_2 8 = 3$, then $\log_8 2 = \frac{1}{3}$

The statement is TRUE

Answer 1.

(i) $\log 36$

$$\begin{aligned}\log 36 &= \log (2 \times 2 \times 3 \times 3) = \log (2^2 \times 3^2) \\ &= \log 2^2 + \log 3^2 = 2\log 2 + 2\log 3.\end{aligned}$$

(ii) $\log 54$

$$\begin{aligned}\log 54 &= \log (2 \times 3 \times 3 \times 3) = \log (2 \times 3^3) \\ &= \log 2 + \log 3^3 = \log 2 + 3 \log 3\end{aligned}$$

(iii) $\log 144$

$$\log 144 = \log (2^4 \times 3^2) = \log 2^4 + \log 3^2 = 4 \log 2 + 2 \log 3$$

(iv) $\log 216$

$$\log 216 = \log (2^3 \times 3^3) = \log 2^3 + \log 3^3 = 3 \log 2 + 3 \log 3$$

(v) $\log 648$

$$\log 648 = \log (2^3 \times 3^4) = \log 2^3 + \log 3^4 = 3 \log 2 + 4 \log 3$$

(vi)

$$\begin{aligned}\log 12^8 &= \log (3 \times 2^2)^8 \\ &= 8 \log (3 \times 2^2) \\ &= 8 [\log 3 + \log 2^2] \\ &= 8 [\log 3 + 2 \log 2]\end{aligned}$$

Ex 10.2

Answer 2.

(i) $\log 20$

$$\log 20 = \log (2^2 \times 5) = \log 2^2 + \log 5 = 2 \log 2 + \log 5$$

(ii) $\log 80$

$$\log 80 = \log (2^4 \times 5) = \log 2^4 + \log 5 = 4 \log 2 + \log 5$$

(iii) $\log 125$

$$\log 125 = \log 5^3 = 3 \log 5$$

(iv) $\log 160$

$$\log 160 = \log (2^5 \times 5) = \log 2^5 + \log 5 = 5 \log 2 + \log 5$$

(v) $\log 500$

$$\log 500 = \log (2^2 \times 5^3) = \log 2^2 + \log 5^3 = 2 \log 2 + 3 \log 5$$

(vi)

$$\log 250 = \log (5^3 \times 2)$$

$$= \log 5^3 + \log 2$$

$$= 3 \log 5 + \log 2$$

Answer 3.

(i) $\log \sqrt[3]{144}$

$$= \log (144)^{\frac{1}{3}}$$

$$= \frac{1}{3} \log 144 = \frac{1}{3} \log (2^4 \times 3^2) = \frac{1}{3} \log 2^4 + \frac{1}{3} \log 3^2 = \frac{4}{3} \log 2 + \frac{2}{3} \log 3$$

(ii) $\log \sqrt[5]{216}$

$$= \log (216)^{\frac{1}{5}}$$

$$= \frac{1}{5} \log 216 = \frac{1}{5} \log (2^3 \times 3^3) = \frac{1}{5} \log 2^3 + \frac{1}{5} \log 3^3 = \frac{3}{5} \log 2 + \frac{3}{5} \log 3$$

(iii) $\log \sqrt[4]{648}$

$$= \log (648)^{\frac{1}{4}}$$

$$= \frac{1}{4} \log 648 = \frac{1}{4} \log (2^3 \times 3^4) = \frac{1}{4} \log 2^3 + \frac{1}{4} \log 3^4 = \frac{3}{4} \log 2 + \frac{4}{4} \log 3 = \frac{3}{4} \log 2 + \log 3$$

(iv) $\log \frac{26}{51} - \log \frac{91}{119}$

$$= \log \frac{2 \times 13}{3 \times 17} - \log \frac{7 \times 13}{7 \times 17}$$

$$= \log \frac{2 \times 13}{3 \times 17} - \log \frac{13}{17} = \log 13 + \log 2 - \log 3 - \log 17 - \log 13 + \log 17 = \log 2 - \log 3$$

(vi)

$$\begin{aligned} & \log \frac{225}{16} - 2 \log \frac{5}{9} + \log \left(\frac{2}{3} \right)^5 \\ &= \log \frac{225}{16} - 2 \log \frac{5}{9} + 5 \log \frac{2}{3} \\ &= \log 225 - \log 16 - 2[\log 5 - \log 9] + 5[\log 2 - \log 3] \\ &= \log(5^2 \times 3^2) - \log 2^4 - 2[\log 5 - \log 3^2] + 5[\log 2 - \log 3] \\ &= \log 5^2 + \log 3^2 - 4 \log 2 - 2[\log 5 - 2 \log 3] + 5[\log 2 - \log 3] \\ &= 2 \log 5 + 2 \log 3 - 4 \log 2 - 2 \log 5 + 4 \log 3 + 5 \log 2 - 5 \log 3 \\ &= \log 2 + \log 3 \end{aligned}$$

Answer 4.

(i) $F = G \frac{m_1 m_2}{d^2}$

Considering log on both the sides, we get

$$\begin{aligned} \log F &= \log \left(G \frac{m_1 m_2}{d^2} \right) = \log(G m_1 m_2) - \log d^2 \\ &= \log G + \log m_1 + \log m_2 - 2 \log d \end{aligned}$$

(ii) $E = \frac{1}{2} m v^2$

Considering log on both the sides, we get

$$\begin{aligned} \log E &= \log \left(\frac{1}{2} m v^2 \right) = \log \frac{1}{2} + \log m + \log v^2 = \log 1 - \log 2 + \log m + 2 \log v \\ &= \log m + 2 \log v - \log 2 \end{aligned}$$

(iii) $n = \sqrt{\frac{M \cdot g}{m \cdot \ell}}$

$$\Rightarrow n = \left(\frac{M \cdot g}{m \cdot \ell} \right)^{1/2}$$

Considering log on both sides,

$$\begin{aligned} \log n &= \log \left(\frac{M \cdot g}{m \cdot \ell} \right)^{1/2} = \frac{1}{2} \log \left(\frac{M \cdot g}{m \cdot \ell} \right) = \frac{1}{2} [\log(Mg) - \log(m \cdot \ell)] \\ &= \frac{1}{2} (\log M + \log g - \log m - \log \ell) \end{aligned}$$

$$(iv) \quad V = \frac{4}{3} \pi r^3$$

Considering log on both the sides, we get

$$\begin{aligned} \log V &= \log \left(\frac{4}{3} \pi r^3 \right) = \log 4 + \log \pi + \log r^3 - \log 3 \\ &= \log 2^2 + \log \pi + 3 \log r - \log 3 = 2 \log 2 - \log 3 + \log \pi + 3 \log r \end{aligned}$$

$$(v) \quad V = \frac{1}{D\ell} \sqrt{\frac{T}{\pi r}}$$

$$\Rightarrow V = \frac{1}{D\ell} \left(\frac{T}{\pi r} \right)^{1/2}$$

Considering log on both the sides, we get

$$\begin{aligned} \log V &= \log \left[\frac{1}{D\ell} \left(\frac{T}{\pi r} \right)^{1/2} \right] = \log \left(\frac{1}{D\ell} \right) + \log \left(\frac{T}{\pi r} \right)^{1/2} = (\log 1 - \log D - \log \ell) + \frac{1}{2} \log \left(\frac{T}{\pi r} \right) \\ &= (0 - \log D - \log \ell) + \frac{1}{2} (\log T - \log \pi - \log r) \\ &= \frac{1}{2} (\log T - \log \pi - \log r) - \log D - \log \ell \end{aligned}$$

Answer 5.

$$(i) \quad \log 18 + \log 25 - \log 30$$

$$\begin{aligned} &= \log (2 \times 3^2) + \log 5^2 - \log (2 \times 3 \times 5) \\ &= \log 2 + \log 3^2 + 2 \log 5 - \{\log 2 + \log 3 + \log 5\} \\ &= \log 2 + 2 \log 3 + 2 \log 5 - \log 2 - \log 3 - \log 5 \\ &= \log 3 + \log 5 = \log (3 \times 5) = \log 15 \end{aligned}$$

$$(ii) \quad \log 144 - \log 72 + \log 150 - \log 50$$

$$\begin{aligned} &= \log (2^4 \times 3^2) - \log (2^3 \times 3^2) + \log (2 \times 3 \times 5^2) - \log (2 \times 5^2) \\ &= \log 2^4 + \log 3^2 - \{\log 2^3 + \log 3^2\} + \log 2 + \log 3 + \log 5^2 \\ &\quad - \{\log 2 + \log 5^2\} \\ &= 4 \log 2 + 2 \log 3 - 3 \log 2 - 2 \log 3 + \log 2 + \log 3 + 2 \\ &\quad \log 5 - \log 2 - 2 \log 5 \\ &= \log 2 + \log 3 = \log (2 \times 3) = \log 6 \end{aligned}$$

$$(iii) \quad 2 \log 3 - \frac{1}{2} \log 16 + \log 12$$

$$\begin{aligned} &= 2 \log 3 - \frac{1}{2} \log 2^4 + \log (2^2 \times 3) \\ &= 2 \log 3 - \frac{1}{2} \times 4 \log 2 + \log 2^2 + \log 3 \\ &= 2 \log 3 - 2 \log 2 + 2 \log 2 + \log 3 = 3 \log 3 = \log 3^3 = \log 27 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & 2 + \frac{1}{2} \log 9 - 2 \log 5 \\
 &= 2 + \frac{1}{2} \log 3^2 - 2 \log 5 \\
 &= 2 \log 10 + \frac{1}{2} \times 2 \log 3 - 2 \log 5 \\
 &= \log 10^2 + \log 3 - \log 5^2 \\
 &= \log 100 + \log 3 - \log 25 \\
 &= \log \frac{100 \times 3}{25} = \log 12
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & 2 \log \frac{9}{5} - 3 \log \frac{3}{5} + \log \frac{16}{20} \\
 &= 2 \log 9 - 2 \log 5 - 3 \log 3 + 3 \log 5 + \log 16 - \log 20 \\
 &= 2 \log(3^2) - 2 \log 5 - 3 \log 3 + 3 \log 5 + \log(4^2) - \log(5 \times 4) \\
 &= 4 \log 3 - 2 \log 5 - 3 \log 3 + 3 \log 5 + 2 \log 4 - \log 5 - \log 4 \\
 &= (4 - 3) \log 3 + (-2 - 1 + 3) \log 5 + \log 4 \\
 &= \log 3 + \log 4 \\
 &= \log(3 \times 4) = \log 12
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & 2 \log \frac{15}{18} - \log \frac{25}{162} + \log \frac{4}{9} \\
 &= 2 \log \frac{5}{2 \times 3} - \log \frac{5^2}{2 \times 3^4} + \log \frac{2^2}{3^2} \\
 &= 2 \log 5 - 2 \log 2 - 2 \log 3 - \{\log 5^2 - \log 2 - \log 3^4\} + \log 2^2 - \log 3^2 \\
 &= 2 \log 5 - 2 \log 2 - 2 \log 3 - 2 \log 5 + \log 2 + 4 \log 3 + 2 \log 2 - 2 \log 3 \\
 &= \log 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & 2 \log \frac{16}{25} - 3 \log \frac{8}{5} + \log 90 \\
 &= 2 \log \frac{2^4}{5^2} - 3 \log \frac{2^3}{5} + \log (2 \times 5 \times 3^2) \\
 &= 2 \log 2^4 - 2 \log 5^2 - 3 \{\log 2^3 - \log 5\} + \log 2 + \log 5 + \log 3^2 \\
 &= 2 \times 4 \log 2 - 2 \times 2 \log 5 - 3 \times 3 \log 2 + 3 \log 5 + \log 2 + \log 5 + 2 \log 3 \\
 &= 8 \log 2 - 4 \log 5 - 9 \log 2 + 3 \log 5 + \log 2 + \log 5 + 2 \log 3 \\
 &= 2 \log 3 = \log 3^2 = \log 9
 \end{aligned}$$

$$(viii) \frac{1}{2} \log 25 - 2 \log 3 + \log 36$$

$$= \frac{1}{2} \log 5^2 - 2 \log 3 + \log(2^2 \times 3^2)$$

$$\frac{1}{2} \times 2 \log 5 - 2 \log 3 + \log 2^2 + \log 3^2$$

$$= \log 5 + 2 \log 2$$

$$= \log 5 + \log 2^2 = \log 5 + \log 4 = \log(5 \times 4) = \log 20$$

$$(ix) \log \frac{81}{8} - 2 \log \frac{3}{5} + 3 \log \frac{2}{5} + \log \frac{25}{9}$$

$$= \log \frac{3^4}{2^3} - 2 \log \frac{3}{5} + 3 \log \frac{2}{5} + \log \frac{5^2}{3^2}$$

$$= \log 3^4 - \log 2^3 - 2 \log 3 + 2 \log 5 + 3 \log 2 - 3 \log 5 + \log 5^2 - \log 3^2$$

$$= 4 \log 3 - 3 \log 2 - 2 \log 3 + 2 \log 5 + 3 \log 2 - 3 \log 5 + 2 \log 5 - 2 \log 3$$

$$= \log 5$$

$$(x) 3 \log \frac{5}{8} + 2 \log \frac{8}{15} - \frac{1}{2} \log \frac{25}{81} + 3$$

$$= 3 \log \frac{5}{2^3} + 2 \log \frac{2^3}{3 \times 5} - \frac{1}{2} \log \frac{5^2}{3^4} + 3 \log 10$$

$$= 3 \log 5 - 3 \log 2^3 + 2 \log 2^3 - 2 \log 3 - 2 \log 5 - \frac{1}{2} \log 5^2 + \frac{1}{2} \log 3^4 + 3 \log(2 \times 5)$$

$$= 3 \log 5 - 3 \times 3 \log 2 + 2 \times 3 \log 2 - 2 \log 3 - 2 \log 5 - \frac{1}{2} \times 2 \log 5 + \frac{1}{2} \times 4 \log 3 + 3 \log 2 + 3 \log 5$$

$$= 3 \log 5 - 9 \log 2 + 6 \log 2 - 2 \log 3 - 2 \log 5 - \log 5 + 2 \log 3 + 3 \log 2 + 3 \log 5$$

$$= 3 \log 5 = \log 5^3 = \log 125$$

Answer 6.

$$(i) 2 \log 5 + \log 8 - \frac{1}{2} \log 4$$

$$2 \log 5 + \log 8 - \frac{1}{2} \log 4$$

$$= 2 \log 5 + \log 2^3 - \frac{1}{2} \log 2^2$$

$$= 2 \log 5 + 3 \log 2 - \frac{1}{2} \times 2 \log 2$$

$$= 2 \log 5 + 3 \log 2 - \log 2$$

$$= 2 \log 5 + 2 \log 2 = 2(\log 5 + \log 2) = 2 \log(5 \times 2) = 2 \log 10$$

$$= 2 \times 1 = 2$$

$$(ii) 2 \log 7 + 3 \log 5 - \log \frac{49}{8}$$

$$2 \log 7 + 3 \log 5 - \log \frac{49}{8}$$

$$= 2 \log 7 + 3 \log 5 - \log 49 + \log 8$$

$$= 2 \log 7 + 3 \log 5 - \log 7^2 + \log 2^3$$

$$= 2 \log 7 + 3 \log 5 - 2 \log 7 + 3 \log 2$$

$$= 3 \log 5 + 3 \log 2 = 3(\log 5 + \log 2) = 3 \log(5 \times 2) = 3 \log 10$$

$$= 3 \times 1 = 3$$

$$\begin{aligned}
\text{(iii)} \quad & 3 \log \frac{32}{27} + 5 \log \frac{125}{24} - 3 \log \frac{625}{243} + \log \frac{2}{75} \\
& 3 \log \frac{32}{27} + 5 \log \frac{125}{24} - 3 \log \frac{625}{243} + \log \frac{2}{75} \\
& = 3 \log \frac{2^5}{3^3} + 5 \log \frac{5^3}{2^3 \times 3} - 3 \log \frac{5^4}{2 \times 3^4} + \log \frac{2}{3 \times 5^2} \\
& = 3 \log 2^5 - 3 \log 3^3 + 5 \log 5^3 - 5 \log 2^3 - 5 \log 3 - 3 \log 5^4 + 3 \log 2 + 3 \log 3^4 \\
& + \log 2 - \log 3 - \log 5^2 \\
& = 3 \times 5 \log 2 - 3 \times 3 \log 3 + 5 \times 3 \log 5 - 5 \times 3 \log 2 - 5 \log 3 - 3 \log 5^4 + 3 \log 2 + 3 \log 3^4 \\
& + \log 2 - \log 3 - \log 5^2 \\
& = 15 \log 2 - 9 \log 3 + 15 \log 5 - 15 \log 2 - 5 \log 3 - 12 \log 5 + 3 \log 2 + 12 \log 3 + \log 2 - \log 3 - 2 \log 5 \\
& = \log 5 + \log 2 = \log (5 \times 2) = \log 10 = 1
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad & 12 \log \frac{3}{2} + 7 \log \frac{125}{27} - 5 \log \frac{25}{36} - 7 \log 25 + \log \frac{16}{3} \\
& 12 \log \frac{3}{2} + 7 \log \frac{125}{27} - 5 \log \frac{25}{36} - 7 \log 25 + \log \frac{16}{3} \\
& = 12 \log \frac{3}{2} + 7 \log \frac{5^3}{3^3} - 5 \log \frac{5^2}{2^2 \times 3^2} - 7 \log 5^2 + \log \frac{2^4}{3} \\
& = 12 \log 3 - 12 \log 2 + 7 \log 5^3 - 7 \log 3^3 - 5 \log 5^2 + 5 \log 2^2 + 5 \log 3^2 - 7 \log 5^2 + \log 2^4 - \log 3 \\
& = 12 \log 3 - 12 \log 2 + 21 \log 5 - 21 \log 3 - 10 \log 5 + 10 \log 2 + 10 \log 3 - 14 \log 5 + 4 \log 2 - \log 3 \\
& = 2 \log 2 + 2 \log 5
\end{aligned}$$

Answer 7.

$$\begin{aligned}
\text{(i)} \quad & \log(3 - x) - \log(x - 3) = 1 \\
& \Rightarrow \log \left(\frac{3-x}{x-3} \right) = 1 = \log 10 \\
& \Rightarrow \left(\frac{3-x}{x-3} \right) = 10 \\
& \Rightarrow 3 - x = 10(x - 3) \\
& \Rightarrow 3 - x = 10x - 30 \Rightarrow 11x = 33 \Rightarrow x = 3
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad & \log(x^2 + 36) - 2 \log x = 1 \\
& \Rightarrow \log(x^2 + 36) - \log x^2 = 1 \\
& \Rightarrow \log \left(\frac{x^2 + 36}{x^2} \right) = 1 = \log 10 \\
& \Rightarrow \left(\frac{x^2 + 36}{x^2} \right) = 10 \\
& \Rightarrow x^2 + 36 = 10x^2 \\
& \Rightarrow 9x^2 = 36 \\
& \Rightarrow x^2 = 4 \Rightarrow x = 2
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad & \log 7 + \log (3x - 2) = \log (x+3) + 1 \\
& \Rightarrow \log 7 + \log (3x - 2) - \log (x+3) = 1 \\
& \Rightarrow \log \frac{7 \cdot (3x-2)}{x+3} = \log 10 \\
& \Rightarrow \frac{7 \cdot (3x-2)}{x+3} = 10 \\
& \Rightarrow 21x - 14 = 10(x + 3) \\
& \Rightarrow 21x - 10x = 30 + 14 \\
& \Rightarrow 11x = 44 \\
& \Rightarrow x = 44 / 11 = 4
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad & \log (x+1) + \log (x-1) = \log 11 + 2 \log 3 \\
& \Rightarrow \log [(x+1)(x-1)] = \log 11 + \log 3^2 \\
& \Rightarrow \log \{ x^2 - 1 \} = \log (11 \cdot 9) \\
& \Rightarrow \log \{ x^2 - 1 \} = \log 99 \\
& \Rightarrow x^2 - 1 = 99 \\
& \Rightarrow x^2 = 100 \\
& \text{So, } x = 10 \text{ or } -10 \\
& \text{Negative value is rejected} \\
& \text{So, } x = 10
\end{aligned}$$

$$\begin{aligned}
\text{(v)} \quad & \log_4 x + \log_4 (x-6) = 2 \\
& \Rightarrow \log_4 \{x(x-6)\} = 2 \log_4 4 \\
& \Rightarrow \log_4 \{x^2 - 6x\} = \log_4 4^2 \\
& \Rightarrow x^2 - 6x = 16 \\
& \Rightarrow x^2 - 6x - 16 = 0 \\
& \Rightarrow x^2 - 8x + 2x - 16 = 0 \\
& \Rightarrow x(x-8) + 2(x-8) = 0 \\
& \Rightarrow (x-8)(x+2) = 0 \\
& \Rightarrow x = 8 \text{ or } -2 \\
& \text{Negative value is rejected} \\
& \text{So, } x = 8
\end{aligned}$$

$$\begin{aligned}
\text{(vi)} \quad & \log_8 (x^2 - 1) - \log_8 (3x + 9) = 0 \\
& \Rightarrow \log_8 \left(\frac{x^2 - 1}{3x + 9} \right) = \log_8 1 \\
& \Rightarrow \frac{x^2 - 1}{3x + 9} = 1 \\
& \Rightarrow x^2 - 1 = 3x + 9 \\
& \Rightarrow x^2 - 3x - 10 = 0 \\
& \Rightarrow x^2 - 5x + 2x - 10 = 0 \\
& \Rightarrow x(x - 5) + 2(x - 5) = 0 \\
& \Rightarrow (x - 5)(x + 2) = 0 \\
& \Rightarrow x = 5 \text{ or } x = -2
\end{aligned}$$

$$\begin{aligned}
\text{(vii)} \quad & \log(x+1) + \log(x-1) = \log 48 \\
& \Rightarrow \log \{(x+1)(x-1)\} = \log 48 \\
& \Rightarrow \log(x^2 - 1) = \log 48 \\
& \Rightarrow x^2 - 1 = 48 \\
& \Rightarrow x^2 = 49 \\
& \Rightarrow x = 7 \text{ (neglecting the negative value)}
\end{aligned}$$

(viii)

$$\begin{aligned}
& \log_2 x + \log_4 x + \log_{16} x = \frac{21}{4} \\
& \therefore \frac{1}{\log_x 2} + \frac{1}{\log_x 2^2} + \frac{1}{\log_x 2^4} = \frac{21}{4} \\
& \therefore \frac{1}{\log_x 2} + \frac{1}{2\log_x 2} + \frac{1}{4\log_x 2} = \frac{21}{4} \\
& \therefore \frac{1}{\log_x 2} \left(1 + \frac{1}{2} + \frac{1}{4} \right) = \frac{21}{4} \\
& \therefore \frac{1}{\log_x 2} \left(\frac{7}{4} \right) = \frac{21}{4} \\
& \therefore \log_x 2 = \frac{7}{4} \cdot \frac{4}{21} \\
& \therefore \log_x 2 = \frac{1}{3} \\
& \therefore x^{\frac{1}{3}} = 2 \\
& \therefore x = 2^3 = 8
\end{aligned}$$

Answer 8.

(i) $\log(x+5) = 1$

$$= \log 10$$

$$\Rightarrow x + 5 = 10$$

$$\Rightarrow x = 10 - 5 = 5$$

(ii) $\frac{\log 27}{\log 243} = x$

$$\Rightarrow \frac{\log 3^3}{\log 3^5} = x$$

$$\Rightarrow \frac{3\log 3}{5\log 3} = x$$

$$\Rightarrow x = \frac{3}{5}$$

(iii) $\frac{\log 81}{\log 9} = x$

$$\Rightarrow \frac{\log 3^4}{\log 3^2} = x$$

$$\Rightarrow \frac{4\log 3}{2\log 3} = x$$

$$\Rightarrow x = 2$$

(iv) $\frac{\log 121}{\log 11} = \log x$

$$\Rightarrow \frac{\log 11^2}{\log 11} = \log x$$

$$\Rightarrow \frac{2\log 11}{\log 11} = \log x$$

$$\Rightarrow 2 = \log x$$

$$\Rightarrow 2\log 10 = \log x \quad (\text{since } \log 10 = 1)$$

$$\Rightarrow \log 10^2 = \log x$$

$$\therefore x = 10^2 = 100$$

(v) $\frac{\log 125}{\log 5} = \log x$

$$\Rightarrow \frac{\log 5^3}{\log 5} = \log x$$

$$\Rightarrow \frac{3\log 5}{\log 5} = \log x$$

$$\Rightarrow 3 = \log x$$

$$\Rightarrow 3\log 10 = \log x \quad (\text{since } \log 10 = 1)$$

$$\Rightarrow \log 10^3 = \log x$$

$$\therefore x = 10^3 = 1000$$

$$(vi) \frac{\log 128}{\log 32} = x$$

$$\Rightarrow \frac{\log 2^7}{\log 2^5} = x$$

$$\Rightarrow \frac{7\log 2}{5\log 2} = x$$

$$\Rightarrow x = \frac{7}{5} = 1.4$$

$$(vii) \frac{\log 1331}{\log 11} = \log x$$

$$\Rightarrow \frac{\log 11^3}{\log 11} = \log x$$

$$\Rightarrow \frac{3\log 11}{\log 11} = \log x$$

$$\Rightarrow 3 = \log x$$

$$\Rightarrow 3\log 10 = \log x \quad (\text{since } \log 10 = 1)$$

$$\Rightarrow \log 10^3 = \log x$$

$$\Rightarrow \therefore x = 10^3 = 1000$$

$$(viii) \frac{\log 289}{\log 17} = \log x$$

$$\Rightarrow \frac{\log 17^2}{\log 17} = \log x$$

$$\Rightarrow \frac{2\log 17}{\log 17} = \log x$$

$$\Rightarrow 2 = \log x$$

$$\Rightarrow 2\log 10 = \log x \quad (\text{since } \log 10 = 1)$$

$$\Rightarrow \log 10^2 = \log x$$

$$\Rightarrow \therefore x = 10^2 = 100$$

Answer 9.

$$\begin{aligned} \log_{10} 3 + 1 &= \log_{10} 3 + \log_{10} 10 \\ &= \log_{10} (3 \times 10) \\ &= \log_{10} 30 \end{aligned}$$

Answer 10A.

False, since $\log xy = \log x + \log y$

Answer 10B.

True, since $\log 1 = 0$ and anything multiplied by 0 is 0.

Answer 10C.

False, since $\log_b a = \frac{1}{\log_a b}$.

Answer 10D.

True.

$$\frac{\log 49}{\log 7} = \log y$$

$$\Rightarrow \frac{\log 7^2}{\log 7} = \log y$$

$$\Rightarrow \frac{2\log 7}{\log 7} = \log y$$

$$\Rightarrow 2(1) = \log y$$

$$\Rightarrow 2\log_{10} 10 = \log y$$

$$\Rightarrow \log_{10} 10^2 = \log_{10} y$$

$$\Rightarrow \log_{10} 100 = \log_{10} y$$

$$\Rightarrow y = 100$$

Answer 11A.

$$\log 16 = a, \log 9 = b \text{ and } \log 5 = c$$

$$\log 4^2 = a, \log 3^2 = b \text{ and } \log 5 = c$$

$$2\log 4 = a, 2\log 3 = b \text{ and } \log 5 = c$$

$$\log 4 = \frac{a}{2}, \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\text{Consider, } \log 12 = \log(4 \times 3)$$

$$= \log 4 + \log 3$$

$$= \frac{a}{2} + \frac{b}{2}$$

$$= \frac{a+b}{2}$$

Answer 11B.

$$\log 16 = a, \log 9 = b \text{ and } \log 5 = c$$

$$\log 4^2 = a, \log 3^2 = b \text{ and } \log 5 = c$$

$$2\log 4 = a, 2\log 3 = b \text{ and } \log 5 = c$$

$$\log 4 = \frac{a}{2}, \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\begin{aligned} \text{Consider, } \log 75 &= \log(5^2 \times 3) \\ &= \log 5^2 + \log 3 \\ &= 2\log 5 + \log 3 \\ &= 2(c) + \frac{b}{2} \\ &= \frac{4c + b}{2} \end{aligned}$$

Answer 11C.

$$\log 16 = a, \log 9 = b \text{ and } \log 5 = c$$

$$\log 4^2 = a, \log 3^2 = b \text{ and } \log 5 = c$$

$$2\log 4 = a, 2\log 3 = b \text{ and } \log 5 = c$$

$$\log 4 = \frac{a}{2}, \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\begin{aligned} \text{Consider, } \log 720 &= \log(4^2 \times 3^2 \times 5) \\ &= \log 4^2 + \log 3^2 + \log 5 \\ &= 2\log 4 + 2\log 3 + \log 5 \\ &= 2\left(\frac{a}{2}\right) + 2\left(\frac{b}{2}\right) + c \\ &= a + b + c \end{aligned}$$

Answer 11D.

$$\log 16 = a, \log 9 = b \text{ and } \log 5 = c$$

$$\log 4^2 = a, \log 3^2 = b \text{ and } \log 5 = c$$

$$2\log 4 = a, 2\log 3 = b \text{ and } \log 5 = c$$

$$\log 4 = \frac{a}{2}, \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\begin{aligned} \text{Consider, } \log 2.25 &= \log\left(\frac{225}{100}\right) \\ &= \log 225 - \log 100 \\ &= \log(3^2 \times 5^2) - \log(4 \times 5^2) \\ &= \log 3^2 + \log 5^2 - (\log 4 + \log 5^2) \\ &= \log 3^2 + \log 5^2 - \log 4 - \log 5^2 \\ &= 2\log 3 - \log 4 \\ &= 2\left(\frac{b}{2}\right) - \frac{a}{2} \\ &= \frac{2b - a}{2} \end{aligned}$$

Answer 11E.

$$\log 16 = a, \log 9 = b \text{ and } \log 5 = c$$

$$\log 4^2 = a, \log 3^2 = b \text{ and } \log 5 = c$$

$$2\log 4 = a, 2\log 3 = b \text{ and } \log 5 = c$$

$$\log 4 = \frac{a}{2}, \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\text{Consider, } \log 2\frac{1}{4} = \log\left(\frac{9}{4}\right)$$

$$= \log 9 - \log 4$$

$$= \log 3^2 - \log 4$$

$$= 2\log 3 - \log 4$$

$$= 2\left(\frac{b}{2}\right) - \frac{a}{2}$$

$$= \frac{2b - a}{2}$$

Answer 12.

$$\log x = p + q \text{ and } \log y = p - q$$

$$\log \frac{10x}{y^2} = \log 10x - \log y^2$$

$$\Rightarrow \log \frac{10x}{y^2} = \log 10 + \log x - 2\log y$$

$$\Rightarrow \log \frac{10x}{y^2} = 1 + p + q - 2(p - q)$$

$$\Rightarrow \log \frac{10x}{y^2} = 1 - p + 3q$$

Answer 13.

$$\begin{aligned} \log \frac{a^3}{b^2} &= \log a^3 - \log b^2 = 3\log a - 2\log b \\ &= 3p - 2q \end{aligned}$$

Answer 14.

$$\begin{aligned}\log \frac{x^2}{10y} &= \log x^2 - \log 10y = 2\log x - (\log 10 + \log y) \\ &= 2\log x - \log y - 1 = 2(A + B) - (A - B) - 1 = A + 3B - 1\end{aligned}$$

Answer 15.

(i) 10^{a-1} in terms of x

$$\log x = a \Rightarrow x = 10^a$$

$$\therefore 10^{a-1} = \frac{10^a}{10} = \frac{x}{10}$$

(ii) 10^{2b} in terms of y

$$\log y = b \Rightarrow y = 10^b$$

$$\therefore 10^{2b} = (10^b)^2 = y^2$$

Answer 16.

(i) 3^{2x-3} in terms of m

$$\log_3 m = x \Rightarrow m = 3^x$$

$$\therefore 3^{2x-3} = \frac{3^{2x}}{3^3} = \frac{(3^x)^2}{27} = \frac{m^2}{27}$$

(ii) $3^{1-2y+3x}$ in terms of m and n

$$\log_3 m = x \Rightarrow m = 3^x$$

$$\log_3 n = y \Rightarrow n = 3^y$$

$$\begin{aligned}\therefore 3^{1-2y+3x} &= 3 \cdot 3^{-2y} \cdot 3^{3x} = 3 \cdot (3^y)^{-2} \cdot (3^x)^3 \\ &= 3 \cdot n^{-2} \cdot m^3 = \frac{3m^3}{n^2}\end{aligned}$$

Answer 17A.

$$2\log x + 1 = 40$$

$$\Rightarrow 2\log x = 39$$

$$\Rightarrow \log x^2 = 39$$

$$\Rightarrow x^2 = 10^{39}$$

$$\Rightarrow x = 10^{\frac{39}{2}}$$

Answer 17B.

$$\begin{aligned}2\log x + 1 &= 40 \\ \Rightarrow 2\log x + \log 10 &= 40 \\ \Rightarrow 2\log 10x &= 40 \\ \Rightarrow \log 2 \times 5x &= 20 \\ \Rightarrow \log 2 + \log 5x &= 20 \\ \Rightarrow \log 5x &= 20 - \log 2 \\ \Rightarrow \log 5x &= 20 - 0.3010 \quad \dots\dots (\text{Since } \log 2 = 0.3010) \\ \Rightarrow \log 5x &= 19.6989\end{aligned}$$

Answer 18A.

$$\begin{aligned}\log_{10} 25 &= x \\ \Rightarrow \log_{10} 5^2 &= x \\ \Rightarrow 2\log_{10} 5 &= x \\ \Rightarrow \log_{10} 5 &= \frac{x}{2}\end{aligned}$$

Answer 18B.

$$\begin{aligned}\log_{10} 27 &= y \\ \Rightarrow \log_{10} 3^3 &= y \\ \Rightarrow 3\log_{10} 3 &= y \\ \Rightarrow \log_{10} 3 &= \frac{y}{3}\end{aligned}$$

Answer 19.

(i) $\log 18$

$$\begin{aligned}\log 18 &= \log (2 \times 3^2) = \log 2 + \log 3^2 = \log 2 + 2 \log 3 \\ &= 0.3010 + (2 \times 0.4771) = 1.2552\end{aligned}$$

(ii) $\log 45$

$$\begin{aligned}\log 45 &= \log (3^2 \times 5) = \log 3^2 + \log 5 = 2 \log 3 + \log 5 \\ &= (2 \times 0.4771) + 0.6990 = 1.6532\end{aligned}$$

(iii) $\log 540$

$$\begin{aligned}\log 540 &= \log (2^2 \times 3^3 \times 5) = \log 2^2 + \log 3^3 + \log 5 = 2 \log 2 \\ &\quad + 3 \log 3 + \log 5 \\ &= (2 \times 0.3010) + (3 \times 0.4771) + 0.6990 = 2.7323\end{aligned}$$

(iv) $\log \sqrt{72}$

$$\begin{aligned}\log \sqrt{72} &= \log (72)^{1/2} = \frac{1}{2} \log 72 = \frac{1}{2} \log (2^3 \times 3^2) \\ &= \frac{1}{2} \log 2^3 + \frac{1}{2} \log 3^2 = \frac{3}{2} \log 2 + \frac{2}{2} \log 3 = \frac{3}{2} \log 2 + \log 3 \\ &= \left(\frac{3}{2} \times 0.3010 \right) + 0.4771 = 0.9286\end{aligned}$$

Answer 20.

$$2\log y - \log x - 3 = 0$$

$$\Rightarrow \log x = 2\log y - 3$$

$$\Rightarrow \log x = \log y^2 - 3\log 10 \quad [\because \log 10 = 1]$$

$$\Rightarrow \log x = \log y^2 - \log 10^3$$

$$\Rightarrow \log x = \log \left(\frac{y^2}{1000} \right)$$

$$\therefore x = \frac{y^2}{1000}$$

Answer 21.

$$(i) \log 60$$

$$\log 60 = \log (2 \times 3 \times 10) = \log 2 + \log 3 + \log 10 = x + y + 1$$

$$(ii) \log 1.2$$

$$\log 1.2 = \log \left(\frac{12}{10} \right) = \log 12 - \log 10 = \log (2^2 \times 3) - 1$$

$$= \log 2^2 + \log 3 - 1$$

$$= 2 \log 2 + \log 3 - 1 = 2x + y - 1$$

Answer 22.

$$(i) \log 8$$

$$\log 4 = 0.6020 \Rightarrow \log 2^2 = 0.6020 \Rightarrow 2 \log 2 = 0.6020 \Rightarrow \log 2 =$$

$$\frac{0.6020}{2} = 0.3010$$

$$\therefore \log 8 = \log 2^3 = 3 \log 2 = 3 \times 0.3010 = 0.9030$$

$$(ii) \log 2.5$$

$$\log 2.5 = \log \left(\frac{10}{4} \right) = \log 10 - \log 4 = 1 - 0.6020 = 0.3980$$

Answer 23.

$$(i) \log 4$$

$$\log 8 = \log 2^3 = 3 \log 2 = 0.90 \Rightarrow \log 2 = 0.90 / 3 = 0.3$$

$$\therefore \log 4 = \log 2^2 = 2 \log 2 = 2 \times 0.30 = 0.60$$

$$(ii) \log \sqrt{32}$$

$$\log 8 = \log 2^3 = 3 \log 2 = 0.90 \Rightarrow \log 2 = 0.90 / 3 = 0.3$$

$$\log \sqrt{32} = \frac{1}{2} \log 32 = \frac{1}{2} \log 2^5 = \frac{5}{2} \log 2 = \frac{5}{2} \times 0.30 = 0.75$$

Answer 24.

$$\log 27 = \log 3^3 = 3 \log 3 = 1.431 \Rightarrow \log 3 = \frac{1.431}{3} = 0.477$$

$$\therefore \log 9 = \log 3^2 = 2 \log 3 = 2 \times 0.477 = 0.954$$

$$\log 300 = \log(3 \times 100) = \log(3 \times 10^2) = \log 3 + 2 \log 10 = 0.477 + 2 = 2.477$$

Answer 25.

$$x^2 + y^2 = 6xy$$

$$\Rightarrow x^2 + y^2 - 2xy = 6xy - 2xy$$

$$\Rightarrow (x - y)^2 = 4xy$$

$$\Rightarrow \left(\frac{x - y}{2}\right)^2 = xy$$

$$\Rightarrow \left(\frac{x - y}{2}\right) = \sqrt{xy}$$

Considering $\log$ both sides, we get

$$\log \left(\frac{x - y}{2}\right) = \log(xy)^{1/2}$$

$$\Rightarrow \log \left(\frac{x - y}{2}\right) = \frac{1}{2} \log(xy)$$

$$\Rightarrow \log \left(\frac{x - y}{2}\right) = \frac{1}{2} [\log x + \log y]$$

Answer 26.

$$x^2 + y^2 = 7xy$$

$$\Rightarrow x^2 + y^2 + 2xy = 7xy + 2xy$$

$$\Rightarrow (x + y)^2 = 9xy$$

$$\Rightarrow \left(\frac{x + y}{3}\right)^2 = xy$$

$$\Rightarrow \left(\frac{x + y}{3}\right) = \sqrt{xy}$$

Considering $\log$ both sides, we get

$$\log \left(\frac{x + y}{3}\right) = \log(xy)^{1/2}$$

$$\Rightarrow \log \left(\frac{x + y}{3}\right) = \frac{1}{2} \log(xy)$$

$$\Rightarrow \log \left(\frac{x + y}{3}\right) = \frac{1}{2} [\log x + \log y]$$

Answer 27.

$$\frac{\log x}{\log 5} = \frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$$

Considering the first equality

$$\frac{\log x}{\log 5} = \frac{\log 36}{\log 6}$$

$$\Rightarrow \frac{\log x}{\log 5} = \frac{\log 6^2}{\log 6} = \frac{2\log 6}{\log 6} = 2$$

$$\Rightarrow \log x = 2\log 5 = \log 5^2 = \log 25$$

$$\therefore x = 25$$

Considering the second equality

$$\frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$$

$$\Rightarrow \frac{\log 6^2}{\log 6} = \frac{2\log 6}{\log 6} = 2 = \frac{\log 64}{\log y}$$

$$\Rightarrow \log y = \frac{\log 64}{2} = \frac{\log 8^2}{2} = \frac{2\log 8}{2} = \log 8$$

$$\therefore y = 8$$

Answer 28.

$$\log x^2 - \log \sqrt{y} = 1$$

$$\Rightarrow \log \left(\frac{x^2}{\sqrt{y}} \right) = \log 10$$

$$\Rightarrow \frac{x^2}{\sqrt{y}} = 10$$

$$\Rightarrow \sqrt{y} = \frac{x^2}{10}$$

Squaring both sides, we get

$$y = \left(\frac{x^2}{10} \right)^2 = \frac{x^4}{100}$$

$$\text{Now, when } x = 2, y = \frac{2^4}{100} = \frac{16}{100} = \frac{4}{25}$$

Answer 29.

(i) x

$$2\log x + 1 = \log 360$$

$$\Rightarrow \log x^2 + \log 10 = \log 360$$

$$\Rightarrow \log(10x^2) = \log 360$$

$$\Rightarrow 10x^2 = 360$$

$$\Rightarrow x^2 = \frac{360}{10} = 36$$

$$\Rightarrow x = \sqrt{36} = \pm 6$$

As negative value is rejected,

$$\therefore x = 6$$

(ii) $\log(2x - 2)$

$$2\log x + 1 = \log 360$$

$$\Rightarrow \log x^2 + \log 10 = \log 360$$

$$\Rightarrow \log(10x^2) = \log 360$$

$$\Rightarrow 10x^2 = 360$$

$$\Rightarrow x^2 = \frac{360}{10} = 36$$

$$\Rightarrow x = \sqrt{36} = \pm 6$$

As negative value is rejected,

$$\therefore x = 6$$

$$\therefore \log(2x - 2) = \log(2.6 - 2) = \log 10 = 1$$

(iii) $\log(3x^2 - 8)$

$$2\log x + 1 = \log 360$$

$$\Rightarrow \log x^2 + \log 10 = \log 360$$

$$\Rightarrow \log(10x^2) = \log 360$$

$$\Rightarrow 10x^2 = 360$$

$$\Rightarrow x^2 = \frac{360}{10} = 36$$

$$\Rightarrow x = \sqrt{36} = \pm 6$$

As negative value is rejected,

$$\therefore x = 6$$

$$\begin{aligned}\therefore \log(3x^2 - 8) &= \log\{3(6)^2 - 8\} = \log(108 - 8) = \log 100 = \log 10^2 \\ &= 2\log 10 = 2 \times 1 = 2\end{aligned}$$

Answer 30.

$$\begin{aligned}
x + \log 4 + 2 \log 5 + 3 \log 3 + 2 \log 2 &= \log 108 \\
\Rightarrow x &= \log 108 - \log 4 - 2 \log 5 - 3 \log 3 - 2 \log 2 \\
&= \log (2^2 \cdot 3^3) - \log 2^2 - \log 5^2 - \log 3^3 - \log 2^2 \\
&= \log \left(\frac{2^2 \cdot 3^3}{2^2 \cdot 5^2 \cdot 3^3 \cdot 2^2} \right) = \log \left(\frac{1}{100} \right) \\
\Rightarrow x &= \log 1 - \log 100 = 0 - 2 = -2 \\
\therefore x &= -2
\end{aligned}$$

Answer 21A.

$$\begin{aligned}
&\log a^2 + \log a^{-1} \\
&= 2 \log a + (-1) \log a \\
&= 2 \log a - \log a \\
&= \log a (2 - 1) \\
&= \log a
\end{aligned}$$

Answer 31B.

$$\begin{aligned}
&\log b \div \log b^2 \\
&= \log b \div 2 \log b \\
&= \frac{\log b}{2 \log b} \\
&= \frac{1}{2}
\end{aligned}$$

Answer 32A.

$$\begin{aligned}
\frac{\log \sqrt{8}}{8} &= \frac{\log 2\sqrt{2}}{8} \\
&= \frac{\log 2\sqrt{2}}{8} \\
&= \frac{1}{8} (\log 2 + \log \sqrt{2}) \\
&= \frac{1}{8} \left(\log 2 + \log 2^{\frac{1}{2}} \right) \\
&= \frac{1}{8} \log 2 + \frac{1}{8} \log 2^{\frac{1}{2}} \\
&= \frac{1}{8} \log 2 + \frac{1}{2} \cdot \frac{1}{8} \log 2 \\
&= \frac{1}{8} \log 2 + \frac{1}{16} \log 2 \\
&= \frac{2}{16} \log 2 + \frac{1}{16} \log 2 \\
&= \frac{3}{16} \log 2
\end{aligned}$$

Answer 32B.

$$\begin{aligned}
& \frac{\log \sqrt{27} + \log 8 + \log \sqrt{1000}}{\log 120} \\
&= \frac{\log (27)^{\frac{1}{2}} + \log 2^3 + \log 1000^{\frac{1}{2}}}{\log (3 \times 2^2 \times 10)} \\
&= \frac{\log (3)^{3 \times \frac{1}{2}} + \log 2^3 + \log (10)^{3 \times \frac{1}{2}}}{\log 3 + \log 2^2 + \log 10} \\
&= \frac{\frac{3}{2} \log 3 + 3 \log 2 + \frac{3}{2} \log (10)}{\log 3 + 2 \log 2 + \log 10} \\
&= \frac{\frac{3}{2} \log 3 + \frac{3}{2} (2 \log 2) + \frac{3}{2} (1)}{\log 3 + 2 \log 2 + 1} \\
&= \frac{\frac{3}{2} [\log 3 + 2 \log 2 + 1]}{\log 3 + 2 \log 2 + 1} \\
&= \frac{3}{2}
\end{aligned}$$

Answer 32C.

$$\begin{aligned}
& \frac{\log \sqrt{125} - \log \sqrt{27} - \log \sqrt{8}}{\log 6 - \log 5} \\
&= \frac{\log (125)^{\frac{1}{2}} - \log (27)^{\frac{1}{2}} - \log (8)^{\frac{1}{2}}}{\log 6 - \log 5} \\
&= \frac{\log (5)^{3 \times \frac{1}{2}} - \log (3)^{3 \times \frac{1}{2}} - \log (2)^{3 \times \frac{1}{2}}}{\log 6 - \log 5} \\
&= \frac{\frac{3}{2} \log (5) - \frac{3}{2} \log (3) - \frac{3}{2} \log (2)}{\log (2 \times 3) - \log 5} \\
&= \frac{\frac{3}{2} [\log (5) - \log (3) - \log (2)]}{\log 2 + \log 3 - \log 5} \\
&= \frac{\frac{3}{2} [\log (5) - \log (3) - \log (2)]}{-[\log 5 - \log 3 - \log 2]} \\
&= -\frac{3}{2}
\end{aligned}$$

Answer 33.

$$\begin{aligned}
& \text{Consider } \log(5^{a+b-c}) \\
&= (a+b-c)\log 5 = \left(\log \frac{3}{5} + \log \frac{5}{4} - 2\log \sqrt{\frac{3}{4}}\right)\log 5 \\
&= \left(\log \frac{3}{5} + \log \frac{5}{4} - \log \left[\sqrt{\frac{3}{4}}\right]^2\right)\log 5 = \left(\log \frac{3}{5} + \log \frac{5}{4} - \log \frac{3}{4}\right)\log 5 \\
&= \log \left(\frac{\frac{3}{5} \times \frac{5}{4}}{\frac{3}{4}}\right)\log 5 = \log 1 \times \log 5 = 0 \quad [\because \log 1 = 0] \\
\therefore 5^{a+b-c} &= 10^0 = 1
\end{aligned}$$

Answer 34.

$$\begin{aligned}
\text{(i)} \quad & 3 \log x - 2 \log y = 2 \\
& \Rightarrow \log x^3 - \log y^2 = 2 \log 10 \\
& \Rightarrow \log \left(\frac{x^3}{y^2}\right) = \log 10^2 = \log 100 \\
& \Rightarrow \left(\frac{x^3}{y^2}\right) = 100 \\
& \Rightarrow x^3 = 100 y^2 \\
\text{(ii)} \quad & 2 \log x + 3 \log y = \log a \\
& \Rightarrow \log x^2 + \log y^3 = \log a \\
& \Rightarrow \log (x^2 \cdot y^3) = \log a \\
& \Rightarrow x^2 y^3 = a \\
\text{(iii)} \quad & m \log x - n \log y = 2 \log 5 \\
& \Rightarrow \log x^m - \log y^n = \log 5^2 \\
& \Rightarrow \log \left(\frac{x^m}{y^n}\right) = \log 5^2 \\
& \Rightarrow \left(\frac{x^m}{y^n}\right) = 5^2 = 25 \\
& \Rightarrow x^m = 25 y^n
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad & 2\log x + \frac{1}{2}\log y = 1 \\
\Rightarrow & \log x^2 + \log \sqrt{y} = \log 10 \\
\Rightarrow & \log (x^2 \sqrt{y}) = \log 10 \\
\Rightarrow & x^2 \sqrt{y} = 10 \\
\text{(v)} \quad & 5\log m - 1 = 3\log n \\
\Rightarrow & \log m^5 - \log 10 = \log n^3 \\
\Rightarrow & \log \left(\frac{m^5}{10} \right) = \log n^3 \\
\Rightarrow & \left(\frac{m^5}{10} \right) = n^3 \\
\Rightarrow & m^5 = 10 n^3
\end{aligned}$$

Answer 35.

$$\begin{aligned}
\log (1+2+3) &= \log 6 \\
&= \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3 \\
\text{No, this property is not true for any numbers } x, y, z \\
\text{For example, } \log (1+3+5) &= \log 9 \\
\log 1 + \log 3 + \log 5 &= \log (1 \times 3 \times 5) = \log 15 \\
\log (1+3+5) &\neq \log 1 + \log 3 + \log 5
\end{aligned}$$

Answer 36.

$$\begin{aligned}
\text{L.H.S.} &= (\log a)^2 - (\log b)^2 \\
&= (\log a + \log b)(\log a - \log b) \quad \left\{ \text{using identity } m^2 - n^2 = (m+n)(m-n) \right\} \\
&= \log(ab) \log \left(\frac{a}{b} \right) = \log \left(\frac{a}{b} \right) \cdot \log(ab) = \text{R.H.S}
\end{aligned}$$

Answer 37.

$$\begin{aligned}
\text{Given } & a\log b + b\log a - 1 = 0 \\
\Rightarrow & \log(b)^a + \log(a)^b - \log 10 = 0 \\
\Rightarrow & \log(b^a \cdot a^b) = \log 10 \\
\therefore & b^a \cdot a^b = 10
\end{aligned}$$

Answer 38.

$$\begin{aligned}
& \log(a+1) = \log(4a-3) - \log 3 \\
\Rightarrow & \log(a+1) + \log 3 = \log(4a-3) \\
\Rightarrow & \log\{3(a+1)\} = \log(4a-3) \\
\Rightarrow & 3(a+1) = 4a-3 \\
\Rightarrow & 3a+3 = 4a-3 \\
\Rightarrow & 4a-3a = 3+3 \\
\therefore & a = 6
\end{aligned}$$

Answer 39.

$$\begin{aligned}
\text{L.H.S} &= \log_{10} 125 \\
&= \log_{10} \left(\frac{1000}{8} \right) \\
&= \log_{10} 1000 - \log_{10} 8 \\
&= \log_{10} (10)^3 - \log_{10} (2)^3 \\
&= 3\log_{10} 10 - 3\log_{10} 2 \\
&= 3 \times 1 - 3\log_{10} 2 \\
&= 3(1 - \log_{10} 2) = \text{R.H.S.}
\end{aligned}$$

Answer 40.

$$\begin{aligned}
\text{LHS} &= \frac{\log_p x}{\log_{pq} x} \\
&= \frac{\left(\frac{\log x}{\log p} \right)}{\left(\frac{\log x}{\log pq} \right)} \\
&= \frac{\log x}{\log p} \times \frac{\log pq}{\log x} \\
&= \frac{\log pq}{\log p} \\
&= \frac{\log p + \log q}{\log p} \\
&= 1 + \frac{\log q}{\log p} \\
&= 1 + \log_p q \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

Answer 41A.

$$\begin{aligned}
\text{LHS} &= \frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30} \\
&= \frac{1}{\frac{\log 30}{\log 2}} + \frac{1}{\frac{\log 30}{\log 3}} + \frac{1}{\frac{\log 30}{\log 5}} \\
&= \frac{\log 2}{\log 30} + \frac{\log 3}{\log 30} + \frac{\log 5}{\log 30} \\
&= \frac{1}{\log 30} (\log 2 + \log 3 + \log 5) \\
&= \frac{1}{\log(2 \times 3 \times 5)} (\log 2 + \log 3 + \log 5) \\
&= \frac{(\log 2 + \log 3 + \log 5)}{(\log 2 + \log 3 + \log 5)} \\
&= 1 \\
&= \text{LHS}
\end{aligned}$$

Hence proved.

Answer 41B.

$$\begin{aligned}
\text{LHS} &= \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} \\
&= \log_{36} 8 + \log_{36} 9 + \log_{36} 18 \\
&= \frac{\log 8}{\log 36} + \frac{\log 9}{\log 36} + \frac{\log 18}{\log 36} \\
&= \frac{1}{\log 36} (\log 8 + \log 9 + \log 18) \\
&= \frac{1}{\log 36} (\log 2^3 + \log 3^2 + \log(2 \times 3^2)) \\
&= \frac{1}{\log(2^2 \times 3^2)} (\log 2^3 + \log 3^2 + \log 2 + \log 3^2) \\
&= \frac{1}{\log(2^2 \times 3^2)} (3\log 2 + 2\log 3 + \log 2 + 2\log 3) \\
&= \frac{1}{2\log 2 + 2\log 3} (4\log 2 + 4\log 3) \\
&= \frac{4}{2(\log 2 + \log 3)} (\log 2 + \log 3) \\
&= 2 \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

Answer 42.

$$a = \log \frac{p^2}{qr}, b = \log \frac{q^2}{rp}, c = \log \frac{r^2}{pq}$$

Consider,

$$a + b + c$$

$$= \log \frac{p^2}{qr} + \log \frac{q^2}{rp} + \log \frac{r^2}{pq}$$

$$= \log p^2 - \log qr + \log q^2 - \log rp + \log r^2 - \log pq$$

$$= 2\log p - (\log q + \log r) + 2\log q - (\log r + \log p) + 2\log r - (\log p + \log q)$$

$$= 2\log p - \log q - \log r + 2\log q - \log r - \log p + 2\log r - \log p - \log q$$

$$= 0$$

Answer 43.

$$a = \log 20, \quad b = \log 25 \text{ and}$$

$$2 \log (p - 4) = 2a - b$$

$$\Rightarrow 2 \log (p - 4) = 2\log 20 - \log 25$$

$$\Rightarrow \log (p - 4)^2 = \log 20^2 - \log 25$$

$$\Rightarrow \log (p - 4)^2 = \log \left(\frac{400}{25} \right)$$

$$\Rightarrow (p - 4)^2 = \frac{400}{25}$$

$$\Rightarrow p^2 - 8p + 16 = 16$$

$$\Rightarrow p^2 - 8p = 0$$

$$\Rightarrow p(p - 8) = 0$$

$$\Rightarrow p = 0 \text{ or } p = 8$$