Chapter 17. Pythagoras Theorem

Ex 17.1

Answer 1.

Base = 5cm, Hypotenuse = 13cm

By Pythagoras theorem,

$(\text{perpendicular})^2 = (13\text{cm})^2 - (5\text{cm})^2$

$(\text{perpendicular})^2 = 169\text{cm}^2 - 25\text{cm}^2$

$(\text{perpendicular})^2 = 144\text{cm}^2$

$(\text{perpendicular})^2 = (12\text{cm})^2$

$\therefore$ perpendicular=12cm

Area of the triangle = $13\text{cm}^2 \times$ (Base $\times$ Perpendicular)

$= \frac{1}{2} \times 5\text{cm} \times 12\text{cm}$

$= 30\text{cm}^2$

Answer 2.

The two sides (excluding hypotenuse) of a right - angled triangle are given as 24cm and 7cm

$(\text{hypotenuse})^2 = (24\text{cm})^2 + (7\text{cm})^2$

$(\text{hypotenuse})^2 = 576\text{cm}^2 + 49\text{cm}^2$

$(\text{hypotenuse})^2 = 625\text{cm}^2$

$(\text{hypotenuse})^2 = (25\text{cm})^2$

Thus, the length of the hypotenuse of the triangle is 25cm.

Answer 3.

Hypotenuse = 65cm

One side = 16cm

Let the other side be of length x cm

By Pythagoras theorem,

$(65cm)^2 = (16cm)^2 + (x\ cm)^2$

$(x\ cm)^2 = 4225cm^2 - 256cm^2$

$= 3969cm^2$

$= (63cm)^2$

$\Rightarrow x = 63cm$

Area of the triangle $= \frac{1}{2} \times (\text{Base} \times \text{Height})$

$= \frac{1}{2} \times 16cm \times 63cm$

$= 504\ cm^2$

Answer 4.

Let O be the original position of the man.

From the figure, it is clear that B is the final position of the man.

ΔAOB is right – angled at A.

By Pythagoras theorem,

$OB^2 = OA^2 + AB^2$

$OB^2 = (10m)^2 + (24m)^2$

$OB^2 = 100m^2 + 576m^2$

$OB^2 = 676m^2$

$OB^2 = (26m)^2$

$OB = 26m$

Thus, the man is at a distance of 26m from the starting point.

Answer 5.

Let AC be the ladder and A be the position of the window.

Then, AC = 25m, AB = 20m

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow (25m)^2 = (20m)^2 + BC^2$

$\Rightarrow BC^2 = 625m^2 - 400m^2$

$BC^2 = 225m^2$

$BC^2 = (15m)^2$

$\Rightarrow BC = 15m$

Thus, the distance of the foot of the ladder from the building is 15m.

Answer 6.

Hypotenuse = p cm

One side = q cm

Let the length of the third side be x cm.

Using Pythagoras theorem,

$x^2 = p^2 - q^2 = (p + q)(p - q)$

$= (p + q) \times 1 \qquad [\because p - q = 1, \text{given}]$

$= p + q$

$\therefore x = \sqrt{p + q}$

Thus, the length of the third side of the triangle is $\sqrt{p + q}$ cm.

Answer 7.

let O be the foot of the ladder. Let AO be the position of the ladder when it touches the window at A which is 9m high and CO be the position of the ladder when it touches the window at C which is 12m high.
Using Pythagoras theorem,

In ΔAOB,

$BO^2 = AO^2 - AB^2$

$BO^2 = (15m)^2 - (9m)^2$

$BO^2 = 225m^2 - 81m^2$

$BO^2 = 144m^2$

$BO^2 = (12m)^2$

$BO = 12m$

Using Pythagoras theorem in ΔCOB,

$DO^2 = CO^2 - CD^2$

$DO^2 = (15m)^2 - (12m)^2$

$DO^2 = 225m^2 - 144m^2$

$DO^2 = 81m^2$

$DO = 9m$

Width of the street = DO + BO = 9m + 12m = 21m

Answer 8.

Let AC be the ladder and A be the position of the window which is 8m above the ground.
Now, the ladder is shifted such that its foot is at point D which is 8m away from the wall.

$\therefore$ BD = 8m

At this instance, the position of the ladder is DE.

$\therefore$ AC = DE

Using Pythagoras theorem in ΔABC,

$AC^2 = AB^2 + BC^2$

$= (8m)^2 + (6m)^2$

$= 64m^2 + 36m^2$

$= 100m^2$

$= (10m)^2$

$\therefore$ AC = DE = 10m

Using Pythagoras theorem in ΔDBE,

$BE^2 = DE^2 - BD^2$

$\Rightarrow BE^2 = (10m)^2 - (8m)^2$

$= 100m^2 - 64m^2$

$= 36m^2$

$= (6m)^2$

$\Rightarrow BE = 6m$

Thus, the required height up to which the ladder reaches is 6m above the ground.

Answer 9.

Let AB and CD be the two poles of height 14m and 9m respectively.

It is given that BD = 12m

$\therefore CE = 12m$

Now, AE = AB − BE

= 14m − 9m = 5m

Using Pythagoras theorem in ΔACE,

$AC^2 = AE^2 + CE^2$

$= (5m)^2 + (12m)^2$

$= 25m^2 = 144m^2$

$= 169m^2$

$= 13m^2$

$\Rightarrow AC = 13m$

Thus, the distance between the tops of the poles is 13m

Answer 10.

It is given that the diagonals of a rhombus are of length 14cm and 10cm respectively

$\therefore d_1 = 24cm, d_2 = 10cm$

The diagonals of a rhombus bisect each other

$$\therefore \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = side^2$$

$\Rightarrow side^2 = 12^2 + 5^2 = 144 + 25 = 13^2$

$\Rightarrow$Side = 13

Thus, each side of the rhombus is of length 13cm

Answer 11.

Side of the rhombus = 10cm

One diagonal, d_1 = 16cm

Let d_2 be the other diagonal of the rhombus

The diagonals of a rhombus bisect each other

$$\therefore \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = side^2$$

$$8^2 + \left(\frac{d_2}{2}\right)^2 = 100$$

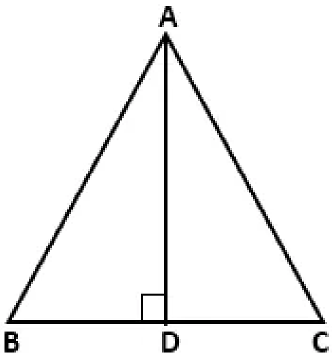
$$\Rightarrow \left(\frac{d_2}{2}\right)^2 = 100 - 64 = (6)^2$$

$$\Rightarrow \frac{d_2}{2} = 6$$

$$\Rightarrow d_2 = 12$$

Thus, the other diagonal of the rhombus is of length 12cm

Answer 12.



Since triangles ABD and ACD are right triangles right-angled at D,

$AB^2 = AD^2 + BD^2$ ....(i)

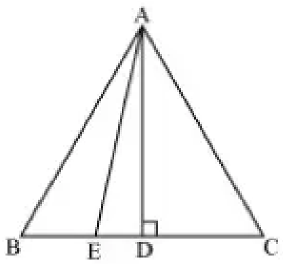
$AC^2 = AD^2 + CD^2$ ....(ii)

Subtracting (ii) from (i), we get

$AB^2 - AC^2 = BD^2 - CD^2$

$\Rightarrow AB^2 + CD^2 = AC^2 + BD^2$

Answer 13.



Let side of equilateral triangle be a. And AE be the altitude of ΔABC

So, BE = EC = $\frac{BC}{2} = \frac{a}{2}$

And, AE = $\frac{a\sqrt{3}}{2}$

Given that BD = $\frac{1}{3}$BC = $\frac{a}{3}$

So, DE = BD-BE = $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Now, in ΔADE by applying Pythagoras theorem

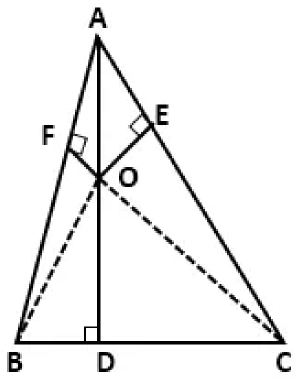
$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right) = \frac{28a^2}{36}$$

Or, $9\ AD^2 = 7\ AB^2$.

Answer 14.



a. In right triangles OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$

$OB^2 = BD^2 + OD^2$

$OC^2 = CE^2 + OE^2$

Adding all these results, we get

$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$

$\Rightarrow AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

b. In right triangles ODB and ODC, we have

$OB^2 = OD^2 + BD^2$

$OC^2 = OD^2 + CD^2$

$\therefore OB^2 - OC^2 = (OD^2 + BD^2) - (OD^2 + CD^2)$

$\Rightarrow OB^2 - OC^2 = BD^2 - CD^2 \quad ....(i)$

Similarly, we have

$OC^2 - OA^2 = CE^2 - AE^2 \quad ....(ii)$

$OA^2 - OB^2 = AF^2 - BF^2 \quad ....(iii)$

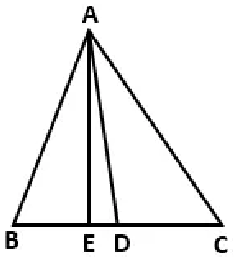
Adding (i), (ii) and (iii), we get

$(OB^2 - OC^2) + (OC^2 - OA^2) + (OA^2 - OB^2) = (BD^2 - CD^2) + (CE^2 - AE^2) + (AF^2 - BF^2)$

$\Rightarrow (BD^2 + CE^2 + AF^2) - (AE^2 + CD^2 + BF^2) = 0$

$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer 15.



We have $\angle AED = 90°$,

$\therefore$ $\angle ADE < 90°$ and $\angle ADC > 90°$

i.e. $\angle ADE$ is acute and $\angle ADC$ is obtuse.

a. In ΔADC, $\angle ADC$ is an obtuse angle.

$\therefore AC^2 = AD^2 + DC^2 + 2 \times DC \times DE$

$\Rightarrow AC^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 + 2 \times \frac{1}{2}BC \times DE$

$\Rightarrow AC^2 = AD^2 + \frac{1}{4}BC^2 + BC \times DE$

$\Rightarrow AC^2 = AD^2 + BC \times DE + \frac{1}{4}BC^2 \quad ....(i)$

b. In ΔABD, $\angle ADE$ is an acute angle.

$\therefore AB^2 = AD^2 + BD^2 - 2 \times BD \times DE$

$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 - 2 \times \frac{1}{2}BC \times DE$

$\Rightarrow AB^2 = AD^2 + \frac{1}{4}BC^2 - BC \times DE$

$\Rightarrow AB^2 = AD^2 - BC \times DE + \frac{1}{4}BC^2 \quad ....(ii)$

c. Adding (i) and (ii), we have

$AC^2 + AB^2 = AD^2 + BC \times DE + \frac{1}{4}BC^2 + AD^2 - BC \times DE + \frac{1}{4}BC^2$

$\Rightarrow AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2 \quad ....(iii)$

d. Subtracting (ii) from (i), we have

$AC^2 - AB^2 = AD^2 + BC \times DE + \frac{1}{4}BC^2 - AD^2 + BC \times DE - \frac{1}{4}BC^2$

$\Rightarrow AC^2 - AB^2 = 2BC \times DE$

e. From (iii), we have

$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$

$\Rightarrow AB^2 + AC^2 = 2AD^2 + \frac{1}{2}(2 \times CD)^2$

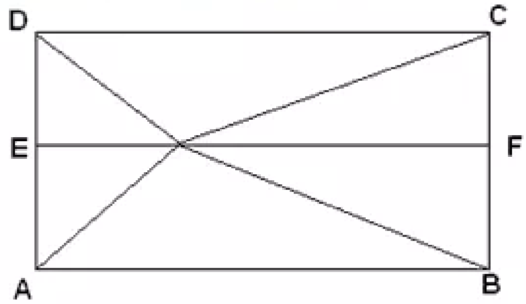
$\Rightarrow AB^2 + AC^2 = 2AD^2 + \frac{1}{2} \times 4CD^2$

$\Rightarrow AB^2 + AC^2 = 2AD^2 + 2CD^2$

$\Rightarrow AB^2 + AC^2 = 2(AD^2 + CD^2)$

Answer 16.

Let ABCD be the given rectangle and let O be a point within it.
Join OA, OB, OC and OD.



Through O, draw EOF ∥ AB. Then, ABFE is a rectangle.

In right triangles ΔOEA and ΔOFC, we have

$OA^2=OE^2+AE^2$ and $OC^2=OF^2+CF^2$

$\Rightarrow OA^2+OC^2=(OE^2+AE^2)+(OF^2+CF^2)$

$\Rightarrow OA^2+OC^2=OE^2+OF^2+AE^2+CF^2$ ......(i)

Now, in right triangles ΔOFB and ΔODE, we have

$OB^2=OF^2+FB^2$ and $OD^2=OE^2+DE^2$

$\Rightarrow OB^2+OD^2=(OF^2+FB^2)+(OE^2+DE^2)$

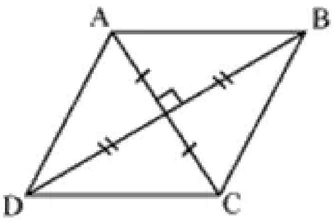
$\Rightarrow OB^2+OD^2=OE^2+OF^2+DE^2+BF^2$

$\Rightarrow OB^2+OD^2=OE^2+OF^2+CF^2+AE^2$ [$\because$ DE=CF and AE=BF]....(ii)

From (i) and (ii), we get

$OA^2+OC^2=OB^2+OD^2$

Answer 17.



In ΔAOB, ΔBOC, ΔCOD, ΔAOD

Applying Pythagoras theorem

$AB^2 = AO^2 + OB^2$
$BC^2 = BO^2 + OC^2$
$CD^2 = CO^2 + OD^2$

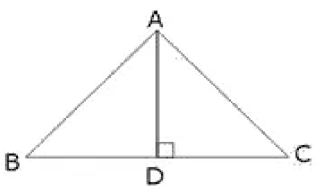
$AD^2 = AO^2 + OD^2$
Adding all these equations,
$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$

$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$ (diagonals bisect each other.)

$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$

$= (AC)^2 + (BD)^2$

Answer 18.



In equilateral triangle AD $\perp$ BC.

$\Rightarrow BD = DC = \frac{BC}{2}$ (In equilateral triangle altitude bisects the opposite side)

In right triangle ABD,

$AB^2 = AD^2 + BD^2$

$= AD^2 + (\frac{BC}{2})^2$

$= \frac{4AD^2 + BC^2}{4}$

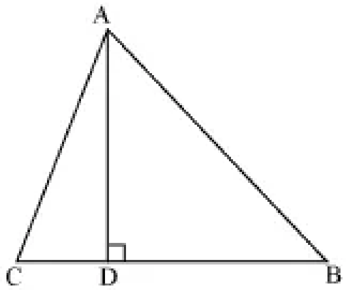
$= \frac{4AD^2 + AB^2}{4}$ (Since AB=BC)

$\Rightarrow 4AB^2 = 4AD^2 + AB^2$

$\Rightarrow 3AB^2 = 4AD^2$

Hence proved.

Answer 19.



In ΔACD

$AC^2 = AD^2 + DC^2$

$AD^2 = AC^2 - DC^2 \qquad (1)$

In ΔABD

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2 \qquad (2)$

From equation (1)and(2)

Therefore $AC^2 - DC^2 = AB^2 - DB^2$

since given that $3DC = DB$

$DC = \frac{BC}{4}$ and $DB = \frac{3BC}{4}$

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

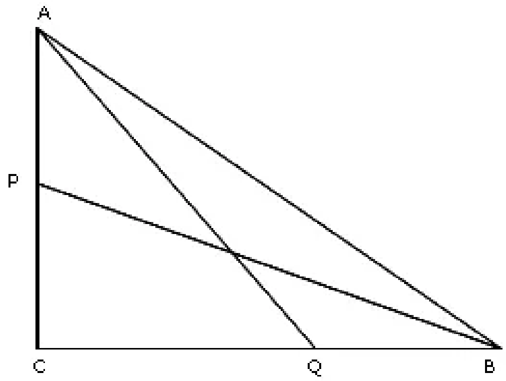
$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$16AC^2 - BC^2 = 16AB^2 - 9BC^2$

$\Rightarrow 16AB^2 - 16AC^2 = 8BC^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

Answer 20.



P divides AC in the ratio 2 : 1

So $CP = \frac{2}{3} AC$ ....(i)

Q divides BC in the ratio 2: 1

$QC = \frac{2}{3} BC$ ...... (ii)

(i) In ΔACQ

Using Pythagoras Theorem we have,

$AQ^2 = AC^2 + CQ^2$

$\Rightarrow AQ^2 = AC^2 + \frac{4}{9} BC^2$ (using (ii))

$\Rightarrow 9AQ^2 = 9AC^2 + 4BC^2$ ......(iii)

(ii) Applying Pythagoras theorem in right triangle BCP, we have

$BP^2 = BC^2 + CP^2$

$\Rightarrow BP^2 = BC^2 + \frac{4}{9} AC^2$ (Using (i))

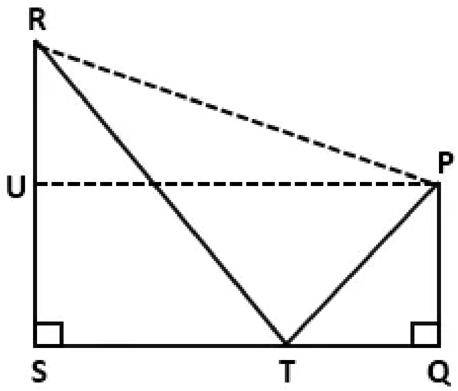
$\Rightarrow 9BP^2 = 9BC^2 + 4AC^2$

Adding (iii) and (iv), we get

$9(AQ^2 + BP^2) = 13(BC^2 + AC^2)$

$\Rightarrow 9(AQ^2 + BP^2) = 13AB^2$

Answer 21.



$PQ = \frac{RS}{3} = 8$ cm

$\Rightarrow PQ = 8$ cm and $RS = 3 \times 8 = 24$ cm

$3ST = 4QT = 48$ cm

$\Rightarrow ST = \frac{48}{3} = 16$ cm and $QT = \frac{48}{4} = 12$ cm

In ΔPTQ,

$PT^2 = PQ^2 + QT^2 = 8^2 + 12^2 = 64 + 144 = 208$

In ΔRTS,

$RT^2 = RS^2 + ST^2 = 24^2 + 16^2 = 576 + 256 = 832$

Now, $PT^2 + RT^2 = 208 + 832 = 1040$ ....(i)

Draw $PU \perp RS$ and Join PR.

$PU = SQ = ST + TQ = 16 + 12 = 28$ cm

$RU = RS - US = RS - PQ = 24 - 8 = 16$ cm

In ΔRUP,

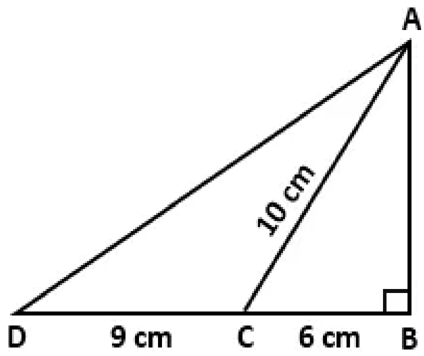
$PR^2 = RU^2 + PU^2 = 16^2 + 28^2 = 256 + 784 = 1040$ ....(ii)

From (i) and (ii), we get

$PT^2 + RT^2 = PR^2$

Thus, $\angle RTP = 90^\circ$

Answer 22.



In $\Delta ABC, \angle B = 90^\circ$

$\therefore AC^2 = AB^2 + BC^2$ ....(Pythagoras Theorem)

$\Rightarrow 10^2 = AB^2 + 6^2$

$\Rightarrow AB^2 = 10^2 - 6^2 = 100 - 36 = 64$

Now, $BD = BC + CD = 6 + 9 = 15$ cm

$\Rightarrow BD^2 = 225$

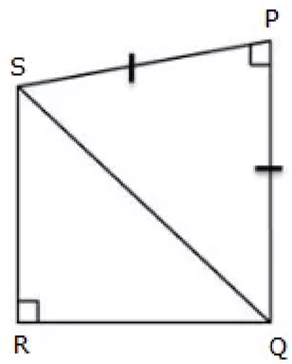
In $\Delta ABD, \angle B = 90^\circ$

$\therefore AD^2 = AB^2 + BD^2$

$\Rightarrow AD^2 = 64 + 225 = 289$

$\Rightarrow AD = 17$ cm

Answer 23.



In $\Delta SRQ, \angle R = 90^\circ$

$\therefore QS^2 = RS^2 + QR^2$ ....(Pythagoras Theorem)

$= 20^2 + 21^2$

$= 400 + 441$

$= 841$

Now, in $\Delta QSP, \angle P = 90^\circ$

$\therefore QS^2 = PQ^2 + PS^2$ ....(Pythagoras Theorem)

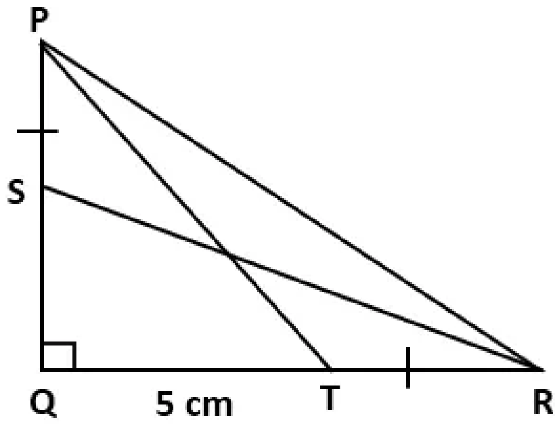
$\Rightarrow QS^2 = PQ^2 + PQ^2$ ....(Given PQ = PS)

$\Rightarrow QS^2 = 2PQ^2$

$\Rightarrow PQ^2 = \frac{QS^2}{2} = \frac{841}{2} = 420.5$

$\Rightarrow PQ = 20.50$ cm

Answer 24.



In ΔPQT, $\angle Q = 90^\circ$

$\therefore PT^2 = PQ^2 + QT^2$ ....(By Pythagoras Theorem)

$\Rightarrow PQ^2 = PT^2 - QT^2 = 13^2 - 5^2 = 169 - 25 = 144$

$\Rightarrow PQ = 12$ cm

Now, PS = TR = a(say)

In ΔSQR, $\angle Q = 90^\circ$

$\therefore SR^2 = QS^2 + QR^2$ ....(By Pythagoras Theorem)

$\Rightarrow SR^2 = (PQ - PS)^2 + (QT + TR)^2$

$\Rightarrow SR^2 = (PQ - PS)^2 + (QT + PS)^2$ ....(Since PS = TR)

$\Rightarrow SR^2 = PQ^2 - 2 \times PQ \times PS + PS^2 + QT^2 + 2 \times QT \times PS + PS^2$

$\Rightarrow 13^2 = 12^2 - 2 \times 12 \times a + a^2 + 5^2 + 2 \times 5 \times a + a^2$

$\Rightarrow 169 = 144 - 24a + a^2 + 25 + 10a + a^2$

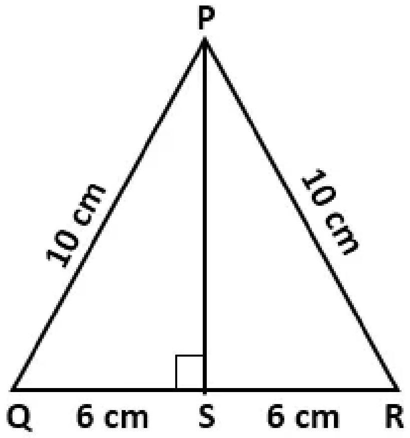
$\Rightarrow 169 = 169 - 14a + 2a^2$

$\Rightarrow 2a^2 = 14a$

$\Rightarrow a = 7$

Hence, PS = 7 cm

Answer 25.



Since, PQR is an isosceles triangle and PS ⊥ QR,
therefore it divides QR into two equal parts.

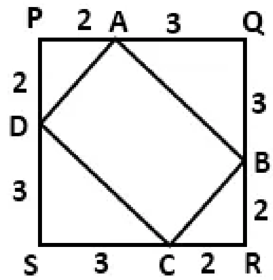
In ΔPSQ, $\angle S = 90^\circ$

$\therefore PQ^2 = PS^2 + QS^2$ ....(By Pythagoras Theorem)

$\Rightarrow PS^2 = PQ^2 - QS^2 = 10^2 - 6^2 = 100 - 36 = 64$

$\Rightarrow PS = 8$ cm

Answer 26.



In $\Delta APD, \angle P = 90^\circ$

$\therefore AD^2 = AP^2 + PD^2 = 2^2 + 2^2 = 4 + 4 = 8$

$\Rightarrow AD = 2\sqrt{2}$ cm

Similarly, we can prove that in ΔBRC,

$BC = 2\sqrt{2}$ cm

$\therefore AD = BC$ ....(i)

In $\Delta AQB, \angle Q = 90^\circ$

$\therefore AB^2 = AQ^2 + BQ^2 = 3^2 + 3^2 = 9 + 9 = 18$

$\Rightarrow AB = 3\sqrt{2}$ cm

Similarly, we can prove that in ΔCSD,

$CD = 3\sqrt{2}$ cm

$\therefore AB = CD$ ....(ii)

Again, in ΔAPD,

$AP = PD$

$\Rightarrow \angle PAD = \angle PDA = 45^\circ$

Also, in ΔAQB,

$AQ = BQ$

$\Rightarrow \angle QAB = \angle QBA = 45^\circ$

Now, $\angle PAD + \angle DAB + \angle QAB = 180^\circ$

$\Rightarrow 45^\circ + \angle DAB + 45^\circ = 180^\circ$

$\Rightarrow \angle DAB = 90^\circ$

Similarly, we can prove that $\angle ABC$, $\angle BCD$ and $\angle ADC$ are 90° each.

Thus, ABCD is a rectangle as opposite sides are equal and each angle is 90°.

Now,

Area of a rectangle ABCD = $AD \times AB = 2\sqrt{2} \times 3\sqrt{2} = 12\ cm^2$

Perimeter of a rectangle ABCD = $AB + BC + CD + AD$

$$= 2\sqrt{2} + 3\sqrt{2} + 2\sqrt{2} + 3\sqrt{2}$$

$$= 10\sqrt{2}\ cm$$