

Chapter 13. Inequalities in Triangles

Ex 13.1

Answer 1A.

In the given $\triangle ABC$ the greatest angle is $\angle B$ and the opposite side to the $\angle B$ is AC .

Hence, the greatest side is AC .

The smallest angle in the $\triangle ABC$ is $\angle A$ and the opposite side to the $\angle A$ is BC .

Hence, the smallest side is BC .

Answer 1B.

In the given $\triangle DEF$ the greatest angle is $\angle F$ and the opposite side to the $\angle F$ is DE .

Hence, the greatest side is DE .

The smallest angle in the $\triangle DEF$ is $\angle D$ and the opposite side to the $\angle D$ is EF .

Hence, the smallest side is EF .

Answer 1C.

In $\triangle XYZ$,

$$\angle X + \angle Y + \angle Z = 180^\circ$$

$$76^\circ + 84^\circ + \angle Z = 180^\circ$$

$$160^\circ + \angle Z = 180^\circ$$

$$\angle Z = 180^\circ - 160^\circ$$

$$\angle Z = 20^\circ$$

Hence, $\angle X = 76^\circ$, $\angle Y = 84^\circ$, $\angle Z = 20^\circ$

In the given $\triangle XYZ$ the greatest angle is $\angle Y$ and the opposite side to the $\angle Y$ is XZ .

Hence, the greatest side is XZ .

The smallest angle in the $\triangle XYZ$ is $\angle Z$ and the opposite side to the $\angle Z$ is XY .

Hence, the smallest side is XY .

Answer 2A.

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$45^\circ + 65^\circ + \angle C = 180^\circ$$

$$110^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 110^\circ$$

$$\angle C = 70^\circ$$

Hence, $\angle A = 45^\circ$, $\angle B = 65^\circ$, $\angle C = 70^\circ$

$$45^\circ < 65^\circ < 70^\circ$$

Hence, ascending order of the angles in the given triangle is $\angle A < \angle B < \angle C$.

Hence, ascending order of sides in triangle BC, AC, AB.

Answer 2B.

In $\triangle DEF$,

$$\angle D + \angle E + \angle F = 180^\circ$$

$$38^\circ + 58^\circ + \angle F = 180^\circ$$

$$96^\circ + \angle F = 180^\circ$$

$$\angle F = 180^\circ - 96^\circ$$

$$\angle F = 84^\circ$$

Hence, $\angle D = 38^\circ$, $\angle E = 58^\circ$, $\angle F = 84^\circ$

$$38^\circ < 58^\circ < 84^\circ$$

Hence, ascending order of the angles in the given triangle is $\angle D < \angle E < \angle F$.

Hence, ascending order of sides in triangle EF, DF, DE.

Answer 3.

- (i) It is known that, in a triangle, the angle opposite to the smallest side is the smallest.

In $\triangle ABC$, AC = 4.2cm is the smallest side.

$\therefore \angle B$ is the smallest angle.

- (ii) It is known that, in a triangle, the angle opposite to the smallest side is the smallest.

In $\triangle PQR$, QR = 5.4cm is the smallest side.

$\therefore \angle P$ is the smallest angle.

- (iii) It is known that, in a triangle, the angle opposite to the smallest side is the smallest.

In $\triangle XYZ$, YZ = 5cm is the smallest side.

$\therefore \angle X$ is the smallest angle.

Answer 4.

In $\triangle ABC$,

$BC = AC$ (given)

$$\Rightarrow \angle A = \angle B = 35^\circ$$

Let $\angle C = x^\circ$

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$35^\circ + 35^\circ + x^\circ = 180^\circ$$

$$70^\circ + x^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 70^\circ$$

$$x^\circ = 110^\circ$$

$$\angle C = x^\circ = 110^\circ$$

Hence, $\angle A = \angle B = 35^\circ$ and $\angle C = 110^\circ$

In $\triangle ABC$, the greatest angle is $\angle C$.

As the smallest angles are $\angle A$ and $\angle B$,
smallest sides are BC and AC .

Answer 5.

It is given that $\angle PBC > \angle QCB$ -----(1)

$$\angle PBC + \angle ABC = 180^\circ \text{ [Linear pair angles]}$$

$$\Rightarrow \angle PBC = 180 - \angle ABC$$

$$\text{Similarly, } \angle QCB = 180^\circ - \angle ACB$$

From (1) and (2)

$$180 - \angle ABC > 180 - \angle ACB$$

$$\Rightarrow -\angle ABC > -\angle ACB$$

$$\Rightarrow \angle ABC < \angle ACB \text{ or } \angle ACB > \angle ABC$$

It is known that, in a triangle, the greater angle has the longer side opposite to it.

$$\therefore AB > AC$$

Answer 6.

Using the exterior angle property in $\triangle ACD$, we have

$$\angle ACB = \angle CDA + \angle CAD$$

$$\Rightarrow \angle ACB > \angle CDA \quad \text{-----(1)}$$

Now, $AB = AC$

$$\Rightarrow \angle ACB = \angle ABC \quad \text{-----(2)}$$

From (1) and (2)

$$\angle ABC > \angle CDA$$

It is known that, in a triangle, the greater angle has the longer side opposite to it.

Now, In $\triangle ABD$, we have $\angle ABC > \angle CDA$

$$\therefore AD > AB$$

Answer 7.

Given: A $\triangle ABC$ in which AD , BE and CF are its medians.

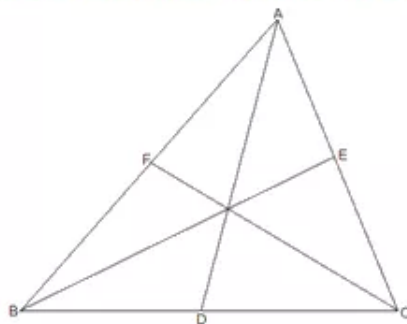
To Prove: We know that the sum of any two sides of a triangle is greater than twice the median bisecting the third side. Therefore,

AD is the median bisecting BC

$$\Rightarrow AB + AC > 2 AD \quad \dots(i)$$

BE is the median bisecting AC $\dots(ii)$

And, CF is the median bisecting AB



$$\Rightarrow BC + AC > 2EF \quad \dots(iii)$$

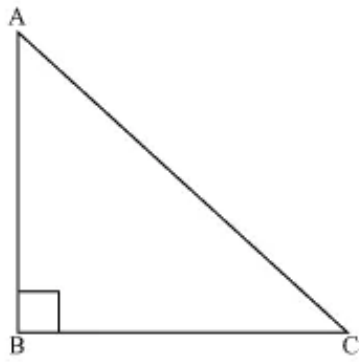
Adding (i), (ii) and (iii), we get

$$(AB + AC) + (AB + BC) + (BC + AC) > 2 \cdot AD + 2 \cdot BE + 2 \cdot CF$$

$$\Rightarrow 2 (AB + BC + AC) > 2 (AD + BE + CF)$$

$$\Rightarrow AB + BC + AC > AD + BE + CF$$

Answer 8.



Let us consider a right angled triangle ABC, right angle at B.

In $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ \quad (\text{angle sum property of a triangle})$$

$$\angle A + 90^\circ + \angle C = 180^\circ$$

$$\angle A + \angle C = 90^\circ$$

Hence, the other two angles have to be acute (i.e. less than 90°).

$\therefore \angle B$ is the largest angle in $\triangle ABC$.

$$\Rightarrow \angle B > \angle A \text{ and } \angle B > \angle C$$

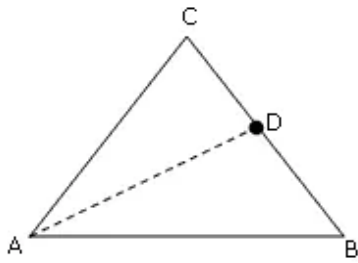
$$\Rightarrow AC > BC \text{ and } AC > AB$$

[In any triangle, the side opposite to the larger (greater) angle is longer]

So, AC is the largest side in $\triangle ABC$.

But AC is the hypotenuse of $\triangle ABC$. Therefore, hypotenuse is the longest side in a right angled triangle.

Answer 9.



Construction: Join AD

In triangle ACD,

$$AC + CD > AD \quad \dots (i)$$

(Sum of two sides of a triangle greater than the third side)

Similarly, in triangle ADB,

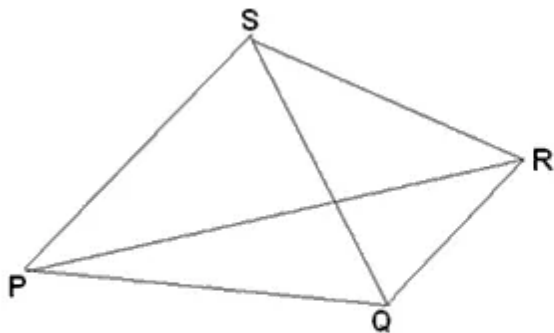
$$AB + BD > AD \quad \dots (ii)$$

Adding (i) and (ii),

$$AC + CD + AB + BD > 2AD$$

$$AB + BC + AC > 2AD \quad (\text{Since, } CD + BD = BC)$$

Answer 10.



Given: PQRS is a quadrilateral. PR and QS are its diagonals.

To Prove: $PQ + QR + SR + PS > PR + QS$

Proof: In $\triangle PQR$

$PQ + QR > PR$ (Sum of two sides of triangle is greater than the third side) Similarly, In $\triangle PSR$, $PS + SR > PR$

In $\triangle PQS$, $PS + PQ > QS$ and in $\triangle QRS$ we have $QR + SR > QS$

Now we have $PQ + QR > PR$

$$PS + SR > PR$$

$$PS + PQ > QS$$

$$QR + SR > QS$$

After adding above inequalities we get

$$2(PQ+QR+PS+SR) > 2(PR+QS)$$

$$\Rightarrow PQ+QR+PS+SR > PR+QS.$$

Answer 11.

In $\triangle AOB$, we have

$$OA + OB > AB \quad \dots (i)$$

In $\triangle BOC$, we have

$$OB + OC > BC \quad \dots (ii)$$

In $\triangle COD$, we have

$$OC + OD > CD \quad \dots (iii)$$

In $\triangle AOD$, we have

$$OA + OD > AD \quad \dots (iv)$$

Adding (i), (ii), (iii) and (iv), we get

$$2(OA + OB + OC + OD) > AB + BC + CD + AD$$

$$\Rightarrow 2[(OA + OC) + (OB + OD)] > AB + BC + CD + AD$$

$$\Rightarrow 2(AC + BD) > AB + BC + CD + AD$$

$$[\because OA + OC = AC \text{ and } OB + OD = BD]$$

$$\Rightarrow AB + BC + CD + AD < 2(AC + BD)$$

Answer 12.

In triangle APR,

$$AP+AR>PR \quad \dots\dots(i)$$

In triangle BPQ,

$$BQ+PB>PQ \quad \dots\dots(ii)$$

In triangle QCR,

$$QC+CR>QR \quad \dots\dots(iii)$$

Adding (i), (ii) and (iii)

$$AP+AR+BQ+PB+QC+CR > PR+PQ+QR$$

$$(AP+PB)+(BQ+QC)+(CR+AR) > PR+QR+PQ$$

$$\Rightarrow AB + BC + AC > PQ + QR + PR.$$

Answer 13.

In $\triangle PQT$, we have

$$PT = PQ \quad \dots (1)$$

In $\triangle PQR$,

$$PQ + QR > PR$$

$$PQ + QR > PT + TR$$

$$PQ + QR > PQ + TR \quad [\text{Using (1)}]$$

$$QR > TR$$

Hence, proved.

Answer 14.

(i). In $\triangle ABC$, we have

$$AB + BC > AC \quad \dots(i)$$

In $\triangle ACD$, we have

$$AD + CD > AC \quad \dots(ii)$$

Adding (i) and (ii), we get

$$AB + BC + AD + CD > 2 AC$$

(ii). In $\triangle ACD$, we have

$$CD + DA > CA$$

$$\Rightarrow CD + DA + AB > CA + AB$$

$$\Rightarrow CD + DA + AB > BC \quad [\because AB + AC > BC]$$

Answer 15.

a. In the given ΔPQR ,

$$PS < PR \quad \dots \left(\begin{array}{l} \text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.} \end{array} \right)$$

$$PN < PR \quad \dots (i) \quad (\because PN < PS)$$

Also,

$$RT < PR \quad \dots \left(\begin{array}{l} \text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.} \end{array} \right)$$

$$RN < PR \quad \dots (ii) \quad (\because RN < RT)$$

Dividing (i) by (ii),

$$\frac{PN}{RN} < \frac{PR}{PR}$$

$$\frac{PN}{RN} < 1$$

$$PN < RN$$

b. In ΔRTQ ,

$$\angle RTQ + \angle TQR + \angle TRQ = 180^\circ$$

$$90^\circ + 60^\circ + \angle TRQ = 180^\circ$$

$$150^\circ + \angle TRQ = 180^\circ$$

$$\angle TRQ = 180^\circ - 150^\circ$$

$$\angle TRQ = 30^\circ$$

$$\angle TRQ = \angle SRN = 30^\circ \quad \dots (iii)$$

In ΔNSR ,

$$\angle RNS + \angle SRN = 90^\circ \quad \dots (\because \angle NSR = 90^\circ)$$

$$\angle RNS + 30^\circ = 90^\circ \quad \dots [\text{from (iii)}]$$

$$\angle RNS = 90^\circ - 30^\circ$$

$$\angle RNS = 60^\circ \quad \dots (iv)$$

$$\angle SRN < \angle RNS \quad \dots (\text{from (iii) and (iv)})$$

$$SN < SR$$

Answer 16.

In $\triangle ACD$,

$$AC = CD \quad \dots (\text{Given})$$

$$\angle CDA = \angle DAC \quad \dots (\triangle ACD \text{ is isosceles triangle.})$$

$$\text{Let } \angle CDA = \angle DAC = x^\circ$$

$$\angle CDA + \angle DAC + \angle ACD = 180^\circ$$

$$x^\circ + x^\circ + 105^\circ = 180^\circ$$

$$2x^\circ + 105^\circ = 180^\circ$$

$$2x^\circ = 180^\circ - 105^\circ$$

$$2x^\circ = 75^\circ$$

$$x = \frac{75^\circ}{2}$$

$$x = 37.5^\circ$$

$$\angle C = \angle DAC = x^\circ = 37.5^\circ \dots (i)$$

$$\angle DAB = \angle DAC + \angle BAC$$

$$125^\circ = 37.5^\circ + \angle BAC \quad \dots \text{from (i)}$$

$$125^\circ - 37.5^\circ = \angle BAC$$

$$87.5^\circ = \angle BAC$$

$$\text{Also, } \angle BCA + \angle ACD = 180^\circ$$

$$\Rightarrow \angle BCA + 105^\circ = 180^\circ$$

$$\Rightarrow \angle BCA = 75^\circ$$

So, in $\triangle BAC$,

$$\angle ACB + \angle BAC + \angle ABC = 180^\circ$$

$$\Rightarrow 75^\circ + 87.5^\circ + \angle ABC = 180^\circ$$

$$\Rightarrow \angle ABC = 17.5^\circ$$

$$\text{As } 87.5^\circ > 17.5^\circ$$

$$\angle BAC > \angle ABC$$

$$\Rightarrow BC > AC$$

$$\Rightarrow BC > CD \quad \dots (\text{Since } AC = CD)$$

Answer 17A.

In $\triangle PQS$,

$$PS < PQ \quad \dots \left(\begin{array}{l} \text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.} \end{array} \right)$$

i.e. $PQ > PS$

$$\text{Also, } QS < QP \quad \dots \left(\begin{array}{l} \text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.} \end{array} \right)$$

i.e. $PQ > QS$

Answer 17B.

In $\triangle PQS$,

$$PS \perp QR \quad \dots (\text{Given})$$

$$PS < PR \quad \dots \left(\begin{array}{l} \text{Of all the straight lines that can} \\ \text{be drawn to a given straight line} \\ \text{from a point outside it, the} \\ \text{perpendicular is the shortest.} \end{array} \right)$$

i.e. $PR > PS$

Answer 17C.

In $\triangle PQR$,

$$PQ + PR > QR \quad \left(\begin{array}{l} \because \text{Sum of two sides of a} \\ \text{triangle is always greater} \\ \text{than third side.} \end{array} \right)$$

In $\triangle PQS$,

$$PQ + QS > PS \quad \left(\begin{array}{l} \because \text{Sum of two sides of a} \\ \text{triangle is always greater} \\ \text{than third side.} \end{array} \right)$$

$\dots (i)$

In $\triangle PRS$,

$$PR + SR > PS \quad \left(\begin{array}{l} \because \text{Sum of two sides of a} \\ \text{triangle is always greater} \\ \text{than third side.} \end{array} \right)$$

$\dots (ii)$

Adding (i) and (ii),

$$PQ + QS + PR + SR > 2PS$$

$$PQ + (QS + SR) + PR > 2PS$$

$$PQ + QR + PR > 2PS$$

$$\text{Since } PQ + PR > QR$$

$$\Rightarrow PQ + QR > 2PS$$

Answer 18.

Note : The question is incomplete.

Question should be :

In the given figure, T is a point on the side PR of an **equilateral** triangle PQR.

Show that :

a. $PT < QT$

b. $RT < QT$

Solution :

a. In $\triangle PQR$,

$$PQ = QR = PR$$

$$\Rightarrow \angle P = \angle Q = \angle R = 60^\circ$$

In $\triangle PQT$,

$$\angle PQT < 60^\circ$$

$$\therefore \angle PQT < \angle P$$

$$\therefore PT < QT$$

b. In $\triangle TQR$,

$$\angle TQR < 60^\circ$$

$$\therefore \angle TQR < \angle R$$

$$\therefore RT < QT$$

Answer 19.

In $\triangle PQR$,

$$PQ + PR > QR \quad \dots \left(\begin{array}{l} \because \text{Sum of the two sides of a} \\ \text{triangle is always greater} \\ \text{than the third side.} \end{array} \right)$$

....(i)

Also, in $\triangle SQR$,

$$SQ + SR > QR \quad \dots \left(\begin{array}{l} \because \text{Sum of the two sides of a} \\ \text{triangle is always greater} \\ \text{than the third side.} \end{array} \right)$$

....(ii)

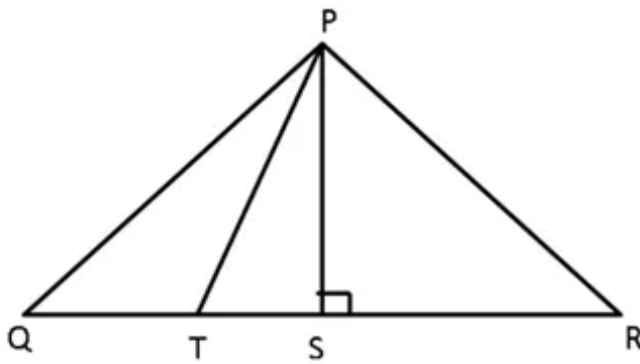
Dividing (i) by (ii),

$$\frac{PQ + PR}{SQ + SR} > \frac{QR}{QR}$$

$$\frac{PQ + PR}{SQ + SR} > 1$$

$$PQ + PR > SQ + SR$$

$$\text{i.e. } SQ + SR < PQ + PR$$

Answer 20.

Let the triangle be $\triangle PQR$.

$PS \perp QR$, the straight line joining vertex P to the line QR.

To prove: $PQ > PT$ and $PR > PT$

In $\triangle PSQ$,

$$PS^2 + SQ^2 = PQ^2 \quad \dots (\text{Pythagoras theorem})$$

$$PS^2 = PQ^2 - SQ^2 \quad \dots (i)$$

In $\triangle PST$,

$$PS^2 + ST^2 = PT^2 \quad \dots (\text{Pythagoras theorem})$$

$$PS^2 = PT^2 - ST^2 \quad \dots (ii)$$

$$PQ^2 - SQ^2 = PT^2 - ST^2 \quad \dots [\text{from (i) and (ii)}]$$

$$PQ^2 - (ST + TQ)^2 = PT^2 - ST^2$$

$$PQ^2 - (ST^2 + 2ST \times TQ + TQ^2) = PT^2 - ST^2$$

$$PQ^2 - ST^2 - 2ST \times TQ - TQ^2 = PT^2 - ST^2$$

$$PQ^2 - PT^2 = TQ^2 + 2ST \times TQ$$

$$PQ^2 - PT^2 = TQ \times (2ST + TQ)$$

As, $TQ \times (2ST + TQ) > 0$ always.

$$PQ^2 - PT^2 > 0$$

$$PQ^2 > PT^2$$

$$PQ > PT$$

Also, $PQ = PR$

$$PR > PT$$

Answer 21.

$\angle AEF > \angle ABC$... (Exterior angle property)

$$\angle AFE = \angle DFC$$

$\angle ACB > \angle DFC$... (Exterior angle property)

$$\Rightarrow \angle ACB > \angle AFE$$

Since $AB = AC$

$$\Rightarrow \angle ACB = \angle ABC$$

So, $\angle ABC > \angle AFE$

$$\Rightarrow \angle AEF > \angle ABC > \angle AFE$$

that is, $\angle AEF > \angle AFE$

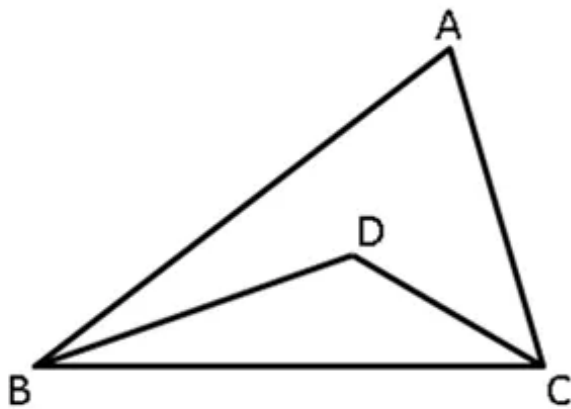
$$\Rightarrow AF > AE$$

Answer 22.

In the $\triangle ABE$ and $\triangle ADE$,
 $AB = AD$ (Given)
 $\angle BAE = \angle DAE$ (AE is the bisector of $\angle BAC$)
 $AE = AE$ (Common side)
 $\therefore \triangle ABE \cong \triangle ADE$ (SAS test)
 $\Rightarrow BE = DE$ (c.p.c.t.)

In $\triangle ABD$,
 $AB = AD$
 $\Rightarrow \angle ABD = \angle ADB$
 $\angle ADB > \angle C$...(Exterior angle property)
 $\Rightarrow \angle ABD > \angle C$

Answer 23.



In the $\triangle ABC$,
 $AB + AC > BC$ $\left(\because \text{Sum of the two sides of triangle is always greater than third side.} \right)$
(i)

Also, in the $\triangle BDC$,
 $BD + DC > BC \dots (ii)$

Dividing (i) by (ii),

$$\frac{AB + AC}{BD + DC} > \frac{BC}{BC}$$

$$\frac{AB + AC}{BD + DC} > 1$$

$$AB + AC > BD + DC$$

i.e. $BD + DC < AB + AC$