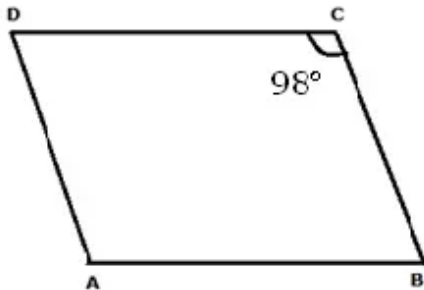


Chapter 19. Quadrilaterals

Ex 19.1

Answer 2.



ABCD is a parallelogram

$$\angle C = 98^\circ$$

$$\therefore \angle A = \angle C = 98^\circ \text{ (opposite angles of a parallelogram are equal)}$$

Now,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ \text{ (Sum of the angles of a quadrilateral = } 360^\circ)$$

$$98^\circ + \angle B + 98^\circ + \angle D = 360^\circ$$

$$\angle B + 196^\circ + \angle D = 360^\circ$$

$$\angle B + \angle D = 360^\circ - 196^\circ$$

$$\angle B + \angle D = 164^\circ$$

$$\text{But } \angle B = \angle D \text{ (opposite angles of a parallelogram are equal)}$$

$$\Rightarrow 2\angle B = 164^\circ$$

$$\Rightarrow \angle B = 82^\circ = \angle D$$

Therefore, $\angle B = 82^\circ$, $\angle A = 98^\circ$

Answer 4.

In $\triangle BDC$,

$$\angle BDC + \angle DCB + \angle CBD = 180^\circ$$

$$2a + 5a + 3a = 180^\circ$$

$$10a = 180^\circ$$

$$\Rightarrow a = 18^\circ$$

$$\angle BDC = 2a = 2 \times 18^\circ = 36^\circ$$

$$\angle DCB = 5a = 5 \times 18^\circ = 90^\circ$$

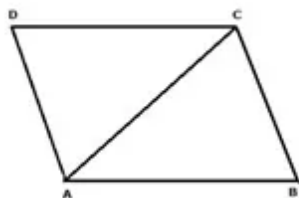
$$\angle CBD = 3a = 3 \times 18^\circ = 54^\circ$$

$$\angle DAB = \angle DCB = 90^\circ \text{ (opposite angles of parallelogram are equal)}$$

$$\angle DBA = \angle BDC = 36^\circ \text{ (alternate angles since } AB \parallel CD \text{)}$$

$$\angle BDA = \angle CBD = 54^\circ \text{ (alternate angles since } AB \parallel CD \text{)}$$

Therefore, $\angle DAB = \angle DCB = 90^\circ$, $\angle DBA + \angle CBD = 90^\circ$, $\angle BDA + \angle BDC = 90^\circ$

Answer 6.

ABCD is a parallelogram.

$$\text{Let } \angle CAB = x^\circ$$

$$\text{Then, } \angle ABC = 5x^\circ \text{ and } \angle BCA = 3x^\circ$$

In $\triangle ABC$,

$$\angle CAB + \angle ABC + \angle BCA = 180^\circ \text{ (sum of angles of triangle = } 180^\circ \text{)}$$

$$x^\circ + 5x^\circ + 3x^\circ = 180^\circ$$

$$9x^\circ = 180^\circ$$

$$x^\circ = 20^\circ$$

$$\Rightarrow \angle CAB = x^\circ = 20^\circ$$

$$\Rightarrow \angle ABC = 5x^\circ = 5 \times 20^\circ = 100^\circ$$

$$\Rightarrow \angle BCA = 3x^\circ = 3 \times 20^\circ = 60^\circ$$

Now,

$$\angle ADC = \angle ABC = 100^\circ \quad (\text{opposite angles of a parallelogram are equal})$$

$$\angle ACD = \angle CAB = 20^\circ \quad (\text{Alternate angles since } BC \parallel AD)$$

$$\angle CAD = \angle BCA = 60^\circ \quad (\text{Alternate angles since } BC \parallel AD)$$

$$\text{Therefore, } \angle ADC = \angle ABC = 100^\circ, \angle ACD + \angle BCA = 80^\circ, \angle CAD + \angle CAB = 80^\circ$$

Answer 7.

PQRS is a parallelogram.

$$\text{Let } \angle RPQ = 3x^\circ$$

$$\text{Then, } \angle PQR = 8x^\circ \text{ and } \angle QRP = 4x^\circ$$

In $\triangle PQR$,

$$\angle RPQ + \angle PQR + \angle QRP = 180^\circ \quad (\text{sum of angles of triangle} = 180^\circ)$$

$$3x^\circ + 8x^\circ + 4x^\circ = 180^\circ$$

$$15x^\circ = 180^\circ$$

$$x^\circ = 12^\circ$$

$$\Rightarrow \angle RPQ = 3x^\circ = 3 \times 12^\circ = 36^\circ$$

$$\Rightarrow \angle PQR = 8x^\circ = 8 \times 12^\circ = 96^\circ$$

$$\Rightarrow \angle QRP = 4x^\circ = 4 \times 12^\circ = 48^\circ$$

Now,

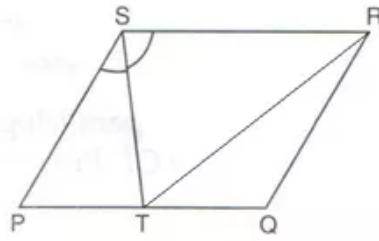
$$\angle PSR = \angle PQR = 96^\circ \quad (\text{opposite angles of a parallelogram are equal})$$

$$\angle RPS = \angle QRP = 48^\circ \quad (\text{Alternate angles since } QR \parallel PS)$$

$$\angle PRS = \angle RPQ = 36^\circ \quad (\text{Alternate angles since } QR \parallel PS)$$

$$\text{Therefore, } \angle PSR = \angle PQR = 96^\circ, \angle RPS + \angle RPQ = 84^\circ, \angle PRS + \angle QRP = 84^\circ$$

Answer 8.



- (i) $\angle PST = \angle TSR$ (i)
 $\angle PTS = \angle TSR$ (ii)(alternate angles $\because SR \parallel PQ$)

From (i) and (ii)

$$\angle PST = \angle PTS$$

Therefore, $PT = PS$

But $PT = QT$ (T is midpoint of PQ)

And $PS = QR$ (PS and QR are opposite and equal sides of a parallelogram)

Hence, $QT = QR$

- (ii) Since $QT = QR$

$$\angle QTR = \angle QRT$$

But $\angle QTR = \angle TRS$ (alternate angles $\because SR \parallel PQ$)

$$\Rightarrow \angle QRT = \angle TRS$$

Therefore, RT bisects $\angle R$

- (iii) $\angle PST = \angle TSR$

$$\angle QRT = \angle TRS$$

$$\angle QRS + \angle PSR = 180^\circ \quad (\text{adjacent angles of } \parallel \text{gm are supplementary})$$

Multiplying by $\frac{1}{2}$

$$\frac{1}{2} \angle QRS + \frac{1}{2} \angle PSR = \frac{1}{2} \times 180^\circ$$

$$\angle TSR + \angle TRS = 90^\circ$$

In $\triangle STR$,

$$\angle TSR + \angle RTS + \angle TRS = 180^\circ$$

$$90^\circ + \angle RTS = 180^\circ$$

$$\angle RTS = 90^\circ$$

Answer 9.

(i) Since $PC = BC$ (AD is half of AB and $BC=AD$ and $DC=AB$)

$$\angle CPB = \angle CBP$$

But $\angle CPB = \angle PBA$ (alternate angles $\because DC \parallel AB$)

$$\Rightarrow \angle CBP = \angle PBA$$

Therefore, BP bisects $\angle ABC$

(ii) $\angle DAP = \angle PAB$

$$\angle CBP = \angle PBA$$

$\angle DAB + \angle CBA = 180^\circ$ (adjacent angles of ||gm are supplementary)

Multiplying by $\frac{1}{2}$

$$\frac{1}{2} \angle DAB + \frac{1}{2} \angle CBA = \frac{1}{2} \times 180^\circ$$

$$\angle PAB + \angle PBA = 90^\circ$$

In $\triangle APB$,

$$\angle PAB + \angle PBA + \angle APB = 180^\circ$$

$$90^\circ + \angle APB = 180^\circ$$

$$\angle APB = 90^\circ$$

Therefore, $\angle APB$ is a right angle.

Answer 10.

In quadrilateral APCQ,

$$AP \parallel QC \quad (\text{since } AB \parallel CD)$$

$$AP = \frac{1}{2} AB \quad (\text{given})$$

$$CQ = \frac{1}{2} CD \quad (\text{given})$$

$$\text{But } AB = CD$$

$$\Rightarrow AP = CQ$$

Therefore, APCQ is a parallelogram.

Answer 11.

(i) In $\triangle SNR$ and $\triangle QMP$

$$\angle SNR = \angle QMP \quad (\text{right angles})$$

$$\angle SRN = \angle MPQ \quad (\text{alternate angles since } PQ \parallel SR)$$

$$\therefore \triangle SNR \sim \triangle QMP$$

$$\angle RSN = \angle PQM \quad \dots\dots\dots(i)$$

In $\triangle SNR$ and $\triangle QMP$

$$\angle SRN = \angle MPQ$$

$$\angle RSN = \angle PQM \quad (\text{from (i)})$$

$$PQ = SR \quad (\text{PQRS is a parallelogram})$$

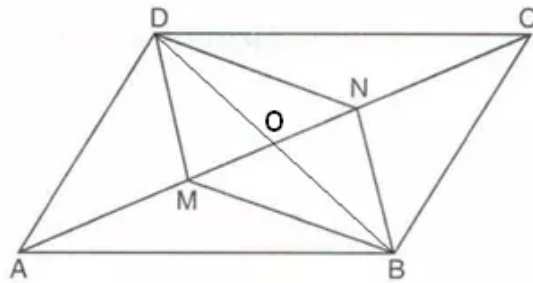
$$\text{Therefore, } \triangle SNR \cong \triangle QMP \quad (\text{ASA axiom})$$

(ii) Since $\triangle SNR \cong \triangle QMP$

$$\text{Hence, } SN = QM$$

Answer 12.

Join BD.



The diagonals of a parallelogram bisect each other.

Therefore, AC and BD bisect each other.

$$\Rightarrow OA = OC$$

$$\text{But } AM = CN$$

$$\text{Therefore, } OA - AM = OC - CN$$

$$\Rightarrow OM = ON$$

Therefore, in quadrilateral BMDN,

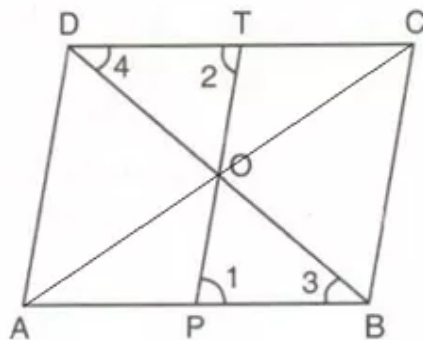
$$OM = ON \text{ and } OD = OB$$

$\Rightarrow$ Diagonals MN and BD bisect each other

$\Rightarrow$ BMDN is a parallelogram.

Answer 14.

Join AC



Since AC and BD are diagonals of a parallelogram, AC and BD bisect each other.

$$\Rightarrow OA = OC \text{ and } OD = OB \quad \dots\dots(i)$$

$$AP = CT$$

$$\text{But } AB = CD$$

$$\Rightarrow PB = DT$$

In $\triangle DOT$ and $\triangle POB$,

$$PB = DT$$

$$\angle 1 = \angle 2 \quad (\text{alternate angles since } AB \parallel CD)$$

$$\angle 3 = \angle 4 \quad (\text{alternate angles since } AB \parallel CD)$$

Therefore, $\triangle DOT \cong \triangle POB$

$$\text{Hence, } OT = OP \quad \dots\dots(ii)$$

From (i) and (ii)

$$OD = OB \text{ and } OT = OP$$

Therefore, PT and BD bisect each other.

Answer 15.

$$PQ = QT$$

$$\text{But } PQ = SR \quad (\text{PQRS is a parallelogram})$$

$$\text{Therefore, } QT = SR$$

In $\triangle SOR$ and $\triangle QOT$

$$QT = SR$$

$$\angle 3 = \angle 4 \quad (\text{vertically opposite angles})$$

$$\angle 1 = \angle 2 \quad (\text{alternate angles since } PQ \parallel SR)$$

Therefore, $\triangle SOR \cong \triangle QOT$

$$\text{Hence, } OS = OT \text{ and } OR = OQ$$

Therefore, ST bisects QR.

Ex 19.2

Answer 1.

In $\triangle QOM$,

$$\angle OQM = 45^\circ \quad (\text{In square diagonals make } 45^\circ \text{ with the sides})$$

$$OQ = MQ$$

$$\Rightarrow \angle QOM = \angle QMO \quad (\text{i) (equal sides have equal angles opposite to them)}$$

$$\angle QOM + \angle QMO + \angle OQM = 180^\circ$$

$$\angle QOM + \angle QOM + 45^\circ = 180^\circ$$

$$2\angle QOM = 180^\circ - 45^\circ$$

$$\angle QOM = 67.5^\circ$$

In $\triangle QOR$,

$$\angle QOR = 90^\circ \quad (\text{diagonals bisect at right angles})$$

$$\angle QOM + \angle MOR = 90^\circ$$

$$67.5^\circ + \angle MOR = 90^\circ$$

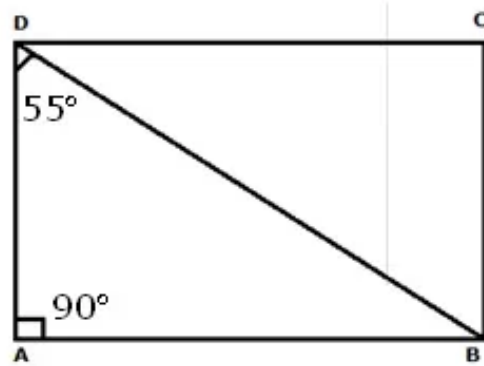
$$\angle MOR = 22.5^\circ$$

In $\triangle ROS$,

$$\angle OSR = 45^\circ \quad (\text{In square diagonals make } 45^\circ \text{ with the sides})$$

$$\Rightarrow \angle QSR = 45^\circ$$

Answer 2.



In $\triangle ABD$,

$$\angle ADB = 55^\circ$$

$$\angle DAB = 90^\circ \quad (\text{in rectangle angle between two sides is } 90^\circ)$$

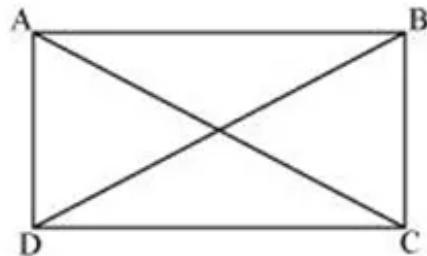
$$\angle ADB + \angle DAB + \angle ABD = 180^\circ$$

$$55^\circ + 90^\circ + \angle ABD = 180^\circ$$

$$\angle ABD = 180^\circ - 145^\circ$$

$$\angle ABD = 35^\circ$$

Answer 3.



Let ABCD be a parallelogram.

In $\triangle ABC$ and $\triangle DCB$,

$$AB = DC \quad (\text{Opposite sides of a parallelogram are equal})$$

$$BC = BC \quad (\text{Common})$$

$$AC = DB \quad (\text{Given})$$

$$\therefore \triangle ABC \cong \triangle DCB \quad (\text{By SSS Congruence rule})$$

$$\Rightarrow \angle ABC = \angle DCB$$

It is known that the sum of the measures of angles on the same side of transversal is 180° .

$$\angle ABC + \angle DCB = 180^\circ \quad (AB \parallel CD)$$

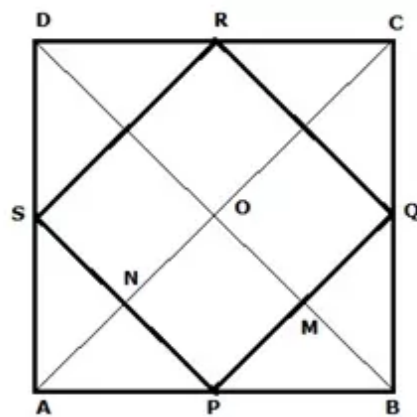
$$\Rightarrow \angle ABC + \angle ABC = 180^\circ$$

$$\Rightarrow 2\angle ABC = 180^\circ$$

$$\Rightarrow \angle ABC = 90^\circ$$

Since ABCD is a parallelogram and one of its interior angles is 90° , ABCD is a rectangle.

Answer 4.



Join AC and BD

In $\triangle ABC$, P and Q are the mid-points of sides AB and BC respectively.

Therefore, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$(i)

In $\triangle ADC$, R and S are the mid-points of sides CD and AD respectively.

Therefore, $RS \parallel AC$ and $RS = \frac{1}{2}AC$(ii)

From (i) and (ii)

$PQ \parallel RS$ and $PQ = RS$ (iii)

Thus, in a quadrilateral PQRS one pair of opposite sides are equal and parallel.

Hence, PQRS is a parallelogram.

Since ABCD is a square

$$AB = BC = CD = DA$$

$$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD; \frac{1}{2}AB = \frac{1}{2}BC$$

$$\Rightarrow PB = RC; BQ = CQ$$

Thus in $\triangle PBQ$ and $\triangle RCQ$,

$$PB = RC$$

$$BQ = CQ$$

$$\angle PBQ = \angle RCQ = 90^\circ$$

Therefore, $\triangle PBQ \cong \triangle RCQ$

$$\text{Hence, } PQ = QR \quad \dots\dots\dots(\text{iv})$$

From (iii) and (iv)

$$PQ = QR = RS$$

But PQRS is a parallelogram

$$\Rightarrow QR = PS$$

$$\Rightarrow PQ = QR = RS = PS \quad \dots\dots\dots(\text{v})$$

Now, $PQ \parallel AC$

$$\Rightarrow PM \parallel NO \quad \dots\dots\dots(\text{vi})$$

Since P and S are the mid-points of AB and AD respectively

$PS \parallel BD$

$$\Rightarrow PN \parallel MO \quad \dots\dots\dots(\text{vii})$$

Thus in quadrilateral PMON,

$$PM \parallel NO \quad (\text{from (vi)})$$

$$PN \parallel MO \quad (\text{from (vii)})$$

So, PMON is a parallelogram

$$\Rightarrow \angle MPN = \angle MON$$

$$\Rightarrow \angle MPN = \angle BOA$$

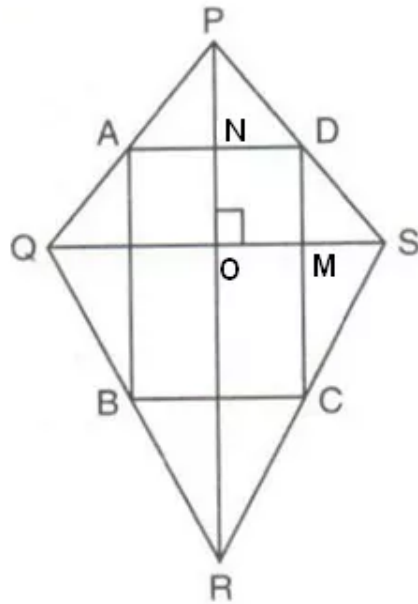
$$\Rightarrow \angle MPN = 90^\circ (\because \text{diagonals})$$

$$\Rightarrow \angle QPS = 90^\circ$$

Thus, PQRS is a quadrilateral such that $PQ=QR=RS=PS$ and $\angle QPS = 90^\circ$

Hence, PQRS is a square.

Answer 5.



In $\triangle PQS$, A and D are mid points of sides QP and PS respectively.

Therefore, $AD \parallel QS$ and $AD = \frac{1}{2} QS$ (i)

In $\triangle QRS$

B and C are the mid points of QR and RS respectively

Therefore, $BC \parallel QS$ and $BC = \frac{1}{2} QS$ (ii)

From equations (i) and (ii),

$AD \parallel BC$ and $AD = BC$

As in quadrilateral ABCD one pair of opposite sides are equal and parallel to each other, so, it is a parallelogram.

The diagonals of quadrilateral PQRS intersect each other at point O.

Now in quadrilateral OMDN

$ND \parallel OM$ ($AD \parallel QS$)

$DM \parallel ON$ ($DC \parallel PR$)

So, OMDN is parallelogram

$\angle MDN = \angle NOM$

$\angle ADC = \angle NOM$

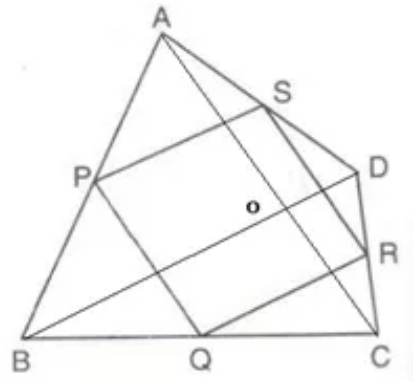
But, $\angle NOM = 90^\circ$ (diagonals are perpendicular to each other)

$\Rightarrow \angle ADC = 90^\circ$

Clearly ABCD is a parallelogram having one of its interior angle as 90° .

Hence, ABCD is rectangle.

Answer 6.



Join AC and BD

In $\triangle ABC$,

P and Q are mid-points of AB and BC respectively.

Therefore, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ (i)

In $\triangle ADC$,

S and R are mid-points of AD and DC respectively.

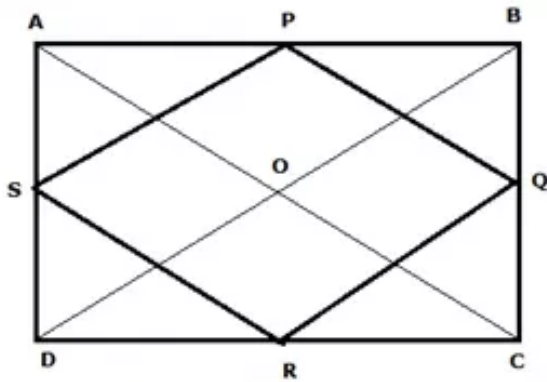
Therefore, $SR \parallel AC$ and $SR = \frac{1}{2}AC$ (ii)

From (i) and (ii)

$PQ \parallel SR$ and $PQ = SR$

Therefore, PQRS is a parallelogram.

Answer 7.



Let us join AC and BD

In $\triangle ABC$

P and Q are the mid-points of AB and BC respectively

Therefore, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (mid-point theorem) ... (1)

Similarly in $\triangle ADC$

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ (mid-point theorem) (2)

Clearly, $PQ \parallel SR$ and $PQ = SR$

As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.

Therefore, $PS \parallel QR$ and $PS = QR$ (opposite sides of parallelogram)... (3)

Now, in $\triangle BCD$, Q and R are mid points of side BC and CD respectively.

Therefore, $QR \parallel BD$ and $QR = \frac{1}{2} BD$ (mid-point theorem) ... (4)

But diagonals of a rectangle are equal

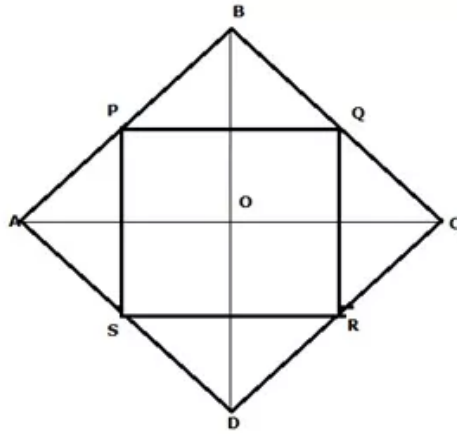
$\Rightarrow AC = BD$... (5)

Now, by using equation (1), (2), (3), (4), (5) we can say that

$PQ = QR = SR = PS$

So, PQRS is a rhombus.

Answer 8.



In $\triangle ABC$, P and Q are mid points of sides AB and BC respectively.

Therefore, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (using mid-point theorem) ... (1)

In $\triangle ADC$

R and S are the mid points of CD and AD respectively

Therefore, $RS \parallel AC$ and $RS = \frac{1}{2} AC$ (using mid-point theorem) ... (2)

From equations (1) and (2), we have

$PQ \parallel RS$ and $PQ = RS$

As in quadrilateral PQRS one pair of opposite sides are equal and parallel to each other, so, it is a parallelogram.

Let diagonals of rhombus ABCD intersect each other at point O.

Now in quadrilateral OMQN

$MQ \parallel ON$ ($PQ \parallel AC$)

$QN \parallel OM$ ($QR \parallel BD$)

So, OMQN is parallelogram

$\angle MQN = \angle NOM$

$\angle PQR = \angle NOM$

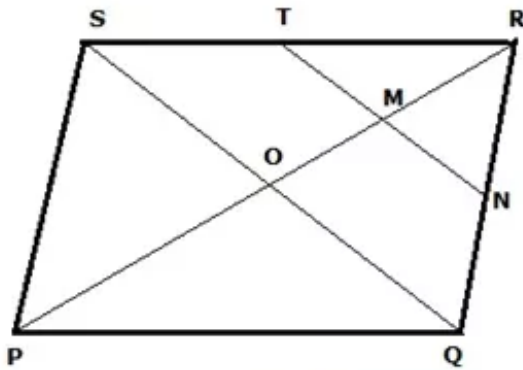
But, $\angle NOM = 90^\circ$ (diagonals of a rhombus are perpendicular to each other)

$\angle PQR = 90^\circ$

Clearly PQRS is a parallelogram having one of its interior angle as 90° .

Hence, PQRS is rectangle.

Answer 9.



Join PR to intersect QS at O

Diagonals of a parallelogram bisect each other.

Therefore, $OP = OR$

$$\text{But } MR = \frac{1}{4}PR$$

$$\therefore MR = \frac{1}{4}(2 \times OR)$$

$$\Rightarrow MR = \frac{1}{2}OR$$

Hence, M is the mid-point of OR.

In $\triangle ROS$, T and M are the mid-points of RS and OR respectively.

Therefore, $TM \parallel OS$

$$\Rightarrow TN \parallel QS$$

Also in $\triangle RQS$, T is the mid-point of RS and $TN \parallel QS$

Therefore, N is the mid-point of QR and $TN = \frac{1}{2}QS$

$$\Rightarrow QN = RN$$

Answer 10.

$$KP = \frac{1}{3}KN \quad (\text{since } KP:PN=1:2)$$

$$MQ = \frac{1}{3}LM \quad (\text{since } LQ:MQ=1:2)$$

$$\text{But } KN = LM \quad (\text{opposite sides of parallelogram } KLMN)$$

$$\Rightarrow \frac{1}{3}KN = \frac{1}{3}LM$$

$$\therefore KP = MQ \dots\dots\dots (i)$$

$$\text{Also, } KN \parallel LM$$

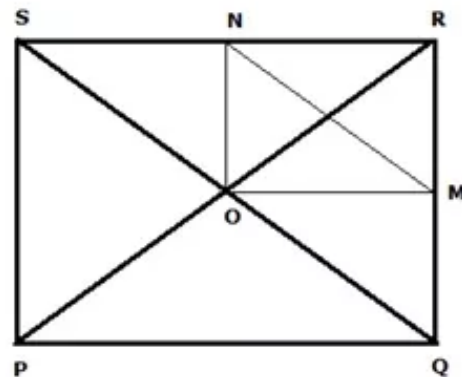
$$\Rightarrow KP \parallel MQ \dots\dots\dots (ii)$$

From (i) and (ii)

$$KP = MQ \text{ and } KP \parallel MQ$$

Hence, KQMP is a parallelogram.

Answer 11.



In $\triangle SRQ$,

N and O are the mid-points of SR and PR respectively.

$$\text{Therefore, } ON \parallel QR \text{ and } ON = \frac{1}{2}QR \text{ i.e. } ON = MR \dots\dots\dots (i)$$

$$PR = SQ \quad (\text{diagonals of rectangle are equal and bisect each other})$$

$$\Rightarrow O \text{ is mid-point of } SQ$$

In $\triangle RQS$,

M and O are the mid-points of QR and SQ respectively.

Therefore, $OM \parallel SR$ and $OM = \frac{1}{2} SR$ i.e. $OM = NR$ (ii)

$\angle MRN = \angle QRS = 90^\circ$ (iii) (PQRS is a rectangle)

From (i), (ii) and (iii)

Therefore, quadrilateral MONR has two opposite pairs of sides equal and parallel and an interior angle as right angle, so it is a rectangle.

In ΔSQR ,

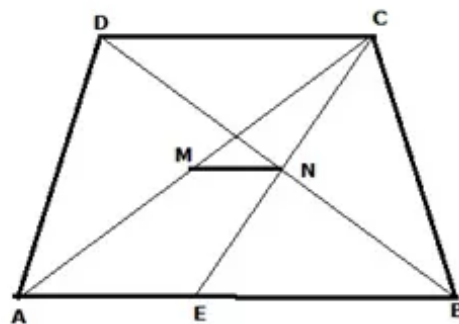
M and N are the mid-points of QR and SR respectively.

Therefore, $MN \parallel SQ$ and $MN = \frac{1}{2} SQ$

But $SQ = PR$

$$\Rightarrow MN = \frac{1}{2} PR$$

Answer 12.



Join AC and BD. M and N are mid-points of AC and BD respectively. Join MN. Draw a line CN cutting AB at E.

Now, in Δs DNC and BNE,

$DN = NB$ (N is the mid-point of BD, given)

$\angle CDN = \angle EBN$ (Alternate angles as $DC \parallel AB$)

$\angle DNC = \angle BNE$ (Vertically opposite angles)

$\Rightarrow \Delta DNC \cong \Delta BNE$ (By A-S-A Test)

$\Rightarrow DC = BE$

By Mid-Point Theorem, in ΔACE , M and N are mid-points

$$MN = \left(\frac{1}{2}\right) AE \text{ and } MN \parallel AE \text{ or } MN \parallel AB$$

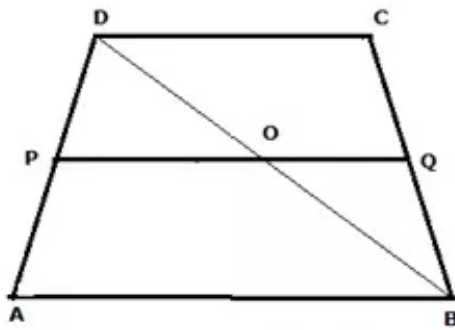
Also, $AB \parallel CD$, therefore, $MN \parallel CD$

$$\Rightarrow MN = \left(\frac{1}{2}\right) [AB - BE]$$

$$\Rightarrow MN = \left(\frac{1}{2}\right) [AB - CD] \quad (\text{since } BE = CD)$$

$$\Rightarrow MN = \left(\frac{1}{2}\right) \times \text{Difference of parallel sides } AB \text{ and } CD$$

Answer 14.



$$PQ \parallel DC \Rightarrow OQ \parallel DC \parallel AB$$

Therefore, Q and O are mid-points of BC and BD respectively.

In $\triangle ABD$,

P and O are mid-points of AD and BD respectively

$$\Rightarrow OP = \frac{1}{2} AB \quad \dots\dots\dots(i)$$

In $\triangle BCD$,

Q and O are mid-points of BC and BD respectively

$$\Rightarrow OQ = \frac{1}{2} CD \quad \dots\dots\dots(ii)$$

Adding (i) and (ii)

$$OP + OQ = \frac{1}{2} AB + \frac{1}{2} CD$$

$$\Rightarrow PQ = \frac{1}{2} (AB + CD)$$