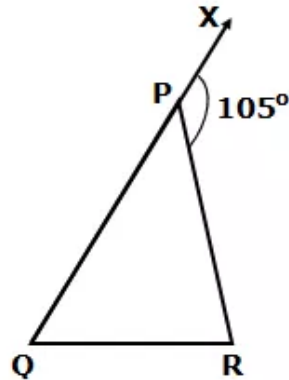


Chapter 11. Triangles and Their Congruency

Ex 11.1

Answer 1.



$$\angle Q : \angle R = 1 : 2$$

$$\text{Let } \angle Q = x^\circ$$

$$\Rightarrow \angle R = 2x^\circ$$

$$\text{Now, } \angle RPX = \angle Q + \angle R \quad \dots [\text{Exterior angle property}]$$

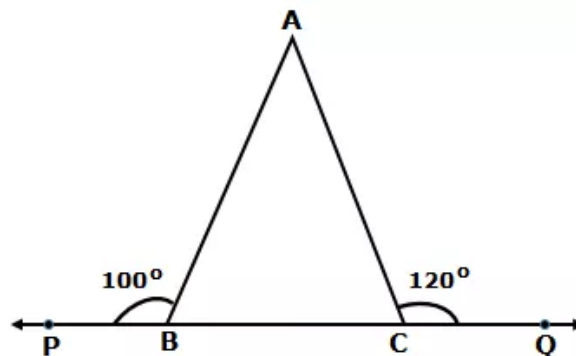
$$\Rightarrow 105^\circ = x^\circ + 2x^\circ$$

$$\Rightarrow 105^\circ = 3x^\circ$$

$$\Rightarrow x^\circ = 35^\circ$$

$$\Rightarrow \angle Q = x^\circ = 35^\circ \text{ and } \angle R = 2x^\circ = 70^\circ$$

Answer 2.



$$\angle ABP + \angle ABC = 180^\circ \quad \dots (\text{Linear pair})$$

$$\Rightarrow 100^\circ + \angle ABC = 180^\circ$$

$$\Rightarrow \angle ABC = 180^\circ - 100^\circ = 80^\circ$$

$$\angle ACQ + \angle ACB = 180^\circ \quad \dots (\text{Linear pair})$$

$$\Rightarrow 120^\circ + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ACB = 180^\circ - 120^\circ = 60^\circ$$

Now, in $\triangle ABC$,

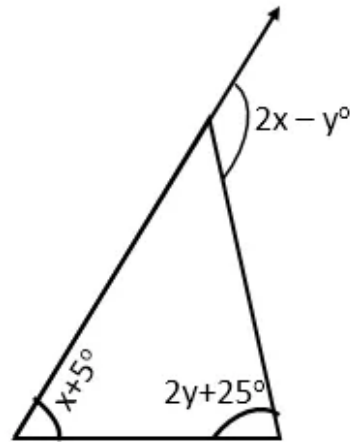
$$\angle A + \angle B + \angle C = 180^\circ \quad \dots (\text{Angle sum property of a triangle})$$

$$\Rightarrow \angle A + 80^\circ + 60^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 80^\circ - 60^\circ = 40^\circ$$

Hence, the angles of a triangle are 40° , 60° and 80° .

Answer 3.



$$(2x - y)^\circ = (x + 5)^\circ + (2y + 25)^\circ \quad \dots(\text{Exterior angle property})$$

$$\Rightarrow 2x^\circ - y^\circ = x^\circ + 5^\circ + 2y^\circ + 25^\circ$$

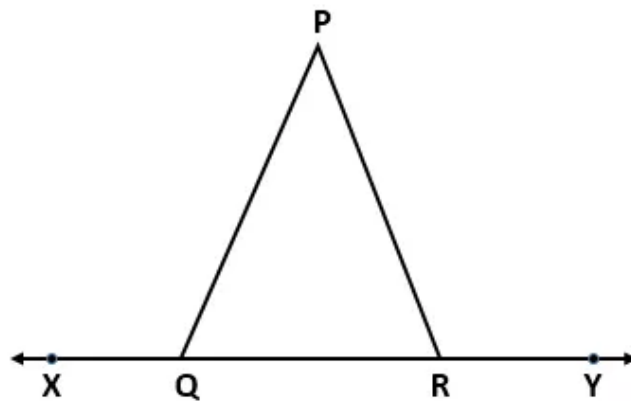
$$\Rightarrow 2x^\circ - x^\circ = 2y^\circ + y^\circ + 30^\circ$$

$$\Rightarrow x^\circ = 3y^\circ + 30^\circ$$

When $y = 15^\circ$, we have

$$x^\circ = 3 \times 15^\circ + 30^\circ = 45^\circ + 30^\circ = 75^\circ$$

Answer 4.



In $\triangle PQR$,

$$\angle P + \angle Q = 130^\circ \quad \dots(\text{given})$$

$$\text{Now, } \angle P + \angle Q = \angle PRY \quad \dots(\text{Exterior angle property})$$

$$\Rightarrow \angle PRY = 130^\circ$$

$$\angle PRY + \angle R = 180^\circ \quad \dots(\text{Linear pair})$$

$$\Rightarrow 130^\circ + \angle R = 180^\circ$$

$$\Rightarrow \angle R = 180^\circ - 130^\circ = 50^\circ$$

Also, $\angle P + \angle R = 120^\circ$ (given)

Now, $\angle P + \angle R = \angle PQX$ (Exterior angle property)

$$\Rightarrow \angle PQX = 120^\circ$$

$$\angle PQX + \angle Q = 180^\circ \quad \text{....(Linear pair)}$$

$$\Rightarrow 120^\circ + \angle Q = 180^\circ$$

$$\Rightarrow \angle Q = 180^\circ - 120^\circ = 60^\circ$$

In $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ \quad \text{....(Angle sum property of a triangle)}$$

$$\Rightarrow \angle P + 60^\circ + 50^\circ = 180^\circ$$

$$\Rightarrow \angle P = 180^\circ - 110^\circ = 70^\circ$$

Thus, the angles of $\triangle PQR$ are as follows:

$$\angle P = 70^\circ, \angle Q = 60^\circ \text{ and } \angle R = 50^\circ$$

Answer 5.

For any triangle, sum of measures of all three angles = 180°

Thus, we have

$$(x + 10)^\circ + (x + 30)^\circ + (x - 10)^\circ = 180^\circ$$

$$\Rightarrow x^\circ + 10^\circ + x^\circ + 30^\circ + x^\circ - 10^\circ = 180^\circ$$

$$\Rightarrow 3x^\circ + 30^\circ = 180^\circ$$

$$\Rightarrow 3x^\circ = 150^\circ$$

$$\Rightarrow x = 50$$

Now,

$$(x + 10)^\circ = (50 + 10)^\circ = 60^\circ$$

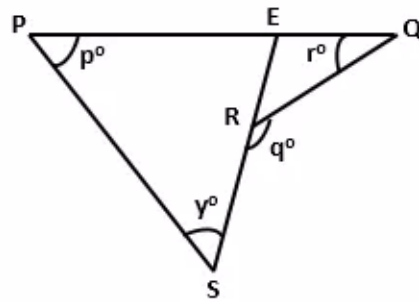
$$(x + 30)^\circ = (50 + 30)^\circ = 80^\circ$$

$$(x - 10)^\circ = (50 - 10)^\circ = 40^\circ$$

Thus, the angles of a triangle are 60° , 80° and 40° .

Answer 6.

SR is produced to meet PQ at E.



In $\triangle PSE$,

$$\angle P + \angle S + \angle PES = 180^\circ$$

....(Angle sum property of a triangle)

$$\Rightarrow p^\circ + y^\circ + \angle PES = 180^\circ$$

$$\Rightarrow \angle PES = 180^\circ - p^\circ - y^\circ$$

....(i)

In $\triangle RQE$,

$$\angle R + \angle Q + \angle REQ = 180^\circ$$

....(Angle sum property of a triangle)

$$\Rightarrow (180^\circ - q^\circ) + r^\circ + \angle REQ = 180^\circ$$

$$\Rightarrow \angle REQ = 180^\circ - (180^\circ - q^\circ) - r^\circ$$

$$\Rightarrow \angle REQ = q^\circ - r^\circ$$

....(ii)

$$\text{Now, } \angle PES + \angle REQ = 180^\circ$$

....(Linear pair)

$$\Rightarrow (180^\circ - p^\circ - y^\circ) + (q^\circ - r^\circ) = 180^\circ$$

....[From (i) and (ii)]

$$\Rightarrow -p^\circ - y^\circ + q^\circ - r^\circ = 0$$

$$\Rightarrow -y^\circ = -q^\circ + p^\circ + r^\circ$$

$$\Rightarrow y^\circ = q^\circ - p^\circ - r^\circ$$

Answer 7.

In $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ \quad \text{....(angle sum property)}$$

$$\Rightarrow 4x^\circ + 5x^\circ + 9x^\circ = 180^\circ$$

$$\Rightarrow 18x^\circ = 180^\circ$$

$$\Rightarrow x = 10$$

$$\Rightarrow \angle P = 4x^\circ = 4 \times 10^\circ = 40^\circ$$

$$\angle Q = 5x^\circ = 5 \times 10^\circ = 50^\circ$$

$$\angle QPR = \angle PRS \quad \text{....(Alternate angles)}$$

$$\text{And, } \angle QPR = 40^\circ$$

$$\Rightarrow \angle PRS = 40^\circ$$

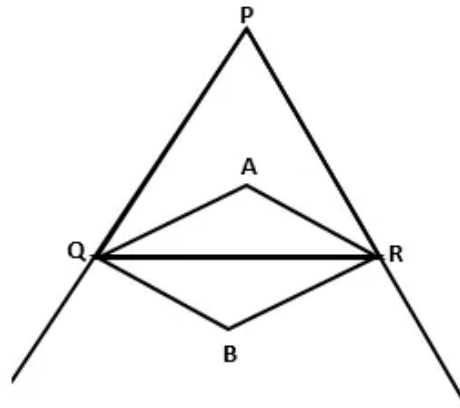
By exterior angle property,

$$\angle PQR + \angle QPR = \angle PRS + y^\circ$$

$$\Rightarrow 40^\circ + 50^\circ = 40^\circ + y^\circ$$

$$\Rightarrow y = 50^\circ$$

Answer 8.



By exterior angle property,

$$\angle RQS = \angle P + \angle R \text{ and } \angle QRT = \angle P + \angle Q$$

Since QB bisects $\angle RQS$,

$$\angle BQR = \frac{1}{2} \angle RQS = \frac{1}{2} (\angle P + \angle R)$$

Also RB bisects $\angle QRT$,

$$\angle BRQ = \frac{1}{2} \angle QRT = \frac{1}{2} (\angle P + \angle Q)$$

In $\triangle QBR$,

$$\angle QBR + \angle BRQ + \angle BQR = 180^\circ$$

$$\Rightarrow \angle QBR + \frac{1}{2} (\angle P + \angle Q) + \frac{1}{2} (\angle P + \angle R) = 180^\circ$$

$$\Rightarrow \angle QBR + \frac{1}{2} (\angle P + \angle Q + \angle P + \angle R) = 180^\circ$$

$$\Rightarrow \angle QBR + \frac{1}{2} (\angle P + 180^\circ) = 180^\circ \quad \dots [\angle P + \angle Q + \angle R = 180^\circ]$$

$$\Rightarrow 2\angle QBR + \angle P + 180^\circ = 360^\circ$$

$$\Rightarrow 2\angle QBR = 180^\circ - \angle P \quad \dots (i)$$

Since QB bisects $\angle PQR$,

$$\angle AQR = \frac{1}{2} \angle PQR$$

Also RA bisects $\angle PRQ$,

$$\angle QRA = \frac{1}{2} \angle PRQ$$

In $\triangle AQR$,

$$\angle AQR + \angle QRA + \angle QAR = 180^\circ$$

$$\Rightarrow \frac{1}{2} \angle PQR + \frac{1}{2} \angle PRQ + \angle QAR = 180^\circ$$

$$\Rightarrow \frac{1}{2} (\angle PQR + \angle PRQ) + \angle QAR = 180^\circ$$

$$\Rightarrow \angle PQR + \angle PRQ + 2\angle QAR = 360^\circ$$

$$\Rightarrow 2\angle QAR = 360^\circ - \angle PQR - \angle PRQ$$

$$\Rightarrow 2\angle QAR = 180^\circ + (180^\circ - \angle PQR - \angle PRQ)$$

$$\Rightarrow 2\angle QAR = 180^\circ + \angle P \quad \dots (ii)$$

Adding (i) and (ii),

$$\Rightarrow 2\angle QAR + 2\angle QBR = 180^\circ + \angle P + 180^\circ - \angle P$$

$$\Rightarrow 2\angle QAR + 2\angle QBR = 360^\circ$$

$$\Rightarrow \angle QAR + \angle QBR = 180^\circ$$

Answer 9.

By exterior angle property,

$$\angle p = \angle PQR + \angle PRQ$$

$$\angle q = \angle QPR + \angle PRQ$$

$$\angle r = \angle PQR + \angle QPR$$

$$\text{Now, } \angle p + \angle q + \angle r = \angle PQR + \angle PRQ + \angle QPR + \angle PRQ + \angle PQR + \angle QPR$$

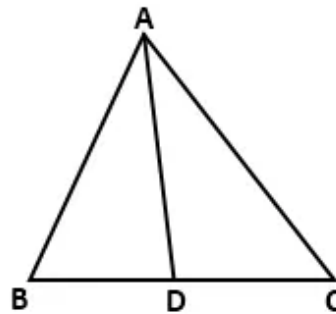
$$\Rightarrow \angle p + \angle q + \angle r = 2\angle PQR + 2\angle PRQ + 2\angle QPR$$

$$\Rightarrow \angle p + \angle q + \angle r = 2(\angle PQR + \angle PRQ + \angle QPR)$$

$$\Rightarrow \angle p + \angle q + \angle r = 2 \times 180^\circ \quad [\text{Angle sum property: } \angle PQR + \angle PRQ + \angle QPR = 180^\circ]$$

$$\Rightarrow \angle p + \angle q + \angle r = 360^\circ$$

Answer 10.



$$\text{Given, } \angle CAD = \angle B \quad \dots(i)$$

By exterior angle property,

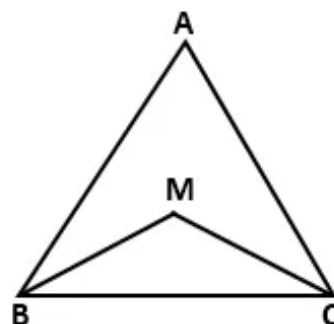
$$\angle ADB = \angle CAD + \angle C$$

$$\text{Also, } \angle ADC = \angle BAD + \angle B$$

$$\Rightarrow \angle ADC = \angle BAD + \angle CAD \quad \dots[\text{From (i)}]$$

$$\Rightarrow \angle ADC = \angle BAC$$

Answer 11.



Since BM and CM are bisectors of $\angle ABC$ and $\angle ACB$,
 $\angle B = 2\angle OBC$ and $\angle C = 2\angle OCB$ (i)

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + 2\angle OBC + 2\angle OCB = 180^\circ \quad \dots[\text{From (i)}]$$

$$\Rightarrow \frac{\angle A}{2} + \angle OBC + \angle OCB = 90^\circ \quad \dots[\text{Dividing both sides by 2}]$$

$$\Rightarrow \angle OBC + \angle OCB = 90^\circ - \frac{\angle A}{2} \quad \dots(\text{ii})$$

Now, in $\triangle BMC$,

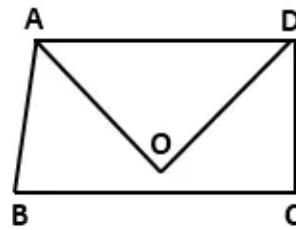
$$\angle OBC + \angle OCB + \angle BMC = 180^\circ$$

$$\Rightarrow 90^\circ - \frac{\angle A}{2} + \angle BMC = 180^\circ \quad \dots[\text{From (ii)}]$$

$$\Rightarrow \angle BMC = 180^\circ - 90^\circ + \frac{\angle A}{2}$$

$$\Rightarrow \angle BMC = 90^\circ + \frac{\angle A}{2}$$

Answer 12.



Since AO and DO are bisectors of $\angle A$ and $\angle D$ of quadrilateral ABCD,
 $\angle A = 2\angle OAD$ and $\angle D = 2\angle ODA$ (i)

In $\triangle AOD$,

$$\angle OAD + \angle ODA + \angle AOD = 180^\circ$$

$$\Rightarrow 2\angle OAD + 2\angle ODA + 2\angle AOD = 360^\circ \quad \dots [\text{Multiplying both sides by 2}]$$

$$\Rightarrow 2\angle OAD + 2\angle ODA = 360^\circ - 2\angle AOD \quad \dots(\text{ii})$$

In quadrilateral ABCD,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 2\angle OAD + \angle B + \angle C + 2\angle ODA = 360^\circ \quad \dots [\text{From (i)}]$$

$$\Rightarrow \angle B + \angle C = 360^\circ - 2\angle OAD - 2\angle ODA$$

$$\Rightarrow \angle B + \angle C = 360^\circ - (2\angle OAD + 2\angle ODA)$$

$$\Rightarrow \angle B + \angle C = 360^\circ - (360^\circ - 2\angle AOD) \quad \dots [\text{From (ii)}]$$

$$\Rightarrow \angle B + \angle C = 360^\circ - 360^\circ + 2\angle AOD$$

$$\Rightarrow \angle B + \angle C = 2\angle AOD$$

Answer 13.

Consider $\triangle ABC$.

Now, $\angle A < \angle B + \angle C$

$$\Rightarrow \angle A + \angle A < \angle A + \angle B + \angle C$$

$$\Rightarrow 2\angle A < 180^\circ$$

$$\Rightarrow \angle A < \frac{180^\circ}{2}$$

$$\Rightarrow \angle A < 90^\circ$$

Similarly, we have

$$\angle B < 90^\circ \text{ and } \angle C < 90^\circ.$$

Hence, the triangle is acute-angled.

Answer 14.

Let the angles of a triangle be $2x$, $4x$ and $6x$.

Then, we have

$$2x + 4x + 6x = 180^\circ$$

$$\Rightarrow 12x = 180^\circ$$

$$\Rightarrow x = 15^\circ$$

$$\Rightarrow 2x = 2 \times 15^\circ = 30^\circ$$

$$4x = 4 \times 15^\circ = 60^\circ$$

$$6x = 6 \times 15^\circ = 90^\circ$$

Since one angle is 90° , the triangle is a right-angled triangle.

Answer 15.

Let ABC be a triangle such that

$$\angle A + \angle B = 139^\circ \quad \dots(i)$$

$$\text{and, } \angle A - \angle B = 5^\circ \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2\angle A = 144^\circ$$

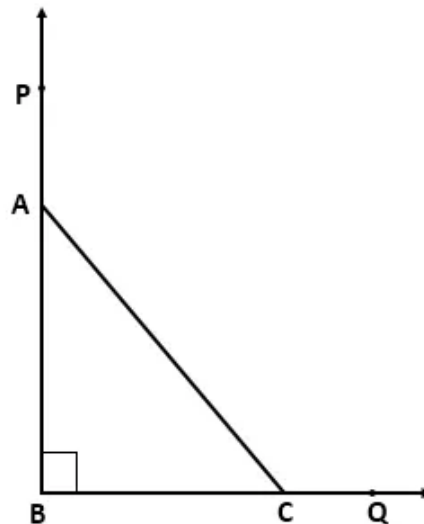
$$\Rightarrow \angle A = 72^\circ$$

From (i), we have

$$\angle B = 139^\circ - 72^\circ = 67^\circ$$

$$\text{Now, 3}^{\text{rd}} \text{ angle} = 180^\circ - (\angle A + \angle B) = 180^\circ - 139^\circ = 41^\circ$$

Thus, the angles of a triangle are 72° , 67° and 41° .

Answer 16.

In $\triangle ABC$, $\angle B = 90^\circ$

And, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

$$\Rightarrow \angle BAC + \angle ACB = 180^\circ - \angle ABC$$

$$\Rightarrow \angle BAC + \angle ACB = 180^\circ - 90^\circ$$

$$\Rightarrow \angle BAC + \angle ACB = 90^\circ \quad \dots(i)$$

By exterior angle property,

$$\angle PAC = \angle ABC + \angle ACB \quad \dots(ii)$$

$$\angle QCA = \angle ABC + \angle BAC \quad \dots(iii)$$

Adding (ii) and (iii), we get

$$\angle PAC + \angle QCA = \angle ABC + \angle ACB + \angle ABC + \angle BAC$$

$$\Rightarrow \angle PAC + \angle QCA = (\angle ACB + \angle BAC) + 2\angle ABC$$

$$\Rightarrow \angle PAC + \angle QCA = 90^\circ + 2 \times 90^\circ \quad \dots[\text{From (i)}]$$

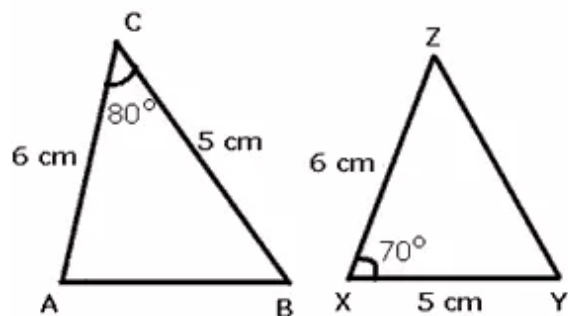
$$\Rightarrow \angle PAC + \angle QCA = 90^\circ + 180^\circ$$

$$\Rightarrow \angle PAC + \angle QCA = 270^\circ$$

Ex 11.2

Answer 1.

(i)



In $\triangle ABC$ and $\triangle XYZ$

$$AC = XZ$$

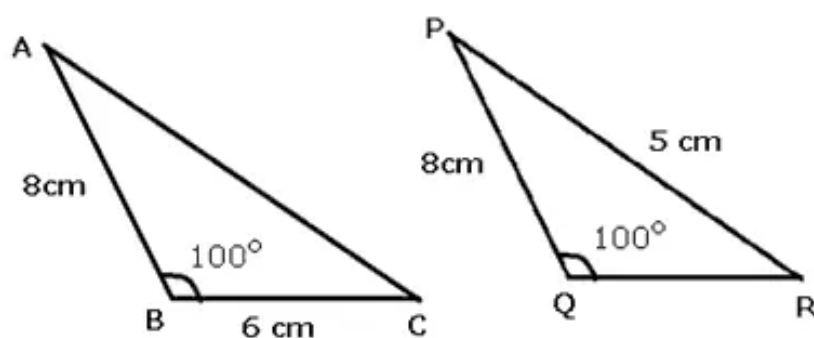
$$BC = XY$$

The included angle $\angle C = 80^\circ$ is not equal to $\angle X$ i.e. 70° .

Now, for $\triangle ABC$ to be congruent to $\triangle XYZ$, AB should be equal to XY and YZ should be equal to BC . Then, $\angle A = \angle Z$ and $\angle X = \angle Y$. So, the measure of $\angle B$ will not be equal to $\angle Y$.

Therefore, $\triangle ABC$ cannot be congruent to $\triangle XYZ$.

(ii)



In $\triangle ABC$ and $\triangle PQR$

$$AB = PQ$$

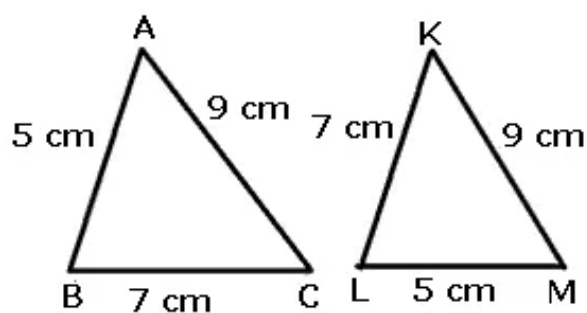
$$\angle B = \angle Q =$$

BC can be equal to QR or AC can be equal to RP

Therefore,

$\triangle ABC$ can be congruent to $\triangle PQR$.

(iii)



In $\triangle ABC$ and $\triangle KLM$

$$AB = LM$$

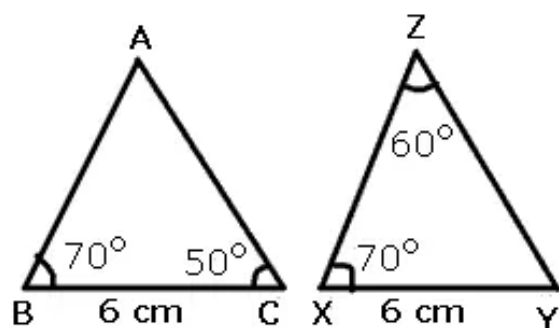
$$BC = KL$$

$$AC = KM$$

Therefore,

$$\triangle ABC \cong \triangle KLM \text{ (SSS criteria)}$$

(iv)



In $\triangle ABC$ and $\triangle XYZ$

$$\angle B = \angle X$$

$$BC = XY$$

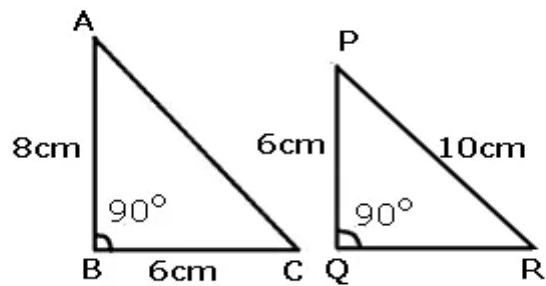
$$\angle Y = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$$

$$\angle C = \angle Y$$

Therefore,

$$\triangle ABC \cong \triangle XYZ \text{ (ASA criteria)}$$

(v)



In $\triangle ABC$ and $\triangle PQR$

$$\angle B = \angle Q$$

$$BC = PQ$$

By Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

$$10^2 = 6^2 + QR^2$$

$$100 = 36 + QR^2$$

$$QR = \sqrt{100 - 36}$$

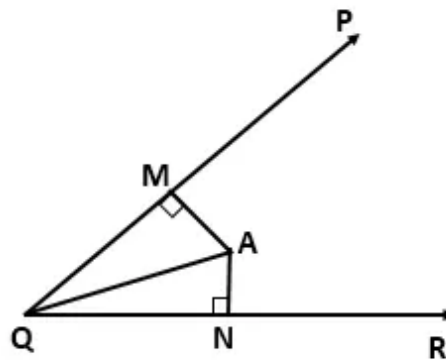
$$QR = \sqrt{64} = 8\text{cm}$$

$$AB = QR$$

Therefore,

$\triangle ABC \cong \triangle PQR$ (SAS and RHS criteria)

Answer 2.



Given,

$AM \perp PQ$ and $AN \perp QR$

$AM = AN$

In $\triangle AQM$ and $\triangle AQN$,

$$AM = AN \quad \dots (\text{given})$$

$$AQ = AQ \quad \dots (\text{common})$$

$$\angle AMQ = \angle ANQ \quad \dots (\text{Each} = 90^\circ)$$

So, by RHS congruence, we have

$$\triangle AQM \cong \triangle AQN$$

$$\Rightarrow \angle AQM = \angle AQN \quad \dots (\text{opct})$$

$$\Rightarrow \angle AQP = \angle AQR$$

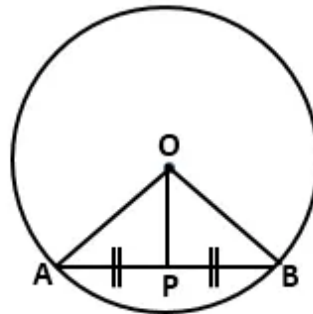
Answer 3.

Given:

In the figure, O is centre of the circle and AB is chord.

P is the mid-point of AB $\Rightarrow$ AP = PB

To prove: $OP \perp AB$



Construction: Join OA and OB

Proof:

In $\triangle OAP$ and $\triangle OBP$

OA = OB[radii of the same circle]

OP = OP[common]

AP = PB[given]

$\therefore$ By Side-Side-Side criterion of congruency,

$\triangle OAP \cong \triangle OBP$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle OPA = \angle OPB$

But $\angle OPA + \angle OPB = 180^\circ$ [linear pair]

$\therefore \angle OPA = \angle OPB = 90^\circ$

Hence $OP \perp AB$.

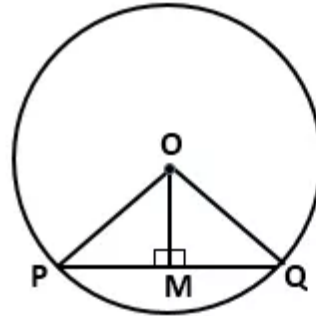
Answer 4.

Given:

In the figure, O is centre of the circle and PQ is a chord.

$OM \perp PQ$

To prove: $PM = QM$



Construction: Join OP and OQ

Proof:

In right triangles $\triangle OPM$ and $\triangle OQM$,

$OP = OQ$ [radii of the same circle]

$OM = OM$ [common]

$\therefore$ By Right angle-Hypotenuse-Side criterion of congruency,

$\triangle OPM \cong \triangle OQM$

The corresponding parts of the congruent triangles are congruent.

$\therefore PM = QM$

Answer 5.

In $\triangle ABC$ and $\triangle PQR$ and

$AB = PQ$

$BC = QR$

$\angle ABX + \angle ABC = \angle PQY + \angle PQR = 180^\circ$

$\angle ABX = \angle PQY$

$\Rightarrow \angle ABC = \angle PQR$

Therefore,

$\triangle ABC \cong \triangle PQR$ (SAS criteria)

Answer 6.

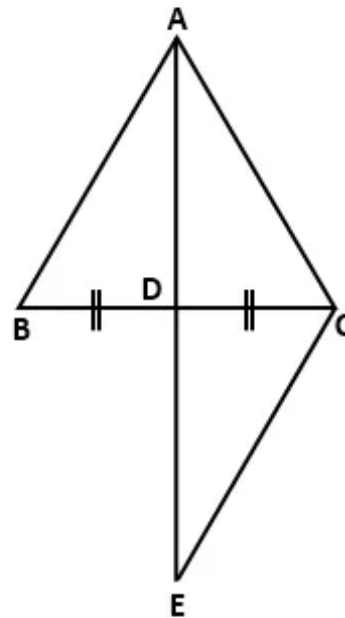
Given:

D is mid-point of BC $\Rightarrow$ BD = DC

DE = AD

To prove:

- a. $\triangle ABD \cong \triangle ECD$
- b. $AB = EC$
- c. $AB \parallel EC$



- a. In $\triangle ABD$ and $\triangle ECD$,
BD = DC(given)

$\angle ADB = \angle CDE$ (vertically opposite angles)

AD = DE(given)

$\therefore$ By Side-Angle-Side criterion of congruence,
 $\triangle ABD \cong \triangle ECD$

- b. The corresponding parts of the congruent triangles are congruent.
 $\therefore AB = EC$

- c. Also, $\angle DAB = \angle DEC$ (cp.ct)
 $\therefore AB \parallel EC$ ($\angle DAB$ and $\angle DEC$ are alternate angles)

Answer 7.

In $\triangle BAP$ and $\triangle CAP$

$\angle BAP = \angle CAP$ (AD is the bisector of $\angle BAC$)

$AP = AP$

$\angle BPD + \angle BPA = \angle CPD + \angle CPA = 180^\circ$

$\angle BPD = \angle CPD$

$\Rightarrow \angle BPA = \angle CPA$

Therefore,

$\triangle CAP \cong \triangle BAP$ (ASA criteria)

Hence, $CP = BP$.

Answer 8.

In $\triangle GCB$ and $\triangle DCE$ and

$\angle 1 + \angle GBC = \angle 2 + \angle DEC = 180^\circ$

$\angle 1 = \angle 2 =$

$\Rightarrow \angle GBC = \angle DEC$

$BC = CE$

$\angle GCB = \angle DCE =$ (vertically opposite angles)

Therefore,

$\triangle GCB \cong \triangle DCE$ (ASA criteria)

Answer 9.

In $\triangle ABC$,

Since $AB = AC$

$\angle C = \angle B$ (angles opposite to the equal sides are equal)

BO and CO are angle bisectors of $\angle B$ and $\angle C$ respectively

Hence, $\angle ABO = \angle OBC = \angle BCO = \angle ACO$

Join AO to meet BC at D

In $\triangle ABO$ and $\triangle ACO$ and

$AO = AO$

$AB = AC$

$\angle C = \angle B =$

Therefore, $\triangle ABO \cong \triangle ACO$ (SAS criteria)

Hence, $\angle BAO = \angle CAO$

$\Rightarrow$ AO bisects angle BAC

In $\triangle ABO$ and $\triangle ACO$

and $AB = AC$

$AO = AO$

$\angle BAD = \angle CAD =$ (proved)

$\triangle ABO \cong \triangle ACO$ (SAS criteria)

Therefore,

$BO = CO$

Answer 10.

In $\triangle ABD$ and $\triangle FEC$

$AB = FE$

$BD = CE$ ($BC = DE$; CD is common)

$\angle B = \angle E$

$\triangle ABD \cong \triangle FEC$ (SAS criteria)

Answer 11.

In $\triangle BMR$ and $\triangle DNR$

$$BM = DN$$

$$\angle BMR = \angle DNR = 90^\circ$$

$$\angle BRM = \angle DRN = \text{(vertically opposite angles)}$$

Hence, $\angle MBR = \angle NDR$ (sum of angles of a triangle = 180°)

$$\triangle BMR \cong \triangle DNR \quad (\text{ASA criteria})$$

Therefore, $BR = DR$

So, AC bisects BD.

Answer 12.

In $\triangle QLM$ and $\triangle RNM$

$$QM = MR$$

$$LM = MN$$

$$\angle QLM = \angle RNM = 90^\circ$$

Therefore, $\triangle QLM \cong \triangle RNM$ (RHS criteria)

Hence, $QL = RN$ (i)

Join PM

In $\triangle PLM$ and $\triangle PNM$ and

$$PM = PM \quad (\text{common})$$

$$LM = MN$$

$$\angle PLM = \angle PNM = 90^\circ$$

Therefore, $\triangle PLM \cong \triangle PNM$ (RHS criteria)

Hence, $PL = PN$ (ii)

From (i) and (ii)

$$PQ = PR$$

Answer 13.

$$\angle 1 = 2\angle 2 \text{ and } \angle 4 = 2\angle 3$$

$$1 = 22 \text{ and } 4 = 23 \angle 1 = \angle 4 \text{ (vertically opposite angles)}$$

$$\Rightarrow 2\angle 2 = 2\angle 3 \text{ or } \angle 2 = \angle 3 \dots\dots\dots(i)$$

$$\angle R = \angle S = \text{ (since } RT = TS \text{ and angle opposite to equal sides are equal)}$$

$$\Rightarrow \angle TRB = \angle TSA = \dots\dots\dots(ii)$$

In $\triangle RBT$ and $\triangle SAT$.

$$RT = TS$$

$$\angle TRB = \angle TSA$$

$$\angle RTB = \angle STA = \text{ (common)}$$

Therefore, $\triangle RBT \cong \triangle SAT$. (ASA criteria)

Answer 14.

In $\triangle CAD$ and $\triangle CBE$

$$CA = CB \text{ (Isosceles triangle)}$$

$$\angle CDA = \angle CEB = 90^\circ$$

$$\angle ACD = \angle BCE = \text{ (common)}$$

Therefore, $\triangle CAD \cong \triangle CBE$ (AAS criteria)

$$\text{Hence, } CE = CD$$

$$\text{But, } CA = CB$$

$$\Rightarrow AE + CE = BD + CD$$

$$\Rightarrow AE = BD$$

Answer 15.

In $\triangle ABC$

$$AB = AC$$

$$AX = AY$$

$$\Rightarrow BX = CY$$

In $\triangle BXC$ and $\triangle CYB$

$$BX = CY$$

$$BC = BC$$

$$\angle B = \angle C \quad (\text{AB} = \text{AC and angles opposite to equal sides are equal})$$

Therefore, $\triangle BXC \cong \triangle CYB$ (SAS criteria)

Hence, $CX = BY$

Answer 16.

Given:

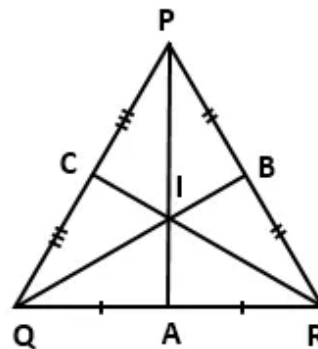
In $\triangle PQR$,

PA is the perpendicular bisector of QR $\Rightarrow QA = RA$

RC is the perpendicular bisector of PQ $\Rightarrow PC = QC$

QB is the perpendicular bisector of PR $\Rightarrow PB = RB$

PA, RC and QB meet at I.



To prove: $IP = IQ = IR$

Proof:

In $\triangle QIA$ and $\triangle RIA$

$$QA = RA \quad \dots [\text{Given}]$$

$$\angle QAI = \angle RAI \quad \dots [\text{Each} = 90^\circ]$$

$$IA = IA \quad \dots [\text{Common}]$$

$\therefore$ By Side-Angle-Side criterion of congruence,

$$\triangle QIA \cong \triangle RIA$$

The corresponding parts of the congruent triangles are congruent.

$$\therefore IQ = IR \quad \dots(i)$$

Similarly, in $\triangle RIB$ and $\triangle PIB$

$$RB = PB \quad \dots[\text{Given}]$$

$$\angle RBI = \angle PBI \quad \dots[\text{Each} = 90^\circ]$$

$$IB = IB \quad \dots[\text{Common}]$$

$\therefore$ By Side-Angle-Side criterion of congruence,

$$\triangle RIB \cong \triangle PIB$$

The corresponding parts of the congruent triangles are congruent.

$$\therefore IR = IP \quad \dots(ii)$$

From (i) and (ii), we have

$$IP = IQ = IR$$

Answer 17.

In $\triangle ADE$ and $\triangle BAC$

$$AE = AC$$

$$AB = AD$$

$$\angle BAD = \angle EAC$$

$$\angle DAC = \angle DAC = \text{DAC (common)}$$

$$\Rightarrow \angle BAC = \angle EAD = \text{EAD}$$

Therefore, $\triangle ADE \cong \triangle BAC$ (SAS criteria)

$$\text{Hence, } BC = DE$$

Answer 18.

Given:

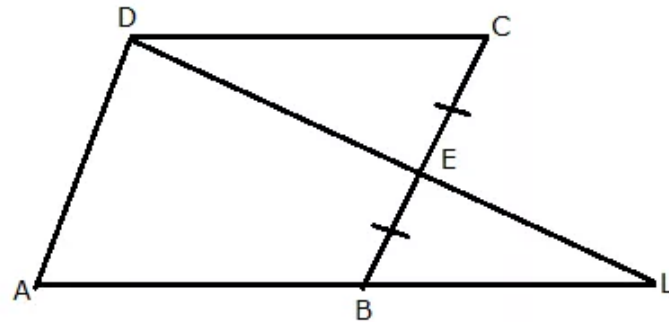
ABCD is a parallelogram, where $BE = CE$

To prove:

a. $\triangle DCE \cong \triangle LBE$

b. $AB = BL$

c. $DC = \frac{AL}{2}$



a. In $\triangle DCE$ and $\triangle LBE$

$\angle DCE = \angle EBL$ [$DC \parallel AB$, alternate angles]

$CE = BE$ [given]

$\angle DEC = \angle LEB$ [vertically opposite angles]

$\therefore$ By Angle-Side-Angle criterion of congruence,
 $\triangle DCE \cong \triangle LBE$

The corresponding parts of the congruent triangles are congruent.

$\therefore DC = LB$ (1)

b. $DC = AB$ (2)[opposite sides of a parallelogram]

From (1) and (2),

$AB = BL$ (3)

c. $AL = AB + BL$

$\Rightarrow AL = AB + AB$ [From (3)]

$\Rightarrow AL = 2AB$

$\Rightarrow AL = 2DC$ [From (2)]

Answer 19.

$$\angle BCD = \angle ADC$$

$$\angle ACB = \angle BDA$$

$$\angle BCD + \angle ACB = \angle ADC + \angle BDA$$

$$\Rightarrow \angle ACD = \angle BDC \quad \angle ACD = \angle BDC$$

In $\triangle ACD$ and $\triangle BCD$

$$\angle ACD = \angle BDC \quad \angle ACD = \angle BDC$$

$$\angle ADC = \angle BCD$$

$$ADC = BCD \quad CD = CD$$

Therefore, $\triangle ACD \cong \triangle BCD$ (ASA criteria)

Hence, $AD = BC$ and $\angle A = \angle B$.

Answer 20.

Since AP and BQ are perpendiculars to the line segment AB , therefore AP and BQ are parallel to each other.

In $\triangle AOP$ and $\triangle BOQ$

$$\angle PAO = \angle QBO = 90^\circ$$

$$\angle APO = \angle BQO \quad (\text{alternate angles})$$

$$AP = BQ$$

Therefore, $\triangle AOP \cong \triangle BOQ$ (ASA criteria)

Hence, $AO = OB$ and $PO = OQ$

Thus, O is the mid-point of line segments AB and PQ .

Answer 21.

CE is median to AB

$$\Rightarrow AE = BE \dots\dots(i)$$

BD is median to AC

$$\Rightarrow AD = DC \dots\dots(ii)$$

But $AB = AC \dots\dots(iii)$

Therefore from (i), (ii) and (iii)

$$BE = CD$$

In $\triangle BEC$ and $\triangle BDC$

$$BE = CD$$

$\angle EBC = \angle DCB$ (angles opposite to equal sides are equal)

$$BC = BC \text{ (common)}$$

Therefore, $\triangle BEC \cong \triangle BDC$ (SAS criteria)

Hence, $BD = CE$

Answer 23.

In $\triangle ABC$ and $\triangle PQR$

$$BC = QR$$

AD and PM are medians of BC and QR respectively

$$\Rightarrow BD = DC = QM = MR$$

In $\triangle ABD$ and $\triangle PQM$

$$AB = PQ$$

$$AD = PM$$

$$BD = QM$$

Therefore, $\triangle ABD \cong \triangle PQM$ (SSS criteria)

Hence, $\angle B = \angle Q$

Now in $\triangle ABC$ and $\triangle PQR$

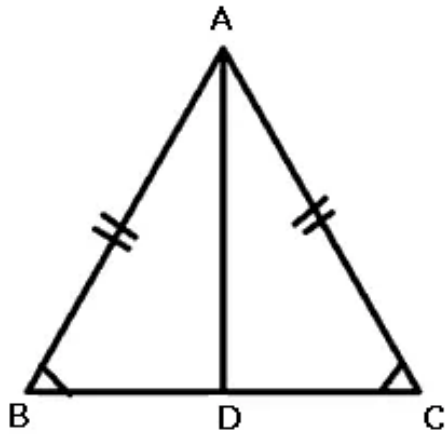
$$AB = PQ$$

$$BC = QR$$

$$\angle B = \angle Q$$

Therefore, $\triangle ABC \cong \triangle PQR$ (SAS criteria)

Answer 24.



Now in $\triangle ABD$ and $\triangle ADC$

$$AB = AC$$

$$AD = AD$$

$$\angle B = \angle C$$

Therefore, $\triangle ABD \cong \triangle ADC$ (SSA criteria)

$$\text{Hence, } BD = DC$$

Thus, AD bisects BC

Answer 25.

Since $AB = AC$

$$\angle ABC = \angle ACB$$

$$\text{But } \angle DBC = \angle DCB$$

$$\Rightarrow \angle ABD = \angle ACD$$

Now in $\triangle ABD$ and $\triangle ADC$

$$AB = AC$$

$$AD = AD$$

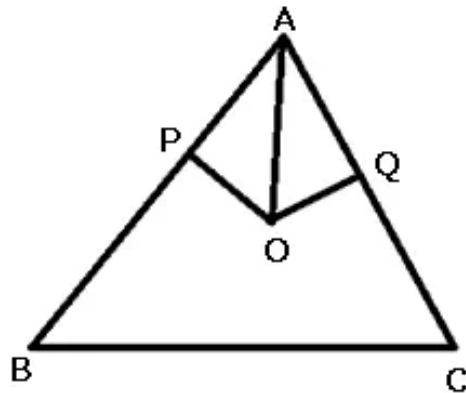
$$\angle ABD = \angle ACD$$

Therefore, $\triangle ABD \cong \triangle ADC$ (SSA criteria)

$$\text{Hence, } \angle BAD = \angle CAD$$

Thus, AD bisects $\angle BAC$

Answer 26.



In $\triangle POA$ and $\triangle QOA$

$$\angle OPA = \angle OQA = 90^\circ$$

$$OP = OQ \text{ (given)}$$

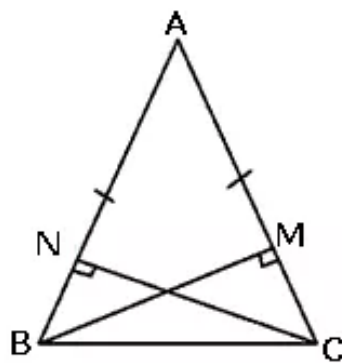
$$AO = AO$$

Therefore, $\triangle POA \cong \triangle QOA$ (SSA criteria)

$$\text{Hence, } \angle PAO = \angle QAO$$

Thus, OA bisects $\angle BAC$

Answer 27.



In $\triangle BNC$ and $\triangle CMB$

$$\angle BNC = \angle CMB = 90^\circ$$

$$\angle NBC = \angle MCB \quad (AB = AC)$$

$$BC = BC$$

Therefore, $\triangle BNC \cong \triangle CMB$ (AAS criteria)

$$\text{Hence, } BM = CN$$

Answer 28.

In $\triangle ABC$

$$AB = AC$$

$$\angle ABC = \angle ACB \quad (\text{equal sides have equal angles opposite to them}) \dots (i)$$

$$\angle GBC = \angle HCB = 90^\circ \dots (ii)$$

Subtracting (i) from (ii)

$$\angle GBA = \angle HCA \dots (iii)$$

In $\triangle GBA$ and $\triangle HCA$

$$\angle GBA = \angle HCA \quad (\text{from iii})$$

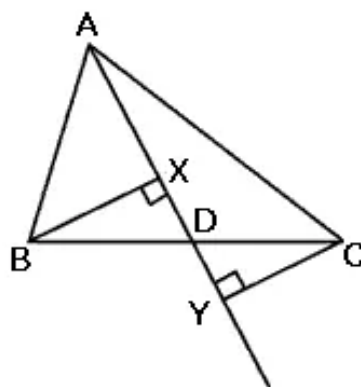
$$\angle BAG = \angle CAH \quad (\text{vertically opposite angles})$$

$$BC = BC$$

Therefore, $\triangle GBA \cong \triangle HCA$ (ASA criteria)

Hence, $BG = CH$ and $AG = AH$

Answer 29.



In $\triangle BXD$ and $\triangle CYD$

$$\angle BXD = \angle CYD \quad (90^\circ)$$

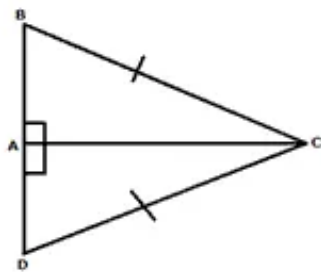
$$\angle XDB = \angle YDC \quad (\text{vertically opposite angles})$$

$$BD = DC \quad (\text{AD is median on BC})$$

Therefore, $\triangle BXD \cong \triangle CYD$ (AAS criteria)

Hence, $BX = CY$

Answer 30.



In $\triangle ABC$ and $\triangle ADC$

$$\angle BAC = \angle DAC \quad (90^\circ)$$

$$BC = DC$$

$$AC = AC \quad (\text{common})$$

Therefore, $\triangle ABC \cong \triangle ADC$ (SSA criteria)

$$\text{Hence, } \angle BCA = \angle DCA$$

Thus, AC bisects $\angle BCD$

Answer 31.

$$\angle PQT = \angle RQU \dots\dots(i)$$

$$\angle TQS = \angle UQS \dots\dots(ii)$$

Adding (i) and (ii)

$$\angle PQS = \angle RQS$$

In $\triangle PQS$ and $\triangle RQS$

$$\angle PQS = \angle RQS$$

$$PQ = RQ \quad (\text{given})$$

$$QS = QS \quad (\text{common})$$

Therefore, $\triangle PQS \cong \triangle RQS$ (SAS criteria)

$$\text{Hence, } \angle QPS = \angle QRS$$

Now in $\triangle PQT$ and $\triangle RQU$

$$\angle QPS = \angle QRS$$

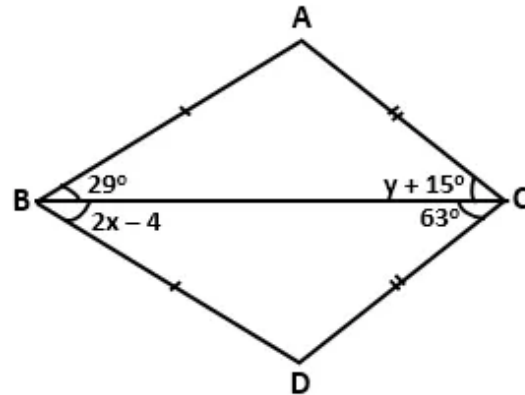
$$PQ = RQ \quad (\text{given})$$

$$\angle PQT = \angle RQU \quad (\text{given})$$

Therefore, $\triangle PQT \cong \triangle RQU$ (ASA criteria)

$$\text{Hence, } QT = QU.$$

Answer 32.



In $\triangle ABC$ and $\triangle DCB$

$$AB = DB \quad \dots [\text{given}]$$

$$AC = DC \quad \dots [\text{given}]$$

$$BC = BC \quad \dots [\text{common}]$$

$\therefore$ By Side-Side-Side criterion of congruence,

$$\triangle ABC \cong \triangle DCB$$

$$\therefore \angle ACB = \angle DCB \quad \dots [\text{c.p.c.t.}]$$

$$\Rightarrow y + 15^\circ = 63^\circ$$

$$\Rightarrow y = 63^\circ - 15^\circ$$

$$\Rightarrow y = 48^\circ$$

$$\text{Now, } \angle ABC = \angle DCB \quad \dots [\text{c.p.c.t.}]$$

$$\Rightarrow 29^\circ = 2x - 4^\circ$$

$$\Rightarrow 2x = 29^\circ + 4^\circ$$

$$\Rightarrow 2x = 33^\circ$$

$$\Rightarrow x = \frac{33^\circ}{2}$$

$$\Rightarrow x = 16.5^\circ$$

Hence, $x = 16.5^\circ$ and $y = 48^\circ$