

Ex 9.1

Q1

We have,

$$\begin{aligned}\sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}} &= \sqrt{\frac{2 \sin^2 \theta}{2 \cos^2 \theta}} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta = \text{RHS}\end{aligned}$$

Q2

LHS,

$$\begin{aligned}\frac{\sin 2\theta}{1 - \cos 2\theta} &= \frac{2 \sin \theta \cos \theta}{2 \sin^2 \theta} \\ &= \frac{\cos \theta}{\sin \theta} \\ &= \cot \theta = \text{RHS}\end{aligned}$$

Q3

LHS,

$$\begin{aligned}\frac{\sin 2\theta}{1 + \cos 2\theta} &= \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta = \text{RHS}\end{aligned}$$

Q4

LHS,

$$\begin{aligned}&\sqrt{2 + \sqrt{2} + 2 \cos 4\theta} \\ &= \sqrt{2 + \sqrt{2}(1 + \cos 4\theta)} \\ &= \sqrt{2 + \sqrt{2} \cdot 2 \cos^2 2\theta} \\ &= \sqrt{2 + 2 \cos 2\theta} \\ &= \sqrt{2(1 + \cos 2\theta)} \\ &= \sqrt{2 \cdot 2 \cos^2 \theta} \\ &= 2 \cos \theta = \text{RHS}\end{aligned}$$

Q5

LHS,

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$$

$$= \frac{2\sin^2 \theta + 2\sin \theta \cdot \cos \theta}{2\cos^2 \theta + 2\sin \theta \cdot \cos \theta}$$

$$= \frac{2\sin \theta (\sin \theta + \cos \theta)}{2\cos \theta (\cos \theta + \sin \theta)}$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta = \text{RHS}$$

Q6

LHS,

$$\frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta}$$

$$= \frac{\sin \theta + 2\sin \theta \cdot \cos \theta}{\cos \theta + (1 + \cos 2\theta)}$$

$$= \frac{\sin \theta (1 + 2\cos \theta)}{\cos \theta + 2\cos^2 \theta}$$

$$= \frac{\sin \theta (1 + 2\cos \theta)}{\cos \theta (1 + 2\cos \theta)}$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

$$= \text{RHS}$$

Q7

LHS,

$$\frac{\cos 2\theta}{1 + \sin 2\theta}$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta}$$

$$\left[\begin{array}{l} \because \cos 2\theta = \cos^2 \theta - \sin^2 \theta \\ \& \sin^2 \theta + \cos^2 \theta = 1 \end{array} \right]$$

$$= \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{(\cos \theta + \sin \theta)^2}$$

$$\left[\because a^2 - b^2 = (a + b)(a - b) \right]$$

$$= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$$

Dividing numerator and denominator by $\cos \theta$

$$= \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$= \tan \left(\frac{\pi}{4} - \theta \right) = \text{RHS}$$

$$\text{Note: } \tan \left(\frac{\pi}{4} - \theta \right) = \frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta}$$

$$= \frac{1 - \tan \theta}{1 + \tan \theta}$$

Q8

$$\frac{\cos \theta}{1 - \sin \theta}$$

$$= \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2}} \quad \left[\begin{array}{l} \cos^2 \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \\ \sin^2 \theta = \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} \end{array} \right]$$

$$= \frac{(\cos \frac{\theta}{2} - \sin \frac{\theta}{2})(\cos \frac{\theta}{2} + \sin \frac{\theta}{2})}{(\cos \frac{\theta}{2} - \sin \frac{\theta}{2})^2}$$

$$= \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}$$

Dividing numerator and denominator by $\cos \frac{\theta}{2}$

$$= \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}}$$

$$= \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right) = \text{RHS}$$

Q9

LHS,

$$\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8}$$

$$= \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \left(\pi - \frac{3\pi}{8} \right) + \cos^2 \left(\pi - \frac{\pi}{8} \right)$$

$$= \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{\pi}{8}$$

$$= 2 \left(\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} \right)$$

$$= 2 \left(\cos^2 \frac{\pi}{8} + \cos^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \right)$$

$$= 2 \left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)$$

$$= 2$$

$$= \text{RHS}$$

Q10

LHS,

$$\begin{aligned} & \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8} \\ &= \sin^2 \frac{\pi}{8} + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \sin^2 \frac{5\pi}{8} + \sin^2 \left(\pi - \frac{\pi}{8} \right) \\ &= \sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} + \sin^2 \left(\pi - \frac{3\pi}{8} \right) + \sin^2 \frac{\pi}{8} \\ &= 1 + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{\pi}{8} \\ &= 1 + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \sin^2 \frac{\pi}{8} \\ &= 1 + \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \\ &= 1 + 1 \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

Q11

LHS,

$$\begin{aligned} & (\cos \lambda + \cos \beta)^2 + (\sin \lambda + \sin \beta)^2 \\ &= \cos^2 \lambda + \cos^2 \beta + 2 \cos \lambda \cos \beta + \sin^2 \lambda + \sin^2 \beta + 2 \sin \lambda + \sin \beta \\ &= (\cos^2 \lambda + \sin^2 \lambda) + (\cos^2 \beta + \sin^2 \beta) + 2 (\cos \lambda \cos \beta + \sin \lambda \sin \beta) \\ &= 1 + 1 + 2 \cos (\lambda - \beta) \\ &= 2 + 2 \cos (\lambda - \beta) \\ &= 2 (1 + \cos (\lambda - \beta)) \\ &= 2 \cdot 2 \cos^2 \left(\frac{\lambda - \beta}{2} \right) \\ &= 4 \cos^2 \left(\frac{\lambda - \beta}{2} \right) \\ &= \text{RHS} \end{aligned}$$

Q12

LHS,

$$\begin{aligned}
& \sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) \\
&= \left[\sin\left(\frac{\pi}{8} + \frac{A}{2}\right) + \sin\left(\frac{\pi}{8} - \frac{A}{2}\right) \right] \left[\sin\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin\left(\frac{\pi}{8} - \frac{A}{2}\right) \right] \\
&= \left[\sin\frac{\pi}{8} \cdot \cos\frac{A}{2} + \cos\frac{\pi}{8} \cdot \sin\frac{A}{2} + \sin\frac{\pi}{8} \cdot \cos\frac{A}{2} - \cos\frac{\pi}{8} \cdot \sin\frac{A}{2} \right] \\
&= \left[\sin\frac{\pi}{8} \cdot \cos\frac{A}{2} + \cos\frac{\pi}{8} \cdot \sin\frac{A}{2} - \sin\frac{\pi}{8} \cdot \cos\frac{A}{2} + \cos\frac{\pi}{8} \cdot \sin\frac{A}{2} \right] \\
&= \left(2\sin\frac{\pi}{8} \cdot \cos\frac{A}{2} \right) \left(2\cos\frac{\pi}{8} \cdot \sin\frac{A}{2} \right) \\
&= 2\sin\frac{\pi}{8} \cdot \cos\frac{\pi}{2} \cdot 2\sin\frac{A}{2} \cdot \cos\frac{A}{2} \\
&= \sin 2 \cdot \frac{\pi}{8} \cdot \sin 2 \cdot \frac{A}{2} \\
&= \sin\frac{\pi}{4} \cdot \sin A \\
&= \frac{1}{\sqrt{2}} \sin A \\
&= \text{RHS}
\end{aligned}$$

Q13

LHS,

$$\begin{aligned}
& 1 + \cos^2 2\theta \\
&= 1 + \left[\cos^2 \theta - \sin^2 \theta \right]^2 \quad \left[\because \cos 2\theta = \cos^2 \theta - \sin^2 \theta \right] \\
&= 1 + \cos^4 \theta + \sin^4 \theta - 2\sin^2 \theta \cdot \cos^2 \theta \\
&= \left[\sin^2 \theta + \cos^2 \theta \right]^2 + \cos^4 \theta + \sin^4 \theta - 2\sin^2 \theta \cos^2 \theta \quad \left[\because \sin^2 \theta + \cos^2 \theta = 1 \right] \\
&= \sin^4 \theta + \cos^4 \theta + 2\sin^2 \theta \cos^2 \theta + \cos^4 \theta + \sin^4 \theta - 2\sin^2 \theta \cos^2 \theta \\
&= 2(\cos^4 \theta + \sin^4 \theta) \\
&= \text{RHS}
\end{aligned}$$

Q14

$$\cos^3 2\theta + 3 \cos 2\theta - 4(\cos^6 \theta - \sin^6 \theta)$$

$$\begin{aligned} \text{RHS} &= 4 \left[(\cos^2 \theta)^3 - (\sin^2 \theta)^3 \right] \\ &= 4(\cos^2 \theta - \sin^2 \theta) [\cos^4 \theta - \sin^4 \theta + \sin^2 \theta \cos^2 \theta] \\ &= 4 \cos^2 \theta \left[(\cos^2 \theta - \sin^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta \right] \\ &= 4 \cos^2 \theta [\cos^2 2\theta + 3 \sin^2 \theta \cos^2 \theta] \\ &= 4 \cos^2 \theta \left[\cos^2 2\theta + 3 \left(\frac{1 - \cos^2 \theta}{2} \right) \left(\frac{1 + \cos^2 \theta}{2} \right) \right] \\ &= 4 \cos^2 \theta \left[\cos^2 2\theta + \frac{3}{4} (1 - \cos^2 \theta) \right] \\ &= \cos^2 \theta [4 \cos^2 2\theta + 3 - 3 \cos^2 \theta] \\ &= \cos^2 \theta [\cos^2 2\theta + 3] \\ &= \cos^3 2\theta + 3 \cos^2 \theta \\ &= \text{LHS} \end{aligned}$$

LHS = RHS

Q15

$$\text{LHS} = (\sin 3A + \sin A) \sin A (\cos 3A - \cos A) \cos A$$

$$\Rightarrow 2 \sin 2A \sin A \cos A + (-2 \sin 2A \sin A \cos A)$$

$$\Rightarrow 2 \sin 2A \cos A \sin A - 2 \sin 2A \cos A \sin A$$

$$\Rightarrow 0 = \text{RHS}$$

$$\left[\begin{array}{l} \because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2} \\ \cos C - \cos D = -2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2} \end{array} \right]$$

Q16

$$\text{LHS} = \cos^2 \left(\frac{\pi}{4} - \theta \right) - \sin^2 \left(\frac{\pi}{4} - \theta \right)$$

$$= \cos 2 \left(\frac{\pi}{4} - \theta \right) \quad \left[\because \cos 2\theta = \cos^2 \theta - \sin^2 \theta \right]$$

$$= \cos \left(\frac{\pi}{2} - 2\theta \right) \quad \left[\because \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta \right]$$

$$= \sin 2\theta$$

$$= \text{RHS}$$

Q17

$$\begin{aligned}
\text{LHS} &= \cos 4A \\
&= \cos 2 \cdot 2A \\
&= 2 \cos^2 2A - 1 \left[\because \cos 2\theta = 2 \cos^2 \theta - 1 \right] \\
&= 2 \left(2 \cos^2 A - 1 \right)^2 - 1 \\
&= 2 \left(4 \cos^4 A - 4 \cos^2 A + 1 \right) - 1 \\
&= 8 \cos^4 A - 8 \cos^2 A + 1 \\
&= 1 - 8 \cos^2 A + 8 \cos^4 A \\
&= \text{RHS}
\end{aligned}$$

Q18

$$\begin{aligned}
\text{LHS} &= \sin 4A \\
&= \sin 2 \cdot 2A \\
&= 2 \sin 2A \cos 2A \\
&= 2 (2 \sin A \cos A) \cdot (\cos^2 A - \sin^2 A) \\
&= 4 \sin A \cos^2 A - 4 \sin^3 A \cos A \\
&= \text{RHS}
\end{aligned}$$

Q19

$$\begin{aligned}
\text{LHS} &= 3 (\sin x - \cos x)^4 + 6 (\sin x + \cos x)^2 + 4 (\sin^6 x + \cos^6 x) \\
&= 3 \left[\sin^4 x - 4 \sin^3 x \cos x + 6 \sin^2 x \cos^2 x - 4 \sin x \cos^3 x + \cos^4 x \right] \\
&\quad - 6 \left[\sin^2 x + 2 \sin x \cos x + \cos^2 x \right] + 4 \left[\sin^6 x + \cos^6 x \right] \\
&\quad \left[\because (a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4 \text{ by binomial expansion} \right] \\
&= 3 \left[\sin^4 x + \cos^4 x - 4 \sin x \cos x (\sin^2 x + \cos^2 x) + 6 \sin^2 x \cos^2 x \right] \\
&\quad + 6 [1 + 2 \sin x \cos x] + 4 \left[(\cos^2 x + \sin^2 x) (\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x) \right] \\
&\quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right] \\
&= 7 \left[\sin^4 x + \cos^4 x \right] + 18 \sin^2 x \cos^2 x - 4 \sin^2 x \cos^2 x + 6 \\
&= 7 \left[\sin^4 x + \cos^4 x + 2 \sin^2 x \cos^2 x \right] + 6 \\
&= 7 \left[\sin^2 x + \cos^2 x \right]^2 + 6 \\
&= 7 + 6 \\
&= 13 \\
&= \text{RHS}
\end{aligned}$$

Q20

$$\begin{aligned}
\text{L.H.S} &= 2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 \\
&= 2\left[(\sin^2 x)^3 + (\cos^2 x)^3\right] - 3(\sin^4 x + \cos^4 x) + 1 \quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right] \\
&= 2\left[(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)\right] - 3(\sin^4 x + \cos^4 x) + 1 \\
&= -[\sin^4 x + \cos^4 x + 2 \sin^2 x \cos^2 x] + 1 \\
&= -[\sin^2 x + \cos^2 x] + 1 \\
&= -1 + 1 \\
&= 0 \\
&= \text{RHS}
\end{aligned}$$

Q21

$$\begin{aligned}
\text{L.H.S} &= \cos^6 A - \sin^6 A \\
&= (\cos^2 A)^3 - (\sin^2 A)^3 \\
&= (\cos^2 A - \sin^2 A)(\cos^4 A + \sin^2 A \cos^2 A + \sin^4 A) \quad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right] \\
&= \cos 2A (\cos^4 A + 2 \sin^2 A \cos^2 A + \sin^4 A - \sin^2 A \cos^2 A) \\
&\quad \left[\because \cos^2 A - \sin^2 A = \cos 2A \text{ \& Adding and subtracting } \sin^2 A \cos^2 A\right] \\
&= \cos 2A \left[(\sin^2 A + \cos^2 A)^2 - \frac{4}{4} \sin^2 A \cos^2 A\right] \\
&= \cos 2A \left[1 - \frac{1}{4} (2 \sin A \cos A)^2\right] \\
&= \cos 2A \left[1 - \frac{1}{4} \sin^2 2A\right] \\
&= \text{RHS}
\end{aligned}$$

Q22

$$\begin{aligned}\text{L.H.S} &= \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) \\&= \frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} + \frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} \\&= \frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} \quad \left[\because \tan \frac{\pi}{4} = 1 \right] \\&= \frac{(1 + \tan^2 \theta + 2 \tan \theta) + (1 + \tan^2 \theta - 2 \tan \theta)}{(1 - \tan \theta)(1 + \tan \theta)} \\&= \frac{2(1 + \tan^2 \theta)}{1 - \tan^2 \theta} \\&= \frac{2 \sec^2 \theta}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} \quad \left[\because \sec^2 \theta = 1 + \tan^2 \theta \right] \\&= \frac{2 \sec^2 \theta \cdot \cos^2 \theta}{\cos^2 \theta - \sin^2 \theta} \quad \left[\because \sec = \frac{1}{\cos \theta} \right] \\&= \frac{2}{\cos 2\theta} \\&= 2 \sec 2\theta \\&= \text{RHS}\end{aligned}$$

Q23

$$\begin{aligned}
\text{L.H.S} &= \cot^2 A - \tan^2 A \\
&= \frac{\cos^2 A}{\sin^2 A} - \frac{\sin^2 A}{\cos^2 A} \\
&= \frac{(\cos^2 A)^2 - (\sin^2 A)^2}{\sin^2 A \cos^2 A} \\
&= \frac{(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)}{(\sin A \cos A)^2} \quad \left[\because a^2 - b^2 = (a+b)(a-b) \right] \\
&= \frac{\cos 2A}{\frac{1}{4}(2 \sin A \cos A)^2} \quad \left[\because \cos 2A = \cos^2 A - \sin^2 A \right] \\
&= \frac{4 \cos 2A}{\sin^2 2A} \\
&= \frac{4 \cos 2A}{\sin 2A} \cdot \frac{1}{\sin 2A} \quad \left[\because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right] \\
&= 4 \cot 2A \cdot \operatorname{cosec} 2A \\
&= \text{RHS}
\end{aligned}$$

Q24

$$\begin{aligned}
\cos 4\theta - \cos 4\alpha &= 2\cos^2 2\theta - 2\cos^2 2\alpha \\
&= 2(\cos 2\theta + \cos 2\alpha)(\cos 2\theta - \cos 2\alpha) \\
&= 2(2\cos^2 \theta - 1 + 1 - 2\sin^2 \alpha)(2\cos^2 \theta - 1 - 2\cos^2 \alpha + 1) \\
&= 8(\cos^2 \theta - \sin^2 \alpha)(\cos^2 \theta - \cos^2 \alpha) \\
&= 8(\cos \theta - \sin \alpha)(\cos \theta + \sin \alpha)(\cos \theta - \cos \alpha)(\cos \theta + \cos \alpha)
\end{aligned}$$

Q25

$$\begin{aligned}
 & \sin 3x + \sin 2x - \sin x \\
 &= (\sin 3x - \sin x) + \sin 2x \\
 &= 2 \cos \left(\frac{3x+x}{2} \right) \sin \left(\frac{3x-x}{2} \right) + 2 \sin x \cos x \dots \dots \dots \left[\because \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \right. \\
 & \qquad \qquad \qquad \left. \sin 2x = 2 \sin x \cos x \right] \\
 &= 2 \cos(2x) \sin(x) + 2 \sin x \cos x \\
 &= 2 \sin x [\cos(2x) + \cos x] \\
 &= 2 \sin x \left[2 \cos \left(\frac{2x+x}{2} \right) + \cos \left(\frac{2x-x}{2} \right) \right] \dots \dots \dots \left[\because \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right] \\
 &= 4 \sin x \cos \frac{3x}{2} \cos \frac{x}{2}
 \end{aligned}$$

Q26

$$\begin{aligned}
 \tan 82 \frac{1^\circ}{2} &= \tan \left(90 - 7 \frac{1^\circ}{2} \right) \\
 &= \cot 7 \frac{1^\circ}{2} \\
 &= \cot A \quad \text{If } A = 7 \frac{1^\circ}{2}
 \end{aligned}$$

Now

$$\begin{aligned}
 \cot A &= \frac{\cos A}{\sin A} \\
 &= \frac{2 \cos^2 A}{2 \sin A \cos A} \\
 &= \frac{1 + \cos^2 A}{\sin^2 A} \\
 \cot A &= \frac{1 + \cos 15}{\sin 15} \\
 &= \frac{1 + \cos(45 - 30)}{\sin 15} \\
 &= \frac{1 + \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right)}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}} \\
 &= \frac{2\sqrt{2} + (\sqrt{3} + 1)}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\
 &= \frac{2\sqrt{2}(\sqrt{3} + 1) + (\sqrt{3} + 1)^2}{3 - 1} \\
 &= \frac{2\sqrt{6} + 2\sqrt{2} + 4 + 2\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \cot A &= \sqrt{6} + \sqrt{2} + 2 + \sqrt{3} \dots \dots \dots (1) \\
 &= \sqrt{2} + 2 + \sqrt{6} + \sqrt{3} \\
 &= \sqrt{2} (\sqrt{2} + 1) + \sqrt{3} (\sqrt{2} + 1)
 \end{aligned}$$

$$\cot A = (\sqrt{2} + 1)(\sqrt{2} + \sqrt{3}) \dots \dots \dots (2)$$

From equation (1) and (2)

$$\begin{aligned}
 \tan 82 \frac{1^\circ}{2} &= \cot 7 \frac{1^\circ}{2} = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6} \\
 &= (\sqrt{2} + 1)(\sqrt{2} + \sqrt{3})
 \end{aligned}$$

Q27

We know that,

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

Put $A = 45^\circ$,

$$\sin 22\frac{1}{2}^\circ = \sqrt{\frac{1 - \cos 45^\circ}{2}}$$

{ since $\sin 22\frac{1}{2}^\circ$, is positive }

$$= \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$

$$\sin 22\frac{1}{2}^\circ = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$

And

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

Put $A = 45^\circ$

$$\cos 22\frac{1}{2}^\circ = \sqrt{\frac{1 + \cos 45^\circ}{2}}$$

$$= \sqrt{\frac{1 + \frac{1}{\sqrt{2}}}{2}}$$

$$\cos 22\frac{1}{2}^\circ = \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}}$$

Now,

$$\cot 22\frac{1}{2}^\circ = \frac{\cos 22\frac{1}{2}^\circ}{\sin 22\frac{1}{2}^\circ}$$

Q28(i)

$$\text{Since } \cos x = -\frac{3}{5} = \frac{b}{h}$$

$$\Rightarrow b = 3, h = 5$$

$$\Rightarrow p = 4$$

Now, x lies on third quad.

$$\begin{aligned}\therefore \sin 2x &= 2 \sin x \cdot \cos x \\ &= 2 \cdot \left(\frac{-4}{5}\right) \cdot \left(\frac{-3}{5}\right) = \frac{24}{25}\end{aligned}$$

$$\therefore \pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Which means $x/2$ lies in second quadrant

$$\begin{aligned}\text{so, } \cos x/2 &= \sqrt{\frac{1 + \cos x}{2}} \\ &= \sqrt{\frac{1 - 3/5}{2}} = \frac{-1}{\sqrt{5}}\end{aligned}$$

$$[\because 1 + \cos 2\theta = 2 \cos^2 \theta]$$

(-ve sign because of second quad.)
where $\cos D$ is -ve

Also,

$$\sin x/2 = \frac{\sin x}{2 \cos x/2}$$

$$[\because \sin 2A = 2 \sin A \cos A]$$

$$\begin{aligned}&= \left(\frac{-4/5}{2 \left(\frac{-1}{\sqrt{5}} \right)} \right) \\ &= \frac{2}{\sqrt{5}}\end{aligned}$$

Q28(ii)

$\therefore x$ lies in IInd quadrant.

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \pi < 2x < 2\pi \Rightarrow 2x \text{ lies in 1st quad.}$$

$$\text{Also, } \cos x = \frac{-3}{5} = \frac{b}{h} \Rightarrow b = 3$$

$$h = 5 \\ \Rightarrow p = 4.$$

$$\text{so, } \sin x = \frac{p}{h} = \frac{4}{5}$$

$$\therefore \sin 2x = 2 \sin x \cos x$$

$$= 2 \cdot \frac{4}{5} \cdot \left(\frac{-3}{5} \right) = \frac{-24}{25}$$

$$\sin \frac{x}{2} = \frac{\sin x}{2 \cos \frac{x}{2}} \text{ or } \sqrt{\frac{1 - \cos x}{2}}$$

$$= \sqrt{\frac{1 - \left(1 - \frac{3}{5}\right)}{2}}$$

$$= \frac{2}{\sqrt{5}}$$

Q29

$\therefore x$ lies in IInd quad.

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Which means $\frac{x}{2}$ lies in first quad.

$$\text{Now, } \sin x = \frac{\sqrt{5}}{3} = \frac{p}{h} \Rightarrow \begin{matrix} p = \sqrt{5} \\ h = 3 \end{matrix} \Rightarrow b = 2$$

$$\text{so, } \cos x = \frac{b}{h} = \frac{-2}{3} \quad (\text{-ve due to II}^{\text{nd}} \text{ quad})$$

Thus,

$$\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}} = \sqrt{\frac{1 - 2/3}{2}} = \frac{1}{\sqrt{6}}$$

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 + 2/3}{2}} = \sqrt{\frac{5}{6}}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}} = \sqrt{5}$$

Q30(i)

Since x lies in IInd quadrant

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}, \text{ which means } \frac{x}{2} \text{ lies in I}^{\text{st}} \text{ quad.}$$

Now,

$$\sin x = \frac{1}{4} = \frac{p}{h} \Rightarrow p = 1 \quad \Rightarrow b = \sqrt{15}$$

$$h = 4$$

$$\text{so, } \cos x = \frac{b}{h} = \frac{-\sqrt{15}}{4} \quad (-\text{ve due to II}^{\text{nd}} \text{ quad})$$

Thus,

$$\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}} = \sqrt{\frac{1 - \frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4 - \sqrt{15}}{8}}$$

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4 + \sqrt{15}}{8}}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{4 + \sqrt{15}}{8}}}{\sqrt{\frac{4 - \sqrt{15}}{8}}} = \sqrt{\frac{4 + \sqrt{15}}{4 - \sqrt{15}}}$$

$$= \sqrt{\frac{(4 + \sqrt{15})(4 + \sqrt{15})}{(4 - \sqrt{15})(4 + \sqrt{15})}}$$

$$= 4 + \sqrt{15}$$

Q30(ii)

Since θ is acute, so $0 \leq 2\theta < \pi$

$$\text{Now, } \cos \theta = \frac{4}{5} = \frac{b}{h} \Rightarrow b = 4 \quad \Rightarrow p = 3$$

$$h = 5$$

$$\therefore \sin \theta = \frac{p}{h} = \frac{3}{5}$$

$$\tan \theta = \frac{p}{b} = \frac{3}{4}$$

$$\text{so, } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \cdot \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{24}{7}$$

Q30(iii)

$$\sin \theta = \frac{4}{5} = \frac{p}{h} \Rightarrow p = 4 \quad \Rightarrow b = 3$$

$$h = 5$$

$$\therefore \cos \theta = \frac{b}{h} = \frac{3}{5}$$

$$\text{Now, } \sin \theta = 2 \sin \theta, \cos \theta = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{-7}{25}$$

$$\text{so, } \sin 4\theta = \sin 2 \cdot 2\theta = 2 \sin 2\theta \cos 2\theta$$

$$= 2 \cdot \frac{24}{25} \cdot \left(\frac{-7}{25}\right)$$

$$= \frac{-336}{625}$$

Q31

$$\begin{aligned}
 \frac{\sqrt{a+b} + \sqrt{a-b}}{\sqrt{a-b} + \sqrt{a+b}} &= \frac{(a+b) + (a-b)}{\sqrt{(a-b)(a+b)}} \\
 &= \frac{2a}{\sqrt{a^2 - b^2}} \\
 &= \frac{2}{\sqrt{1 - \left(\frac{b}{a}\right)^2}} \\
 &= \frac{2}{\sqrt{1 - \tan^2 x}} \dots \dots \dots \left[\because \tan x = \frac{b}{a} \right] \\
 &= \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}} \\
 &= \frac{2 \cos x}{\sqrt{\cos 2x}}
 \end{aligned}$$

Q32

we have,

$$\tan A = \frac{1}{7} \quad \& \quad \tan B = \frac{1}{3}$$

$$\begin{aligned}
 \therefore \cos 2A &= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2} = \frac{47}{50} \\
 &= \frac{47}{50} = \frac{24}{25} \dots \dots \dots (A)
 \end{aligned}$$

Also,

$$\sin 4B = \sin 2 \cdot 2B$$

$$= 2 \sin 2B \cdot \cos 2B$$

$$= 2 \left(\frac{2 \tan B}{1 + \tan^2 B} \right) \left(\frac{1 - \tan^2 B}{1 + \tan^2 B} \right)$$

$$= 4 \left(\frac{\frac{1}{3}}{1 + \frac{1}{9}} \right) \left(\frac{1 - \frac{1}{9}}{1 + \frac{1}{9}} \right)$$

$$= \frac{4 \cdot \frac{1}{3} \cdot \frac{8}{9}}{\frac{10}{9} \times \frac{10}{9}}$$

$$= \frac{32}{100}$$

$$= \frac{8 \times 3}{25} = \frac{24}{25} \dots \dots \dots (B)$$

from (A) & (B)

$$\cos 2A = \sin 4B$$

Q33

LHS,

$$\cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ$$

Divide and multiply by $2 \sin 7^\circ$, we get

$$\frac{1}{2 \sin 7^\circ} \cdot 2 \sin 7^\circ \cdot \cos 7^\circ \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ$$

$$= \frac{2 \sin 14^\circ}{2 \cdot 2 \sin 7^\circ} \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ \quad [\because 2 \sin A \cos A = \sin 2A]$$

$$= \frac{2 \sin 28^\circ}{2 \cdot 4 \sin 7^\circ} \cdot \cos 28^\circ \cdot \cos 56^\circ$$

$$= \frac{2 \sin 56^\circ}{2 \cdot 8 \sin 7^\circ} \cdot \cos 56^\circ$$

$$= \frac{\sin 112^\circ}{16 \sin 7^\circ}$$

$$= \frac{\sin(180^\circ - 68^\circ)}{16 \sin(90^\circ - 83^\circ)}$$

$$= \frac{\sin 68^\circ}{16 \cos 83^\circ}$$

$$= \text{RHS}$$

Q34

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a) \quad (1)$$

$$\text{for } a=b, \sin(2a) = 2 \sin(a) \cos(a) \quad (2)$$

$$\text{let } a = 16\pi/15 \quad (3)$$

$$(\text{so } 2a = 32\pi/15)$$

then using (3) in (2), we have

$$\begin{aligned} \sin(2a) &= 2 \sin(a) \cos(a) \\ &= 2 (2 \sin(a/2) \cos(a/2)) \cos(a) \\ &= 2 (2 (2 \sin(a/4) \cos(a/4)) \cos(a/2)) \cos(a) \\ &= 2 (2 (2 (2 \sin(a/8) \cos(a/8)) \cos(a/4)) \cos(a/2)) \cos(a) \\ &= 16 \sin(a/8) (\cos(a/8) \cos(a/4) \cos(a/2) \cos(a)) \end{aligned}$$

now note $\sin(2a) = \sin(2\pi/15)$ and $\sin(a/8) = \sin(2\pi/15)$

so,

$$\cos(a/8) \cos(a/4) \cos(a/2) \cos(a) = 1/16$$

or, replacing a with $16\pi/15$,

$$\cos(2\pi/15) * \cos(4\pi/15) * \cos(8\pi/15) * \cos(16\pi/15) = 1/16$$

Q35

$$\cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} = \frac{\sin \frac{2^4\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$\left[\because \cos A \cos 2A \cos 2^2A \cos 2^3A, \dots \dots \cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A} \right]$$

$$= \frac{\sin \frac{16\pi}{5}}{16 \sin \frac{\pi}{5}}$$

$$= \frac{\sin \left(3\pi + \frac{\pi}{5} \right)}{16 \sin \frac{\pi}{5}}$$

$$= \frac{1 \left\{ -\sin \left(\frac{\pi}{5} \right) \right\}}{16 \sin \frac{\pi}{5}}$$

$$= \frac{-1}{16}$$

Q36

$$\text{LHS} = \cos \frac{\pi}{65} \cdot \cos \frac{2\pi}{65} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

Divide and Multiply by $2 \sin \frac{\pi}{65}$, we get

$$= \frac{2 \sin \frac{\pi}{65}}{2 \sin \frac{\pi}{65}} \cdot \cos \frac{\pi}{65} \cdot \cos \frac{2\pi}{65} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}$$

$$= \frac{2 \sin \frac{2\pi}{65}}{2 \cdot 2 \sin \frac{\pi}{65}} \cdot \cos \frac{2\pi}{65} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}$$

$$= \frac{2 \sin \frac{4\pi}{65}}{2 \cdot 4 \sin \frac{\pi}{65}} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}$$

$$= \frac{2 \sin \frac{8\pi}{65}}{2 \cdot 8 \sin \frac{\pi}{65}} \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}$$

$$= \frac{2 \sin \frac{16\pi}{65}}{2 \cdot 16 \sin \frac{\pi}{65}} \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}$$

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$$= \frac{2 \sin \frac{32\pi}{65}}{2 \cdot 32 \sin \frac{\pi}{65}} \cos \frac{32\pi}{65}$$

$$= \frac{\sin \frac{64\pi}{65}}{64 \sin \frac{\pi}{65}}$$

$$= \frac{1}{64} \cdot \frac{\sin \left(\pi - \frac{\pi}{65} \right)}{\sin \frac{\pi}{65}}$$

$$= \frac{1}{64} \frac{\sin \frac{\pi}{65}}{\sin \frac{\pi}{65}}$$

$$= \frac{1}{64}$$

$$= \text{RHS}$$

Q37

We have, $2 \tan \alpha = 3 \tan \beta$

$$\Rightarrow \frac{\tan \alpha}{\tan \beta} = \frac{3}{2}$$

Let $\tan \alpha = 3K$ and $\tan \beta = 2K$

$$\text{Now, } \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{3K - 2K}{1 + 3K \cdot 2K} = \frac{K}{1 + 6K^2} \quad \dots (A)$$

Also,

$$\frac{\sin 2\beta}{5 - \cos 2\beta} = \frac{\frac{2 \tan \beta}{1 - \tan^2 \beta}}{5 - \left(\frac{1 - \tan^2 \beta}{1 + \tan^2 \beta} \right)}$$

$$= \frac{\frac{2 \cdot 2K}{1 - 4K^2}}{5 - \left(\frac{1 - 4K^2}{1 + 4K^2} \right)}$$

$$= \frac{4K}{5 + 20K^2 - 1 + 4K^2}$$

$$= \frac{4K}{4 + 24K^2} = \frac{K}{1 + 6K^2} \quad \dots (B)$$

from (A) & (B)

$$\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$$

Q38(i)

We have,

$$\sin \alpha + \sin \beta = a \quad \& \quad \cos \alpha + \cos \beta = b \quad \dots\dots\dots (A)$$

Squaring and adding, we get

$$\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = a^2 + b^2$$

$$\Rightarrow 1 + 1 + 2 (\sin \alpha \sin \beta + \cos \alpha \cos \beta) = a^2 + b^2$$

$$\Rightarrow 2 (\sin \alpha \sin \beta + \cos \alpha \cos \beta) = a^2 + b^2 - 2$$

$$\therefore 2 \cos (\alpha - \beta) = a^2 + b^2 - 2$$

$$\text{Thus, } \cos (\alpha - \beta) = \frac{a^2 + b^2 - 2}{2} \quad \dots\dots\dots (ii)$$

Again,

$$\sin \alpha + \sin \beta = a \quad \Rightarrow \quad 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} = a$$

$$\cos \alpha + \cos \beta = b \quad \Rightarrow \quad 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} = b$$

$$\Rightarrow \quad \tan \frac{\alpha + \beta}{2} = \frac{a}{b} \quad \dots\dots\dots (B)$$

Now,

$$\sin (\alpha + \beta) = \frac{2 \tan \frac{\alpha + \beta}{2}}{1 + \tan^2 \left(\frac{\alpha + \beta}{2} \right)}$$

$$= \frac{2 \frac{a}{b}}{1 + \frac{a^2}{b^2}} = \frac{2ab}{a^2 + b^2}$$

Thus,

$$\sin (\alpha + \beta) = \frac{2ab}{a^2 + b^2}$$

Q38(ii)

We have,

$$\sin \alpha + \sin \beta = a \quad \& \quad \cos \alpha + \cos \beta = b$$

Squaring and adding, we get

$$\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = a^2 + b^2$$

$$\Rightarrow 1 + 1 + 2 (\sin \alpha \sin \beta + \cos \alpha \cos \beta) = a^2 + b^2$$

$$\Rightarrow 2 (\sin \alpha \sin \beta + \cos \alpha \cos \beta) = a^2 + b^2 - 2$$

$$\therefore 2 \cos (\alpha - \beta) = a^2 + b^2 - 2$$

$$\text{Thus, } \cos (\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$$

Q39

We have,

$$2 \tan \frac{\alpha}{2} = \tan \frac{\beta}{2}$$

$$\Rightarrow \frac{\tan \frac{\alpha}{2}}{\tan \frac{\beta}{2}} = \frac{1}{2}$$

$$\text{Let } \tan \frac{\alpha}{2} = K \text{ and } \tan \frac{\beta}{2} = 2K$$

Then,

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{1 - K^2}{1 + K^2} \dots\dots\dots (A)$$

Also,

$$\frac{3 + 5 \cos \beta}{5 + 3 \cos \beta} = \frac{3 + 5 \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}{5 + 3 \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}$$

$$= \frac{3 + 5 \left(\frac{1 - 4K^2}{1 + 4K^2} \right)}{5 + 3 \left(\frac{1 - 4K^2}{1 + 4K^2} \right)}$$

$$= \frac{8 - 8K^2}{8 + 8K^2} = \frac{1 - K^2}{1 + K^2} \dots\dots\dots (B)$$

from (A) & (B)

$$\cos \alpha = \frac{3 + 5 \cos \beta}{5 + 3 \cos \beta}$$

Q40

We have,

$$\cos \theta = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cdot \cos \beta}$$

Now,

$$\cos \theta = \frac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2}$$

$$\Rightarrow \frac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2} = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$$

by componendo and dividendo, we get

$$\frac{(1 - \tan^2 \theta/2) + (1 + \tan^2 \theta/2)}{(1 - \tan^2 \theta/2) - (1 + \tan^2 \theta/2)} = \frac{1 + \cos \alpha \cos \beta + \cos \alpha + \cos \beta}{- (1 + \cos \alpha \cos \beta - \cos \alpha - \cos \beta)}$$

$$\Rightarrow \frac{2}{2 \tan^2 \theta/2} = \frac{(1 + \cos \alpha)(1 + \cos \beta)}{(1 - \cos \alpha)(1 - \cos \beta)}$$

$$\Rightarrow \tan^2 \theta/2 = \frac{(1 - \cos \alpha)(1 - \cos \beta)}{(1 + \cos \alpha)(1 + \cos \beta)}$$

$$= \frac{2 \sin^2 \alpha/2 \cdot 2 \sin^2 \beta/2}{2 \cos^2 \alpha/2 \cdot 2 \cos^2 \beta/2}$$

$$\Rightarrow \tan \theta/2 = \pm \tan \alpha/2 \cdot \tan \beta/2$$

Q41

We have,

$$\sec(\theta + \alpha) + \sec(\theta - \alpha) = 2 \sec \theta.$$

$$\Rightarrow \frac{1}{\cos \theta \cdot \cos \alpha - \sin \theta \sin \alpha} + \frac{1}{\cos \theta \cdot \cos \alpha + \sin \theta \sin \alpha} = \frac{2}{\cos \theta}$$

$$\Rightarrow \frac{2 \cos \theta \cos \alpha}{\cos^2 \theta \cos^2 \alpha - \sin^2 \theta \sin^2 \alpha} = \frac{2}{\cos \theta}$$

$$\Rightarrow \frac{\cos \theta \cos \alpha}{\cos^2 \theta \cos^2 \alpha - (1 - \cos^2 \theta) \sin^2 \alpha} = \frac{1}{\cos \theta}$$

$$\Rightarrow \cos^2 \theta \cos \alpha = \cos^2 \theta (\cos^2 \alpha + \sin^2 \alpha) - \sin^2 \alpha$$

$$\Rightarrow \cos^2 \theta (1 - \cos \alpha) = \sin^2 \alpha$$

$$\Rightarrow \cos^2 \theta = \frac{\sin^2 \alpha}{2 \sin^2 \alpha / 2}$$

$$= \frac{4 \sin^2 \alpha / 2 \cdot \cos^2 \alpha / 2}{2 \sin^2 \alpha / 2}$$

$$\Rightarrow \cos \theta = \pm \sqrt{2} \cos \alpha / 2$$

Q42

We have,

$$\cos \alpha + \cos \beta = \frac{1}{3} \text{ and } \sin \alpha + \sin \beta = \frac{1}{4}$$

Squaring and adding, we get

$$\left(\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta \right) + \left(\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta \right) = \frac{1}{9} + \frac{1}{16}$$

$$\Rightarrow 1 + 1 + 2 (\cos \alpha \cos \beta + \sin \alpha \sin \beta) = \frac{25}{144}$$

$$\Rightarrow 2 \cos (\alpha - \beta) = \frac{25}{144} - 2 = \frac{-263}{144}$$

$$\Rightarrow \cos (\alpha - \beta) = \frac{-263}{288}$$

Now,

$$\cos \left(\frac{\alpha - \beta}{2} \right) = \sqrt{\frac{1 + \cos (\alpha - \beta)}{2}}$$

$$= \sqrt{\frac{1 - \frac{263}{288}}{2}} = \sqrt{\frac{25}{576}}$$

$$= \pm \frac{5}{24}$$

$$\therefore \cos \left(\frac{\alpha - \beta}{2} \right) = \pm \frac{5}{24}$$

Q43

We have,

$$\sin \alpha = \frac{4}{5} \quad \& \quad \cos \beta = \frac{5}{13} \quad \Rightarrow \cos \alpha = \frac{3}{5} \quad \& \quad \sin \beta = \frac{12}{13}$$

$$\therefore \cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

Now,

$$\cos \left(\frac{\alpha - \beta}{2} \right) = \sqrt{\frac{1 + \cos (\alpha - \beta)}{2}}$$

$$= \sqrt{\frac{1 + \frac{63}{65}}{2}}$$

$$= \sqrt{\frac{128}{65 \times 2}} = \sqrt{\frac{64}{65}}$$

$$= \pm \frac{8}{\sqrt{65}}$$

$$\therefore \cos \left(\frac{\alpha - \beta}{2} \right) = \frac{8}{\sqrt{65}}$$

Q44

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

substitute these values in the given equation, it reduces to

$$a(1 - \tan^2 \theta) + b(2 \tan \theta) = c(1 + \tan^2 \theta)$$

$$(c+a) \tan^2 \theta + 2b \tan \theta + c - a = 0$$

As α and β are roots

$$\text{sum of the roots, } \tan \alpha + \tan \beta = \frac{2b}{c+a}$$

$$\text{Product of roots, } \tan \alpha \tan \beta = \frac{c-a}{c+a}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{2b}{c+a - c+a} = \frac{b}{a}$$

Q45

$$\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$$

squaring on both sides gives

$$\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta$$

Bring square terms on one side, we get

$$\cos 2\alpha + \cos 2\beta = -2(-\sin \alpha \sin \beta + \cos \alpha \cos \beta) = -2 \cos(\alpha + \beta)$$

Ex 9.2

Q1

L.H.S,

$$\begin{aligned}
 \sin 5\theta &= \sin(3\theta + 2\theta) \\
 &= \sin 3\theta \cos 2\theta + \cos 3\theta \sin 2\theta \\
 &= (3\sin\theta - 4\sin^3\theta)(1 - 2\sin^2\theta) + (4\cos^3\theta - 3\cos\theta)2\sin\theta\cos\theta \\
 &= 3\sin\theta - 4\sin^3\theta - 6\sin^5\theta + 3\sin^5\theta + (6\cos^4\theta - 6\cos^2\theta)\sin\theta \\
 &= 3\sin\theta - 10\sin^3\theta + 8\sin^5\theta + 8\sin\theta - 10\sin^3\theta + 8\sin^5\theta - 6\sin\theta + 6\sin^3\theta \\
 &= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta = \text{RHS}
 \end{aligned}$$

Q2

Consider the L.H.S of the given equation

$$\begin{aligned}
 4(\cos^3 10^\circ + \sin^3 20^\circ) &= 3(\cos 10^\circ + \sin 20^\circ) \\
 \text{Since } \sin 30^\circ &= \cos 60^\circ = \frac{1}{2} \\
 \text{and } \sin 60^\circ &= \cos 30^\circ = \frac{\sqrt{3}}{2} \\
 \Rightarrow \sin 3 \cdot 20^\circ &= \cos 3 \cdot 10^\circ \\
 \Rightarrow 3\sin 20^\circ - 4\sin^3 20^\circ &= 4\cos^3 10^\circ - 3\cos 10^\circ \\
 \Rightarrow 4(\cos^3 10^\circ - \sin^3 20^\circ) &= 3(\cos 10^\circ + \sin 20^\circ)
 \end{aligned}$$

Q3

$$\cos^3 \theta \sin 3\theta + \sin^3 \theta \cos 3\theta = \frac{3}{4} \sin 4\theta$$

$$\text{L.H.S} = \cos^3 \theta \sin 3\theta + \sin^3 \theta \cos 3\theta$$

$$= \left(\frac{\cos 3\theta + 3\cos\theta}{4} \right) \sin 3\theta + \left(\frac{3\sin\theta - \sin 3\theta}{4} \right) \cos 3\theta \quad \left\{ \begin{array}{l} \sin 3\theta = 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta \end{array} \right.$$

$$= \frac{1}{4} [3(\sin^2\theta \cos 3\theta + \sin\theta \cos 3\theta) + \cos 3\theta \sin^3\theta - \sin 3\theta \cos^3\theta]$$

$$= \frac{1}{4} [3\sin(\pi/2 + \theta) + 0]$$

$$= \frac{3}{4} \sin 4\theta$$

So,

$$\cos^3 \theta \sin 3\theta + \sin^3 \theta \cos 3\theta = \frac{3}{4} \sin 4\theta$$

Q4

We have to prove that

$$\sin 5A = 5 \cos^4 A \sin A - 10 \cos^2 A \sin^3 A + \sin^5 A$$

$$\begin{aligned} \text{L.H.S. } \sin 5A &= \sin (3A + 2A) \\ &= \sin 3A \cos 2A + \cos 3A \sin 2A \\ &= \left(3 \sin A - 4 \sin^3 A \right) \left(2 \cos^2 A - 1 \right) - \left(4 \cos^3 A - 3 \cos A \right) \left(2 \sin A \cos A \right) \\ &= -3 \sin A + 4 \sin^3 A - 6 \sin A \cos^2 A + 3 \sin^3 A \cos^2 A + 8 \cos^4 A \sin A - 6 \cos^2 A \sin A \\ &= 8 \cos^4 A \sin A - 3 \cos^2 A \cos^2 A - 3 \sin A + 4 \sin^3 A \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - 3 \sin A + 4 \cos^4 A \sin A + 4 \sin^3 A + 2 \sin^5 A \cos^2 A \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - 3 \sin A + (1 - \cos^2 A) + 2 \sin^3 A + (3 - \cos^2 A) \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - 3 \sin A + (1 - \cos^2 A) + (1 - \cos^2 A) + 2 \sin^3 A + (2 - \cos^2 A) \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - 3 \sin A + (1 - \cos^2 A) + 2 \sin^3 A + (2 - \cos^2 A) \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - \sin^2 A \left[3(1 + \cos^2 A) - 2(2 - \cos^2 A) \right] \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - \sin^2 A \left[3 - 3 \cos^2 A - 4 + 2 \cos^2 A \right] \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - \sin^2 A \left[\cos^2 A - 1 \right] \\ &= 5 \cos^4 A \sin A - 10 \sin^2 A \cos^2 A - \sin^2 A \\ &= \text{R.H.S.} \end{aligned}$$

Q5

$$\tan A \times \tan (A - 60^\circ) + \tan A \times \tan (A - 60^\circ) + \tan (A + 60^\circ) \tan (A - 60^\circ)$$

$$\begin{aligned} &= \tan(A) \frac{[\tan(A) - \tan(60^\circ)]}{[1 + \tan(A)\tan(60^\circ)]} \\ &+ \tan(A) \frac{[\tan(A) + \tan(60^\circ)]}{[1 - \tan(A)\tan(60^\circ)]} \\ &+ \left\{ \frac{[\tan(A) - \tan(60^\circ)]}{[1 + \tan(A)\tan(60^\circ)]} \right\} \left\{ \frac{[\tan(A) + \tan(60^\circ)]}{[1 - \tan(A)\tan(60^\circ)]} \right\} \end{aligned}$$

$$\begin{aligned} &= \tan(A) \frac{[\tan(A) - \tan(60^\circ)][1 - \tan(A)\tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]} \\ &+ \tan(A) \frac{[\tan(A) + \tan(60^\circ)][1 + \tan(A)\tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]} \\ &+ \frac{[\tan(A) - \tan(60^\circ)][\tan(A) + \tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]} \end{aligned}$$

$$\begin{aligned} &= \tan(A) \frac{[\tan(A) - \sqrt{3}][1 - \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]} \\ &+ \tan(A) \frac{[\tan(A) + \sqrt{3}][1 + \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]} \\ &+ \frac{[\tan(A) - \sqrt{3}][\tan(A) + \sqrt{3}]}{[1 - 3\tan^2(A)]} \end{aligned}$$

$$\begin{aligned} &= \tan(A) \frac{[4\tan(A) - \sqrt{3} - \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]} \\ &+ \tan(A) \frac{[4\tan(A) + \sqrt{3} + \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]} \\ &+ \frac{[\tan^2(A) - 3]}{[1 - 3\tan^2(A)]} \end{aligned}$$

$$= \frac{[9\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}$$

Q6

$$\tan A + \tan (60^\circ + A) - \tan (60^\circ - A) = 3 \tan 3A$$

$$\text{LHS} = \tan A + \tan (60^\circ + A) - \tan (60^\circ - A)$$

$$= \tan A + \frac{\tan 60^\circ + \tan A}{1 - \tan 60^\circ \tan A} - \frac{\tan 60^\circ - \tan A}{1 + \tan 60^\circ \tan A}$$

$$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3} \tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3} \tan A}$$

$$= \tan A + \left[\frac{\sqrt{3} + 3 \tan A + \tan A + \sqrt{3} \tan^2 A + \sqrt{3} + 3 \tan A + \tan A - \sqrt{3} \tan^2 A}{(1 - \sqrt{3} \tan A)(1 + \sqrt{3} \tan A)} \right]$$

$$= \tan A + \frac{8 \tan A}{1 - 3 \tan^2 A}$$

$$= \frac{\tan A - 3 \tan^3 A + 8 \tan A}{1 - 3 \tan^2 A}$$

$$= \frac{9 \tan A - 3 \tan^3 A}{1 - 3 \tan^2 A}$$

$$= 3 \left(\frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \right)$$

$$= 3 \tan 3A$$

so,

$$\tan A + \tan (60^\circ + A) - \tan (60^\circ - A) = 3 \tan 3A$$

Q7

$$\begin{aligned}\text{LHS} &= \cot A + \cot(60^\circ + A) - \cot(60^\circ - A) \\&= \frac{1}{\tan A} + \frac{1}{\tan(60^\circ + A)} - \frac{1}{\tan(60^\circ - A)} \\&= \frac{1}{\tan A} + \frac{1 - \sqrt{3} \tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3} \tan A}{\sqrt{3} - \tan A} \\&= \frac{1}{\tan A} - \frac{8 \tan A}{3 - \tan^2 A} \\&= \frac{3 - \tan^2 A - 8 \tan^2 A}{3 \tan A - \tan^3 A} \\&= \frac{3 - 9 \tan^2 A}{3 \tan A - \tan^3 A} \\&= 3 \left(\frac{1 - 3 \tan^2 A}{3 \tan A - \tan^3 A} \right) \\&= \frac{3}{\tan 3A} \\&= 3 \cot 3A \\&= \text{RHS}\end{aligned}$$

LHS = RHS

Hence proved.

Q8

$$\begin{aligned}\text{LHS} &= \cot A + \cot(60^\circ + A) + \cot(120^\circ + A) \\&= \cot A + \cot(60^\circ + A) - \cot[180^\circ - (120^\circ + A)] \\&\quad \left\{ \text{since } -\cot \theta = \cot(180^\circ - \theta) \right\} \\&= \cot A + \cot(60^\circ + A) - \cot(60^\circ - A) \\&= \frac{1}{\tan A} + \frac{1}{\tan(60^\circ + A)} - \frac{1}{\tan(60^\circ - A)} \\&= \frac{1}{\tan A} + \frac{1 - \sqrt{3} \tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3} \tan A}{\sqrt{3} - \tan A} \\&= \frac{1}{\tan A} - \frac{8 \tan A}{3 - \tan^2 A} \\&= \frac{3 - \tan^2 A - 8 \tan^2 A}{3 \tan A - \tan^3 A} \\&= \frac{3 - 9 \tan^2 A}{3 \tan A - \tan^3 A} \\&= \frac{3(1 - 3 \tan^2 A)}{3 \tan A - \tan^3 A} \\&= \frac{3}{\tan 3A} \\&= 3 \cot 3A\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

Q9

$$\text{LHS} = \sin^3 A + \sin^3 \left(\frac{2\pi}{3} + A \right) + \sin^3 \left(\frac{4\pi}{3} + A \right)$$

$$\left\{ \text{we know that } \sin^3 A = \frac{3 \sin A - \sin 3A}{4} \right\}$$

$$= \left(\frac{3 \sin A - \sin 3A}{4} \right) + \left\{ \frac{3 \sin \left(\frac{2\pi}{3} + A \right) - \sin 3 \left(\frac{2\pi}{3} + A \right)}{4} \right\} + \left\{ \frac{3 \sin \left(\frac{4\pi}{3} + A \right) - \sin 3 \left(\frac{4\pi}{3} + A \right)}{4} \right\}$$

$$= \left[\frac{3 \sin A - \sin 3A}{4} \right] + \left[\frac{3 \sin \left[\pi \left(\frac{2\pi}{3} + A \right) \right] - \sin (2\pi + 3A)}{4} \right] + \left[\frac{3 \sin \left(\pi + \left(\frac{\pi}{3} + A \right) \right) - \sin (4\pi + 3A)}{4} \right]$$

$$= \frac{1}{4} \left\{ [3 \sin A - \sin 3A] + [3 \sin \left(\frac{\pi}{3} - A \right) - \sin 3A] - [3 \sin \left(\frac{\pi}{3} + A \right) + \sin 3A] \right\}$$

$$= \frac{1}{4} [3 \sin A - \sin 3A + 3 \sin \left(\frac{\pi}{3} - A \right) - 3 \sin \left(\frac{\pi}{3} + A \right) - \sin 3A - \sin 3A]$$

$$= \frac{1}{4} [3 \sin A - 3 \sin 3A + 3 \left(\sin \left(\frac{\pi}{3} - A \right) - \sin \left(\frac{\pi}{3} + A \right) \right)]$$

$$= \frac{1}{4} \left[3 \sin A - 3 \sin 3A + 3 \left\{ 2 \cos \frac{\frac{\pi}{3} - A + \frac{\pi}{3} + A}{2} \sin \frac{\frac{\pi}{3} - A - \frac{\pi}{3} - A}{2} \right\} \right]$$

$$= \frac{1}{4} [3 \sin A - 3 \sin 3A + 6 \cos \frac{\pi}{3} \sin(-A)]$$

$$= \frac{1}{4} [3 \sin A - 3 \sin 3A - 3 \sin A]$$

$$= -\frac{3}{4} \sin 3A$$

$$= \text{RHS}$$

$$\text{LHS} = \text{RHS}$$

Q10

$$|\sin \theta \sin (60^\circ - \theta) \sin (60^\circ + \theta)|$$

$$= |\sin \theta (\sin^2 60^\circ - \sin^2 \theta)|$$

$$\left\{ \text{since } \sin (A + B) \sin (A - B) = \sin^2 A - \sin^2 B \right\}$$

$$= \left| \sin \theta \left(\frac{3}{4} - \sin^2 \theta \right) \right|$$

$$= \left| \frac{1}{4} \sin \theta (3 - 4 \sin^2 \theta) \right|$$

$$= \left| \frac{1}{4} \sin 3\theta \right|$$

$$= \frac{1}{4} |\sin 3\theta|$$

$$\leq \frac{1}{4}$$

$$\{ \sin \theta |\sin 3\theta| \leq \}$$

So,

$$|\sin \theta \sin (60^\circ - \theta) \sin (60^\circ + \theta)| \leq \frac{1}{4}$$

Q11

$$\begin{aligned}& \left| \cos \theta \cos (60^\circ - \theta) \cos (60^\circ + \theta) \right| \\&= \left| \cos \theta (\cos^2 60^\circ - \sin^2 \theta) \right| \\& \quad \left\{ \text{since } \cos (A - B) \cos (A + B) = \cos^2 A - \sin^2 B \right\} \\&= \left| \cos \theta \left(\frac{1}{4} - \sin^2 \theta \right) \right|\end{aligned}$$

$$= \left| \cos \theta \frac{1}{4} (1 - 4 \sin^2 \theta) \right|$$

$$= \left| \frac{1}{4} \cos \theta (1 - 4(1 - \cos^2 \theta)) \right|$$

$$= \left| \frac{1}{4} \cos \theta (-3 + 4 \cos^2 \theta) \right|$$

$$= \left| \frac{1}{4} (4 \cos 3\theta - 3 \cos \theta) \right|$$

$$= \left| \frac{1}{4} \cos 3\theta \right|$$

$$\leq \frac{1}{4}$$

$$\left\{ \text{since } |\cos 3\theta| \leq 1 \right\}$$

So,

$$\left| \cos \theta \cos (60^\circ - \theta) \cos (60^\circ + \theta) \right| \leq \frac{1}{4}$$

Ex 9.3

Q1

We have,

$$\begin{aligned} & \sin^2 72^\circ - \sin^2 60^\circ, \\ &= \sin^2 (90^\circ - 18^\circ) - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \cos^2 18^\circ - \frac{3}{4} \\ &= \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 - \frac{3}{4} \quad \left[\because \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} \right] \\ &= \frac{10+2\sqrt{5}}{16} - \frac{3}{4} \\ &= \frac{10+2\sqrt{5}-12}{16} \\ &= \frac{2\sqrt{5}-2}{16} \\ &= \frac{\sqrt{5}-1}{8} \end{aligned}$$

Q2

$$\begin{aligned} \text{L.H.S} &= \sin^2 24^\circ - \sin^2 6^\circ \\ &= \sin (24+6) \sin (24-6) \quad \left[\because \sin (A+B) \sin (A-B) = \sin^2 A - \sin^2 B \right] \\ &= \sin 30^\circ \sin 18^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{5}-1}{4} \quad \left[\because \sin 18^\circ = \frac{\sqrt{5}-1}{4} \right] \\ &= \frac{\sqrt{5}-1}{8} \\ &= \text{RHS} \end{aligned}$$

Q3

$$\begin{aligned}\text{L.H.S} &= \sin^2 42^\circ - \cos^2 78^\circ \\&= \sin^2 (90 - 48) - \cos^2 (90 - 12) \\&= \cos^2 48^\circ - \sin^2 12^\circ \\&= \cos (48 + 12) \cdot \cos (48 - 12) \\&\quad \left[\because \cos (A + B) \cdot \cos (A - B) = \cos^2 A - \sin^2 B \right] \\&= \cos 60^\circ \cdot \cos 36^\circ \\&= \frac{1}{2} \cdot \frac{\sqrt{5} + 1}{4} \quad \left[\because \cos 36^\circ = \frac{\sqrt{5} + 1}{4} \right] \\&= \frac{\sqrt{5} + 1}{8} \\&= \text{RHS}\end{aligned}$$

Q4

$$\begin{aligned}\text{L.H.S} &= \cos 78^\circ \cdot \cos 42^\circ \cdot \cos 36^\circ \\&= \frac{(2 \cos 78^\circ \cdot \cos 42^\circ)}{2} \cdot \cos 36^\circ \\&= \frac{1}{2} (\cos 120^\circ + \cos 36^\circ) \cdot \cos 36^\circ \\&= \frac{1}{2} \left(\frac{-1}{2} + \frac{\sqrt{5} + 1}{4} \right) \frac{\sqrt{5} + 1}{4} \\&= \frac{1}{8} \left[\frac{-2(\sqrt{5} + 1) + 5 + 1 + 2\sqrt{5}}{4} \right] \\&= \frac{1}{8} \left[\frac{4}{4} \right] \\&= \frac{1}{8} \\&= \text{RHS}\end{aligned}$$

Q5

$$\text{L.H.S} = \cos \frac{\pi}{15}, \cos \frac{2\pi}{15}, \cos \frac{4\pi}{15}, \cos \frac{7\pi}{15}$$

$$= \frac{2 \sin \frac{\pi}{15} \cdot \cos \frac{\pi}{15}}{2 \sin \frac{\pi}{15}} \cdot \cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{7\pi}{15} \left[\text{Divide and multiply by } 2 \sin \frac{\pi}{15} \right]$$

$$= \frac{2 \cdot \sin \frac{2\pi}{15}}{2 \cdot 2 \sin \frac{\pi}{15}} \cdot \cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{7\pi}{15}$$

$$= \frac{2 \cdot \sin \frac{4\pi}{15}}{2 \cdot 4 \sin \frac{\pi}{15}} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{7\pi}{15}$$

$$= \frac{2 \sin \frac{8\pi}{15}}{2 \cdot 8 \sin \frac{\pi}{15}} \cdot \cos \left(\frac{7\pi}{15} \right)$$

$$= \frac{\sin \left(\frac{8\pi}{15} + \frac{7\pi}{15} \right) + \sin \left(\frac{8\pi}{15} - \frac{7\pi}{15} \right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{\sin \pi + \sin \frac{\pi}{15}}{16 \sin \frac{\pi}{15}}$$

$$= \frac{\sin \frac{\pi}{15}}{16 \sin \frac{\pi}{15}} \quad [\because \sin \pi = 0]$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

Q6

$$\cos \frac{5\pi}{15} = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$\cos \frac{7\pi}{15} = \cos \left(\pi - \frac{8\pi}{15} \right)$$

$$\cos \frac{7\pi}{15} = -\cos \frac{8\pi}{15}$$

Now LHS = $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}$

$$= \left[\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \left(-\cos \frac{8\pi}{15} \right) \right] \left(\cos \frac{3\pi}{15} \cos \frac{6\pi}{15} \right) \frac{1}{2}$$

$$= -\frac{2^3}{2^4 \sin \frac{\pi}{15}} \left[2 \sin \frac{\pi}{15} \cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \right]$$

$$\times \frac{2}{8 \sin \frac{3\pi}{15}} \left(2 \sin \frac{3\pi}{15} \cos \frac{3\pi}{15} \cos \frac{6\pi}{15} \right)$$

$$= -\frac{2^3}{16 \sin \frac{\pi}{15}} \left[\sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \right] \times \frac{2}{8 \sin \frac{3\pi}{15}} \left(\sin \frac{6\pi}{15} \cos \frac{6\pi}{15} \right)$$

$$= -\frac{2^2}{16 \sin \frac{\pi}{15}} \left[2 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \right] \times \frac{1}{8 \sin \frac{3\pi}{15}} \left(2 \sin \frac{6\pi}{15} \cos \frac{6\pi}{15} \right)$$

$$= -\frac{2}{16 \sin \frac{\pi}{15}} \left[\sin \frac{8\pi}{15} \cos \frac{8\pi}{15} \right] \frac{\sin \frac{12\pi}{15}}{8 \sin \frac{3\pi}{15}}$$

$$= -\frac{1}{16 \sin \frac{\pi}{15}} \left(\sin \frac{16\pi}{15} \right) \frac{\sin \frac{12\pi}{15}}{8 \sin \frac{3\pi}{15}}$$

$$= -\frac{\sin \left(\pi + \frac{\pi}{15} \right)}{128 \sin \frac{\pi}{15}} \times \frac{\sin \left(\pi - \frac{3\pi}{15} \right)}{\sin \frac{3\pi}{15}}$$

$$= -\frac{-\sin \frac{\pi}{15}}{128 \sin \frac{\pi}{15}} \times \frac{\sin \frac{3\pi}{15}}{\sin \frac{3\pi}{15}}$$

$$= \frac{1}{128}$$

Q7

$$\text{L.H.S} = \cos 6^\circ, \cos 42^\circ, \cos 66^\circ, \cos 78^\circ$$

$$= \frac{1}{4} (2 \cos 6^\circ, \cos 66^\circ) (2 \cos 42^\circ, \cos 78^\circ)$$

$$= \frac{1}{4} (\cos 72^\circ + \cos 60^\circ) (\cos 120^\circ + \cos 36^\circ)$$

$$= \frac{1}{4} \left(\sin 18^\circ + \frac{1}{2} \right) \left(-\frac{2}{2} + \frac{\sqrt{5}+1}{4} \right)$$

$$= \frac{1}{4} \left(\frac{\sqrt{5}-1}{4} + \frac{1}{2} \right) \left(\frac{\sqrt{5}+1}{4} - \frac{1}{2} \right)$$

$$= \frac{1}{4} \left(\frac{\sqrt{5}-1+2}{4} \right) \left(\frac{\sqrt{5}+1-2}{4} \right)$$

$$= \frac{1}{64} (\sqrt{5}+1) (\sqrt{5}-1)$$

$$= \frac{1}{64} (\sqrt{5})^2 - 1^2$$

$$= \frac{1}{64} (5-1)$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

Q8

$$\begin{aligned}\text{L.H.S.} &= \sin 6^\circ \cdot \sin 42^\circ \cdot \sin 60^\circ \cdot \sin 78^\circ \\&= \frac{1}{4} [2 \sin 6^\circ \cdot \sin 66^\circ] [2 \sin 42^\circ \cdot \sin 78^\circ] \\&= \frac{1}{4} [\cos 60^\circ - \cos 12^\circ] [\cos 36^\circ - \cos 120^\circ] \\&= \frac{1}{4} \left(\frac{1}{2} - \sin 18^\circ \right) \left(\frac{\sqrt{5}-1}{4} + \frac{1}{2} \right) \\&= \frac{1}{4} \left(\frac{1}{2} - \frac{\sqrt{5}-1}{4} \right) \left(\frac{\sqrt{5}+1}{4} + \frac{1}{2} \right) \\&= \frac{1}{4} \left(\frac{2 - \sqrt{5} + 1}{4} \right) \left(\frac{\sqrt{5} + 1 + 2}{4} \right) \\&= \frac{1}{64} (3^2 - \sqrt{5}^2) \\&= \frac{1}{64} (4 - 5) \\&= -\frac{1}{64} \\&= \text{RHS}\end{aligned}$$

Q9

$$\begin{aligned}\text{L.H.S.} &= \cos 36^\circ \cdot \cos 42^\circ \cdot \cos 60^\circ \cdot \cos 78^\circ \\&= \frac{1}{2} \cos 36^\circ \cdot \cos 60^\circ \cdot (2 \cos 42^\circ \cdot \cos 78^\circ) \\&= \frac{1}{2} \left(\frac{\sqrt{5}+1}{4} \right) \cdot \frac{1}{2} (\cos 120^\circ + \cos 36^\circ) \\&= \frac{(\sqrt{5}+1)}{16} \left(\frac{-1}{2} + \frac{\sqrt{5}+1}{4} \right) \\&= \frac{(\sqrt{5}+1)}{16} \left(\frac{-2 + \sqrt{5} + 1}{4} \right) \\&= \frac{(\sqrt{5}+1)(\sqrt{5}-1)}{64} \\&= \frac{5-1}{64} \\&= \frac{1}{16} \\&= \text{RHS}\end{aligned}$$

Q10

L.H.S,

$$\sin 36^\circ, \sin 72^\circ, \sin 108^\circ, \sin 144^\circ$$

$$= \sin 36^\circ, \sin 72^\circ, \sin 72^\circ, \sin 36^\circ$$

$$= \frac{1}{4} (2 \sin 36^\circ, \sin 72^\circ)^2$$

$$= \frac{1}{4} (2 \sin 36^\circ \cos 18^\circ)^2$$

$$= \frac{4}{4} \left(\frac{\sqrt{10-2\sqrt{5}}}{4}, \frac{\sqrt{10+2\sqrt{5}}}{4} \right)^2$$

$$= \frac{1}{64} (10-2\sqrt{5})(10+2\sqrt{5})$$

$$= \frac{100-20}{64 \times 4}$$

$$= \frac{80}{256}$$

$$= \frac{5}{16}$$

- R.H.S

$$\left[\begin{array}{l} \because \sin 144^\circ = \sin (180^\circ - 36^\circ) = \sin 36^\circ \\ \text{and } \sin 108^\circ = \sin (180^\circ - 72^\circ) = \sin 72^\circ \end{array} \right]$$

$$[\because \sin 72^\circ = \cos 18^\circ]$$