

Chapter 15. Mid-point and Intercept Theorems

Ex 15.1

Answer 1.

In $\triangle ABC$,

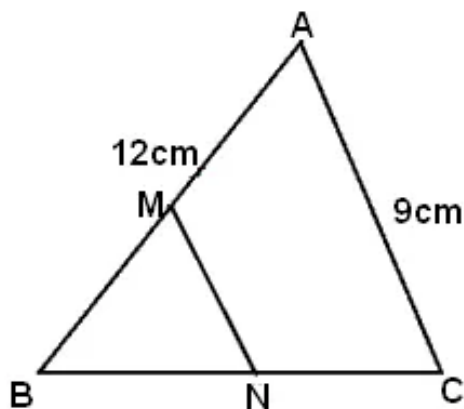
Since D and E are the mid-points of AB and BC respectively

Therefore, by mid-point theorem $DE \parallel AC$ and $DE = \frac{1}{2} AC$

$$(i) DE = \frac{1}{2} AC = \frac{1}{2} \times 8.6 \text{ cm} = 4.3 \text{ cm}$$

$$(ii) \angle DEB = \angle C = 72^\circ \quad (\text{corresponding angles, since } DE \parallel AC)$$

Answer 2.

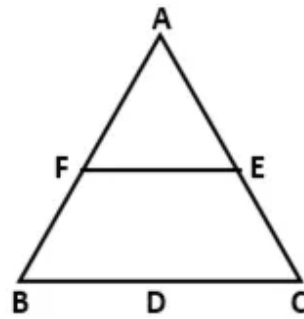


$MN \parallel AC$ and M is mid-point of AB

Therefore, N is mid-point of BC

$$\text{Hence, } MN = \frac{1}{2} AC = \frac{9}{2} \text{ cm} = 4.5 \text{ cm}$$

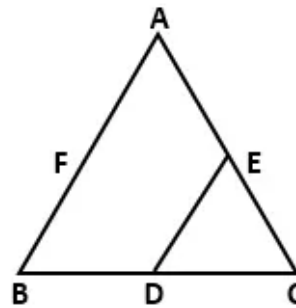
Answer 3A.



F is the mid-point AB and E is the mid-point of AC.

$$\begin{aligned}\therefore FE &= \frac{1}{2}BC \quad \dots(\text{Mid-point Theorem}) \\ &= \frac{1}{2} \times 14 \\ &= 7 \text{ cm}\end{aligned}$$

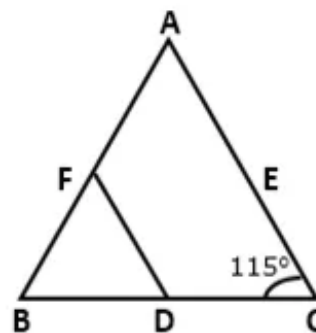
Answer 3B.



D is the mid-point BC and E is the mid-point of AC.

$$\begin{aligned}\therefore DE &= \frac{1}{2}AB \quad \dots(\text{Mid-point Theorem}) \\ &= \frac{1}{2} \times 8 \\ &= 4 \text{ cm}\end{aligned}$$

Answer 3C.



Here, $FD \parallel AC$.

$$\therefore \angle FDB = \angle ACB = 115^\circ \quad \dots(\text{Corresponding angles})$$

Answer 4.

In $\triangle NSR$

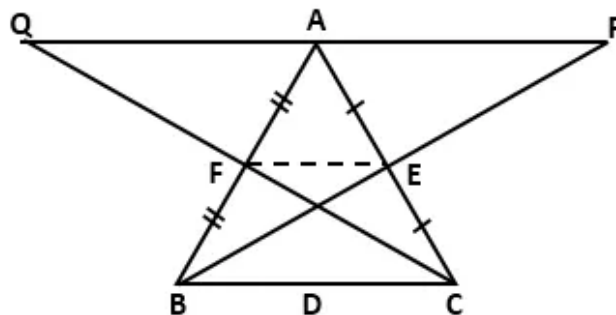
$$MQ = \frac{1}{2}SR$$

But L is the mid-point of SR and $SR = PQ$ (sides of a parallelogram)

$$MQ = \frac{1}{2}PQ$$

$$MQ = PM = LS = LR$$

Therefore, M is the mid-point of PQ.

Answer 5.

Since BE and CF are medians,

F is the mid-point of AB and E is the mid-point of AC.

Now, the line joining the mid-points of any two sides is parallel and half of the third side, we have

In $\triangle ACQ$,

$$EF \parallel AQ \text{ and } EF = \frac{1}{2}AQ \quad \dots(i)$$

In $\triangle ABP$,

$$EF \parallel AP \text{ and } EF = \frac{1}{2}AP \quad \dots(ii)$$

a) From (i) and (ii), we get $AP \parallel AQ$ (both are parallel to EF)

As AP and AQ are parallel and have a common point A, this is possible only if QAP is a straight line.

Hence, proved.

b) From (i) and (ii),

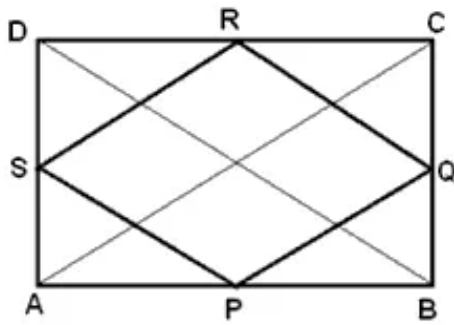
$$EF = \frac{1}{2}AQ \text{ and } EF = \frac{1}{2}AP$$

$$\Rightarrow \frac{1}{2}AQ = \frac{1}{2}AP$$

$$\Rightarrow AQ = AP$$

$\Rightarrow$ A is the mid-point of QP.

Answer 6.



Join AC and BD.

In $\triangle ABC$, P and Q are the mid-points of AB and BC respectively.

$$PQ = \frac{1}{2}AC \dots\dots(i) \text{ and } PQ \parallel AC$$

In $\triangle BDC$, R and Q are the mid-points of CD and BC respectively.

$$QR = \frac{1}{2}BD \dots\dots(ii) \text{ and } QR \parallel BD$$

But $AC = BD$ (diagonals of a rectangle)

From (i) and (ii)

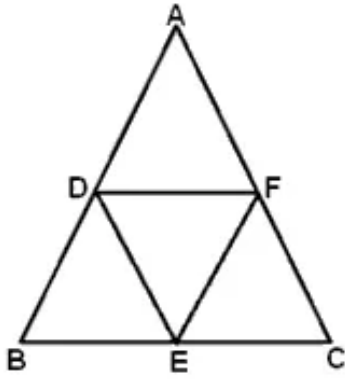
$$PQ = QR$$

Similarly, $QR = RS$, $RS = SP$ and $RS \parallel AC$, $SP \parallel BD$

Hence, $PQ = QR = RS = SP$

Therefore, PQRS is a rhombus.

Answer 7.



E and F are mid-points of BC and AC

Therefore, $EF = \frac{1}{2}AB$(i)

D and F are mid-points of AB and AC

Therefore, $DF = \frac{1}{2}BC$(ii)

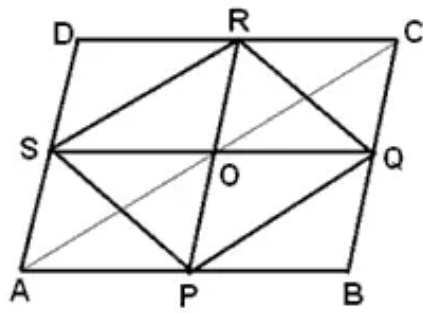
But $AB = BC$

From (i) and (ii)

$EF = DF$

Therefore, $\triangle DEF$ is an isosceles triangle.

Answer 8.



Join AC.

P and Q are mid-points of AB and BC respectively.

$$\therefore PQ \parallel AC, \quad PQ = \frac{1}{2}AC \dots\dots\dots(i)$$

S and R are mid-points of AD and DC respectively.

$$\therefore SR \parallel AC, \quad SR = \frac{1}{2}AC \dots\dots\dots(ii)$$

From (i) and (ii)

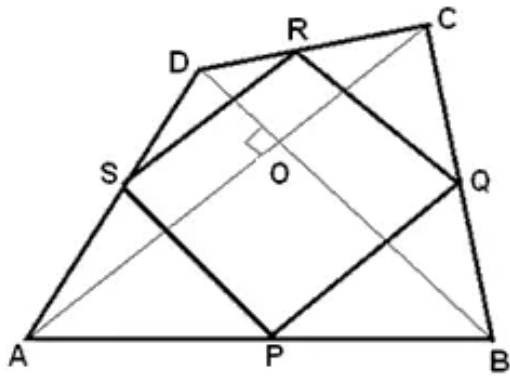
$$PQ = SR$$

Therefore, PQRS is a parallelogram.

Since, diagonals of a parallelogram bisect each other

Therefore, PQ and QS bisect each other.

Answer 9.



P and Q are mid-points of AB and BC.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \dots\dots(i)$$

S and R are mid-points of AD and DC.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2}AC \dots\dots(ii)$$

From (i) and (ii)

$$PQ \parallel SR \text{ and } PQ = SR$$

Therefore, PQRS is a parallelogram.

Further AC and BD intersect at right angles

$$\therefore SP \parallel BD \text{ and } BD \perp AC$$

$$\therefore SP \perp AC$$

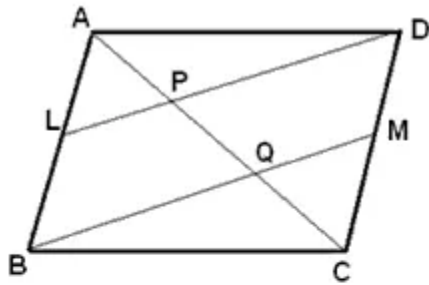
$$\Rightarrow SP \perp SR$$

$$\Rightarrow \angle RSP = 90^\circ$$

$$\therefore \angle RSP = \angle SRQ = \angle RQS = \angle SPQ = 90^\circ$$

Therefore, PQRS is a rectangle.

Answer 10.



Since L and M are the mid-points of AB and DC respectively.

$$BL = \frac{1}{2}AB \text{ and } DM = \frac{1}{2}DC \dots (i)$$

But ABCD is a parallelogram

Therefore, $AB = CD$ and $AB \parallel DC$

$$\Rightarrow BL = DM \text{ and } BL \parallel DM \quad (\text{from (i)})$$

$\Rightarrow$ BLDM is a parallelogram.

$$\Rightarrow DL \parallel BM$$

$$\Rightarrow LP \parallel BQ \dots (ii)$$

It is known that the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In $\triangle ABQ$, L is the mid-point of AB and $LQ \parallel BQ$

Therefore, P is mid-point of AQ

$$\text{Hence, } AP = PQ \dots (iii)$$

Similarly, in $\triangle CPD$, M is the mid-point of CD and $MP \parallel BQ$

Therefore, Q is mid-point of CP

$$\text{Hence, } PQ = QC \dots (iv)$$

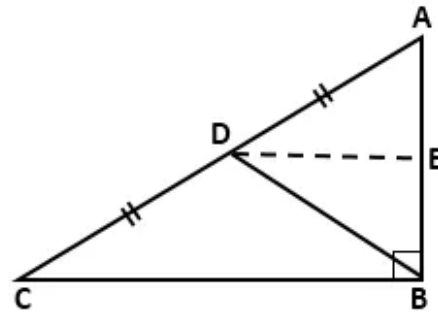
From (iii) and (iv)

$$AP = PQ = QC$$

Therefore, P and Q trisect AC

Thus, DL and BM trisect AC.

Answer 11.



Draw line segment $DE \parallel CB$, which meets AB at point E .

Now, $DE \parallel CB$ and AB is the transversal,

$$\therefore \angle AED = \angle ABC \quad \dots (\text{Corresponding angles})$$

$$\angle ABC = 90^\circ \quad \dots (\text{given})$$

$$\Rightarrow \angle AED = 90^\circ$$

Also, as D is the mid-point of AC and $DE \parallel CB$,

DE bisects side AB ,

$$\text{i.e. } AE = BE \quad \dots (i)$$

In $\triangle AED$ and $\triangle BED$,

$$\angle AED = \angle BED \quad \dots (\text{Each } 90^\circ)$$

$$AE = BE \quad \dots [\text{From (i)}]$$

$$DE = DE \quad \dots (\text{Common})$$

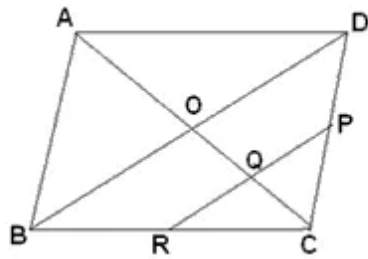
$$\therefore \triangle AED \cong \triangle BED \quad \dots (\text{By SAS Test})$$

$$\Rightarrow AD = BD \quad \dots (\text{C.P.C.T.C})$$

$$\Rightarrow BD = AC$$

$$\Rightarrow BD = \frac{1}{2} AC$$

Answer 12.



(i) Join B and D. Suppose AC and BD cut at O. Then,

$$OC = \frac{1}{2}AC$$

$$\text{Now, } CQ = \frac{1}{4}AC \Rightarrow CQ = \frac{1}{2}OC$$

In $\triangle DCO$, P and Q are the mid-points of DC and OC respectively.

$$\therefore PQ \parallel DO$$

Also, in $\triangle COB$, Q is the mid-point of OC and $PQ \parallel OB$

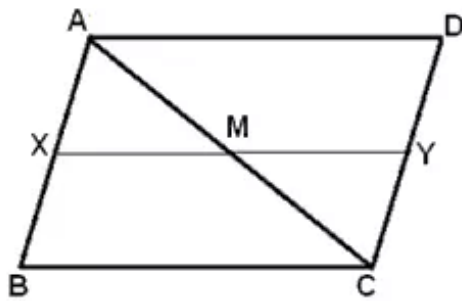
Therefore, R is the mid-point of BC, R being PQ produced.

(ii) In $\triangle BCD$, P and R are the mid-points of DC and BC respectively.

$$\text{Also } PR \parallel BD$$

$$\text{Therefore, } PR = \frac{1}{2}BD$$

Answer 13.



(i) Join XM and MY.

In $\triangle AXM$ and $\triangle CYM$

$$AM = MC \quad (\text{given})$$

$$AX = CY \quad (\text{given})$$

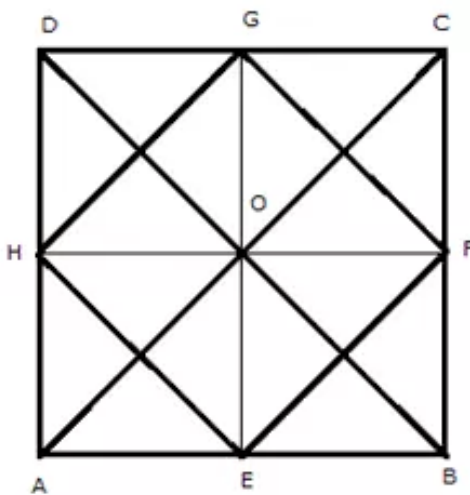
$$\angle XAM = \angle YCM \quad (\text{alternate angles})$$

Therefore, $\triangle AXM \cong \triangle CYM$

(ii) $\angle AMX = \angle CMY$ (Vertically opposite angles)

Therefore, XMY is a straight line.

Answer 14.



Join AC and BD

In $\triangle ACD$, G and H are the mid-points of DC and AD respectively.

$$\text{Therefore, } GH \parallel AC \text{ and } GH = \frac{1}{2}AC \quad \dots\dots(i)$$

In $\triangle ABC$, E and F are the mid-points of AB and BC respectively.

Therefore, $EF \parallel AC$ and $EF = \frac{1}{2}AC$ (ii)

From (i) and (ii)

$EF \parallel GH$ and $EF = GH = \frac{1}{2}AC$ (iii)

Similarly it can be proved that-

$EH \parallel GF$ and $EH = GF = \frac{1}{2}BD$ (iv)

But $AC = BD$ (diagonals of a square are equal)

Dividing both sides by 2,

$$\frac{1}{2}BD = \frac{1}{2}AC \quad (\text{iv})$$

From (iii) and (iv)

$$EF = GH = EH = GF$$

Therefore, EFGH is a parallelogram.

Now in $\triangle GOH$ and $\triangle GOF$

$$OH = OF \quad (\text{diagonals of a parallelogram bisect each other})$$

$$OG = OG \quad (\text{common})$$

$$GH = GF$$

$$\therefore \triangle GOH \cong \triangle GOF$$

$$\therefore \angle GOH = \angle GOF$$

NOW,

$$\angle GOH + \angle GOF = 180^\circ$$

$$\Rightarrow \angle GOH + \angle GOH = 180^\circ$$

$$\Rightarrow 2\angle GOH = 180^\circ$$

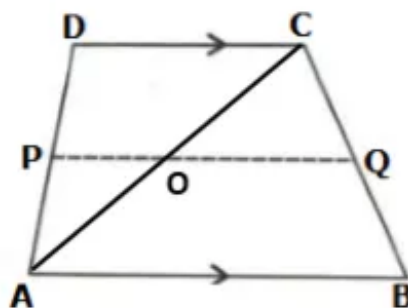
$$\Rightarrow \angle GOH = 90^\circ$$

Therefore, diagonals of parallelogram EFGH bisect each other and are perpendicular to each other.

Thus, EFGH is a square.

Answer 15A.

Let us draw a diagonal AC which meets PQ at O as shown below:



a) Given $AB = 12$ cm and $DC = 10$ cm

In $\triangle ABC$,

$$OQ = \frac{1}{2}AB \quad \dots(\text{Mid-point Theorem})$$

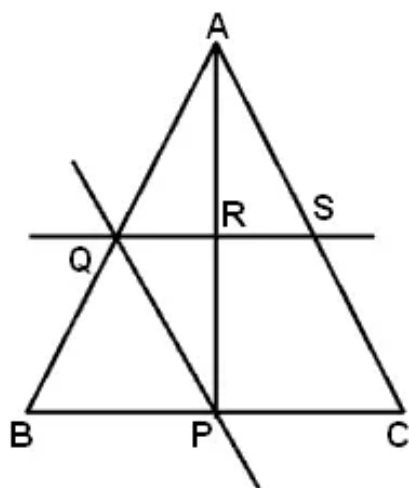
$$\Rightarrow OQ = \frac{1}{2} \times 12 = 6 \text{ cm}$$

In $\triangle ADC$,

$$OP = \frac{1}{2}DC \quad \dots(\text{Mid-point Theorem})$$

$$\Rightarrow OP = \frac{1}{2} \times 10 = 5 \text{ cm}$$

$$\text{Now, } PQ = OP + OQ = 6 + 5 = 11 \text{ cm}$$

Answer 16.

(i) In $\triangle ABC$,

P is the mid-point of BC and PQ is parallel to AC

Therefore, Q is the mid-point of AB.

In $\triangle ABP$,

Q is the mid-point of AB and QR is parallel to BP

Therefore, R is the mid-point of AP.

$$AR = RP$$

$$\text{But } AR + RP = AP$$

$$\Rightarrow AR + AR = AP$$

$$\Rightarrow 2AR = AP \quad \text{or} \quad AP = 2AR$$

(ii) In $\triangle ABC$,

Q and S are the mid-points of AB and AC respectively. Also QS is parallel to BC

$$\text{Therefore, } QS = \frac{1}{2}BC \quad \dots\dots(i)$$

Now, AP is the median, hence it bisects BC and QS

Therefore

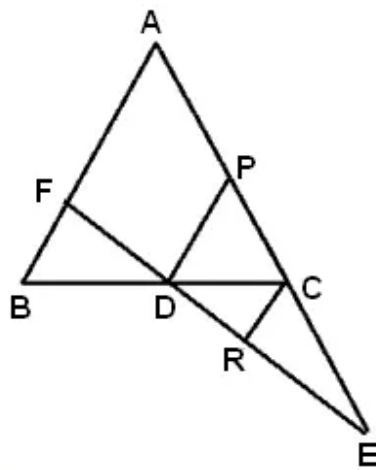
$$\frac{1}{2}QS = QR \Rightarrow QS = 2QR$$

Substituting in (i)

$$\Rightarrow 2QR = \frac{1}{2}BC$$

$$\Rightarrow BC = 4QR$$

Answer 17.



(i) In $\triangle BDF$ and $\triangle DRC$,

$$BD = DC \quad (\text{D is the mid-point of BC})$$

$$CR \parallel PD \parallel AB$$

$$\angle BFD = \angle DRC \quad (\text{alternate angles})$$

$$\angle BDF = \angle RDC \quad (\text{vertically opposite angles})$$

Therefore,

$$\triangle BDF \cong \triangle DRC$$

$$\Rightarrow DF = DR \quad \dots\dots(i)$$

In $\triangle ABC$,

D is the mid-point of BC and $DP \parallel AB$

Therefore, P is the mid-point of AC.

In $\triangle DEP$,

C is the mid-point of PE and $DP \parallel RC \parallel AB$ ($CE = \frac{1}{2}AC$ and P is the mid-point of AC)

Therefore, R is the mid-point of DE.

$$\Rightarrow DR = RE \dots\dots(ii)$$

$$\text{But } EF = DF + DR + RE$$

$$EF = DF + DF + DF$$

$$EF = 3DF$$

(ii) In $\triangle DEP$,

C and R are the mid-points of PE and DE respectively.

Also, $DP \parallel RC$

$$\therefore CR = \frac{1}{2}DP \dots\dots(i)$$

In $\triangle ABC$,

D and P are the mid-points of BC and AC respectively.

Also, $DP \parallel AB$

$$\therefore DP = \frac{1}{2}AB \dots\dots(ii)$$

Substituting the value of DP from (ii) in (i)

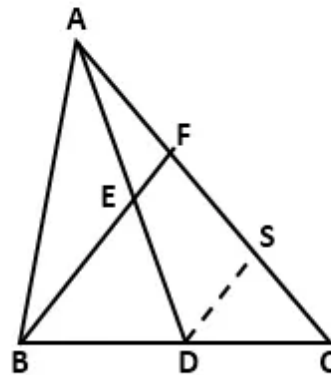
$$\Rightarrow CR = \frac{1}{2}\left(\frac{1}{2}AB\right)$$

$$\Rightarrow CR = \frac{1}{4}AB$$

$$\therefore 4CR = AB$$

Answer 18.

Construction: Draw $DS \parallel BF$, meeting AC at S .



Proof :

In $\triangle BCF$, D is the mid-point of BC and $DS \parallel BF$.

$\therefore S$ is the mid-point of CF .

$$\Rightarrow CS = SF \quad \dots(i)$$

In $\triangle ADS$, E is the mid-point of AD and $EF \parallel DS$.

$\therefore F$ is the mid-point of AS .

$$\Rightarrow AF = FS \quad \dots(ii)$$

From (i) and (ii), we get

$$AF = FS = SC$$

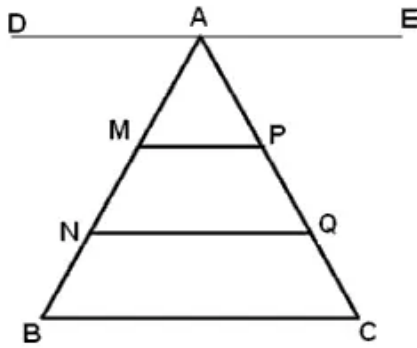
$$\Rightarrow AC = AF + FS + SC$$

$$\Rightarrow AC = AF + AF + AF$$

$$\Rightarrow AC = 3AF$$

$$\Rightarrow \frac{AF}{AC} = \frac{1}{3}$$

$$\Rightarrow AF : AC = 1 : 3$$

Answer 19.

Draw $DE \parallel BC$ through A

$AM = MN = NB$ (given)

$MP \parallel BC$; $NQ \parallel BC$ (given)

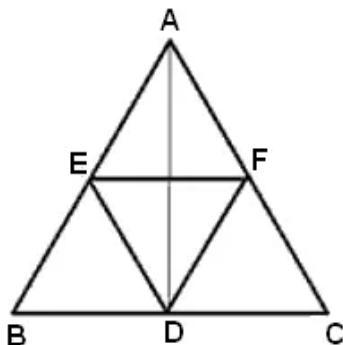
$DE \parallel BC$

i.e. AM, MN and NB are equal intercepts made on transversal AB.

AC is also a transversal; intercepts made on AC are AP, PQ and QC.

Hence, $AP = PQ = QC$

Therefore, P and Q divide AC in three equal parts.

Answer 20.

Since the segment joining the mid-points of two sides of a triangle is parallel to third side and is half of it,

Therefore,

$$DE \parallel AB, DE = \frac{1}{2} AB$$

Also,

$$DF \parallel AC, DF = \frac{1}{2} AC$$

But $AB = AC$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AC$$

$$\Rightarrow DF = DE \quad \dots\dots(i)$$

$$DE = \frac{1}{2} AB \Rightarrow DE = AF \quad \dots\dots(ii)$$

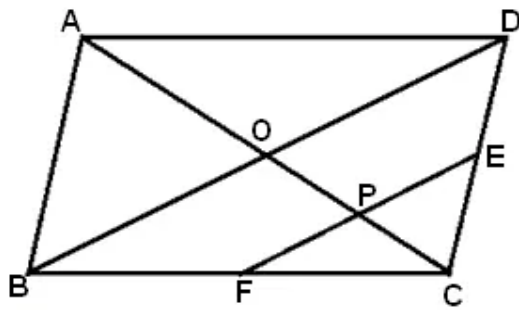
From (i), (ii) and (iii)

$$DE = AE = EF = DF$$

$\Rightarrow$ DEAF is a rhombus.

$\Rightarrow$ Diagonals AD and EF bisect each other at right angles.

$\Rightarrow$ AD perpendicular to EF and AD is bisected by EF.

Answer 21.

(i) Join B and D. Suppose AC and BD cut at O. Then,

$$OC = \frac{1}{2}AC$$

$$\text{Now, } PC = \frac{1}{4}AC \Rightarrow PC = \frac{1}{2}OC$$

In $\triangle DCO$, E and P are the mid-points of DC and OC respectively.

$$\therefore EP \parallel DO$$

Also, in $\triangle COB$, P is the mid-point of OC and $PF \parallel DO \parallel BD$

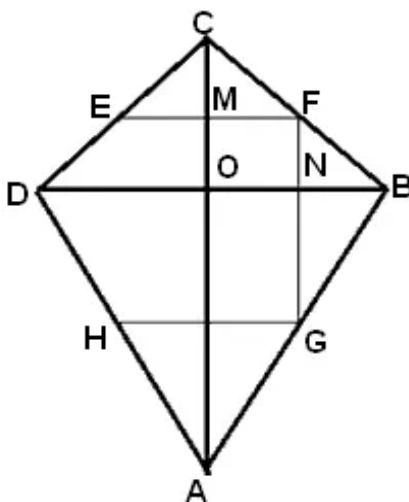
Therefore, F is the mid-point of BC, F being EP produced.

(ii) In $\triangle BCD$, E and F are the mid-points of DC and BC respectively.

Also $EF \parallel BD$

$$\text{Therefore, } EF = \frac{1}{2}BD$$

$$\Rightarrow 2EF = BD$$

Answer 22.

(i) Diagonals of a kite intersect at right angles

$$\therefore \angle MON = 90^\circ \quad \dots\dots(i)$$

In $\triangle BCD$,

E and F are mid-points of CD and BC respectively.

$$\text{Therefore, } EF \parallel DB \text{ and } EF = \frac{1}{2}DB \quad \dots\dots(ii)$$

90°

3 and $HG \parallel DB$ (from (ii), $EF \parallel DB$ and $EF \parallel HG$)

-point of DA.

through G and parallel to FE bisects DA

$EF \parallel DB \Rightarrow MF \parallel ON$

$\therefore \angle MON + \angle MFN = 180^\circ$

$\Rightarrow 90^\circ + \angle MFN = 180^\circ$

$\Rightarrow \angle MFN = 90^\circ$

$\Rightarrow \angle EFG = 90^\circ$

(ii) In $\triangle ABD$,

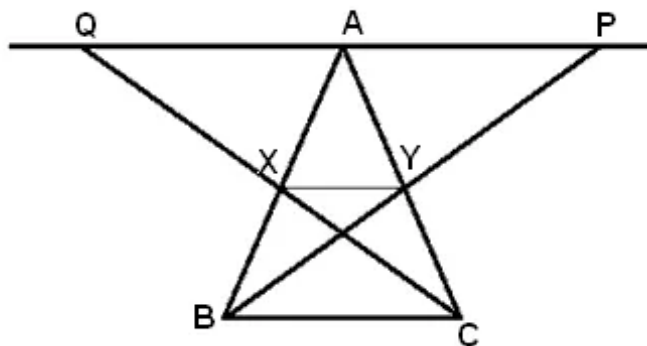
G is the mid-point of AE

Therefore, $HG \parallel DB$

Therefore, H is the mid

Hence, the line drawn t

Answer 23.



Join X and Y

In $\triangle ABP$,

X and Y are the mid-points of AB and AC respectively

Therefore, $XY \parallel BC$

Since $BC \parallel AP$

$\Rightarrow XY \parallel AP$ and $XY \parallel AQ$

$\therefore XY = \frac{1}{2}AP \dots\dots\dots(i)$

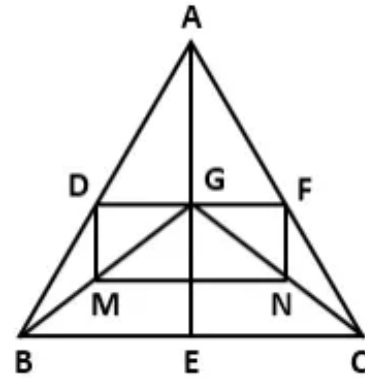
$XY = \frac{1}{2}AQ \dots\dots\dots(ii)$

From (i) and (ii)

$\Rightarrow \frac{1}{2}AP = \frac{1}{2}AQ$

$\Rightarrow AP = AQ$

Answer 24.



- a. Since D and F are mid-points of AB and AC, by Mid-point theorem,

$$BC = 2DF$$

$$\text{Now, } BC = BE + EC$$

$$DF = DG + GF$$

But E is the mid-point of BC,

$$\Rightarrow BE = EC \quad \dots(i)$$

Also, $AG = GE \quad \dots(G \text{ is the mid-point of } AE)$

Consider $\triangle ABE$ and $\triangle ACE$, by mid-point theorem,

$$BE = 2DG \text{ and } EC = 2GF$$

$$\Rightarrow 2DG = 2GF \quad \dots[\text{From (i)}]$$

$$\Rightarrow DG = GF$$

Hence, AE and DF bisect each other.

- b. Consider $\triangle ABC$ and $\triangle GBC$, by mid-point theorem,

$$2DF = BC \text{ and } 2MN = BC$$

$$\Rightarrow DF = MN \quad \dots(i)$$

Consider $\triangle ABG$ and $\triangle ACG$, by mid-point theorem,

$$2DM = AG \text{ and } 2FN = AG$$

$$\Rightarrow DM = FN \quad \dots(ii)$$

From (i) and (ii), it is clear that DMNF is a parallelogram.