

Chapter - 2
Polynomial

Exercise 2.1

Multiple Choice Questions:

Question 1:

Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3 , then the value of k is

- (A) $\frac{4}{3}$
- (B) $\frac{-4}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{-2}{3}$

Solution:

(A) $\frac{4}{3}$

It is given in the question,

-3 is one of the zeros of quadratic polynomial $(k-1)x^2 + kx + 1$.

Putting -3 in the given polynomial,

$$(k-1)(-3)^2 + k(-3) + 1 = 0$$

$$(k-1)9 + k(-3) + 1 = 0$$

$$9k - 9 - 3k + 1 = 0$$

$$6k - 8 = 0$$

$$k = \frac{8}{6}$$

Or,

$$k = \frac{4}{3}$$

2. A quadratic polynomial, whose zeroes are -3 and 4 , is

- (A) $x^2 - x + 12$
- (B) $x^2 + x + 12$
- (C) $\frac{x^2}{2} - \frac{x}{2} - 6$
- (D) $2x^2 + 2x - 24$

Solution:

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$

Justification:

Sum of zeroes, $\alpha + \beta = -3 + 4 = 1$

Product of Zeroes, $\alpha\beta = -3 \times 4 = -12$

So, the quadratic polynomial becomes,

$x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})$

$$= x^2 - (\alpha + \beta)x + (\alpha\beta)$$

$$= x^2 - (1)x + (-12)$$

$$= x^2 - x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

3. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3 , then

- (A) $a = -7, b = -1$
- (B) $a = 5, b = -1$
- (C) $a = 2, b = -6$
- (D) $a = 0, b = -6$

Solution:

(D) $a = 0, b = -6$

The zeroes of the polynomial = 2 and -3 ,

Putting, $x = 2$ in $x^2 + (a+1)x + b$

$$2^2 + (a+1)(2) + b = 0$$

$$4 + 2a + 2 + b = 0$$

$$6 + 2a + b = 0$$

$$2a + b = -6 \quad \dots (i)$$

Now Putting $x = -3$ in equation.

$$(-3)^2 + (a+1)(-3) + b = 0$$

$$9 - 3a - 3 + b = 0$$

$$6 - 3a + b = 0$$

$$-3a + b = -6$$

.... (ii)

Subtracting equation (ii) from (i)

$$2a + b - (-3a + b) = -6 - (-6)$$

$$2a + b + 3a - b = -6 + 6$$

$$5a = 0$$

$$a = 0$$

Putting the value of 'a' in equation (i),

$$2a + b = -6$$

$$2(0) + b = -6$$

$$b = -6$$

4. The number of polynomials having zeroes as -2 and 5 is

(A) 1

(B) 2

(C) 3

(D) more than 3

Solution:

(D) More than 3

Explanation:

It is given in the question, the zeroes of the polynomials are -2 and 5.

The polynomial is the form of

$$p(x) = ax^2 + bx + c.$$

Sum of the zeroes = $-(\text{coefficient of } x) \div \text{coefficient of } x^2$.

$$\text{Sum of the zeroes} = -\frac{b}{a}$$

$$-2 + 5 = -\frac{b}{a}$$

$$3 = -\frac{b}{a}$$

$$b = -3$$

and

$$a = 1$$

Also,

$$\text{Product of zeroes} = \frac{c}{a}$$

$$(-2) \times (5) = \frac{c}{a}$$

$$-10 = c \quad (\text{as, } a=1)$$

Putting the values of a, b and c in the polynomial $p(x) = ax^2 + bx + c$. We get,

$$x^2 - 3x - 10$$

Hence, we can conclude that x can have any value.

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) $-\frac{c}{a}$

(B) $\frac{c}{a}$

(C) 0

(D) $-\frac{b}{a}$

Solution:

(B) $\frac{c}{a}$

Justification:

We have the polynomial,

$$ax^3 + bx^2 + cx + d$$

Sum of product of roots of a cubic equation is given by $\frac{c}{a}$.

It is given that one root = 0

Now, let the other roots be α, β

$$\alpha\beta + \beta(0) + (0)\alpha = \frac{c}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Hence the product of other two roots is $\frac{c}{a}$

6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1 , then the product of the other two zeroes is

- (A) $b - a + 1$
- (B) $b - a - 1$
- (C) $a - b + 1$
- (D) $a - b - 1$

Solution:

(A) $b - a + 1$

Taking,

$$f(x) = x^3 + ax^2 + bx + c$$

Also,

Zero of $f(x)$ is -1 so $f(-1) = 0$

$$(-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$-1 + a - b + c = 0$$

$$a - b + c = 1$$

$$c = 1 + b - a$$

Now,

$$\alpha \cdot \beta \cdot \gamma = \frac{-d}{a}$$

$$[\because c = b, d = c]$$

$$-1\beta\gamma = \frac{-c}{1}$$

$$\beta\gamma = c$$

$$\beta\gamma = 1 + b - a$$

7. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

- (A) both positive
- (B) both negative
- (C) one positive and one negative
- (D) both equal

Solution:

(b) both negative

Taking,

$$f(x) = x^2 + 99x + 127$$

Now,

$$b^2 - 4ac = (99)^2 - 4(1) 127$$

$$(a = 1, b = 99, c = 127)$$

$$b^2 - 4ac = 9801 - 508$$

$$\sqrt{b^2 - 4ac} = \sqrt{9293}$$

$$\sqrt{b^2 - 4ac} = 96.4$$

Now,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-99 \pm 96.4}{2}$$

Therefore, both roots will be negative as $99 > 96.4$

8. The zeroes of the quadratic polynomial $x^2 + kx + k, k \neq 0$,

- (A) cannot both be positive
- (B) cannot both be negative
- (C) are always unequal
- (D) are always equal

Solution:

(A) cannot both be positive

Taking,

$$f(x) = x^2 + kx + k$$

To find the zeroes of $f(x)$, we take,

$$f(x) = 0$$

$$x^2 + kx + k = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-k \pm \sqrt{k^2 - 4k}}{2}$$

$$x = \frac{-k \pm \sqrt{k(k-4)}}{2}$$

For real roots,

$$b^2 - 4ac > 0$$

$$k(k-4) > 0$$

So, solution $k(k-4) > 0$.

Let,

$k = -4$ be any point on number line,

$$x = \frac{-k \pm \sqrt{k(k-4)}}{2}$$

$$x = \frac{-(-4) \pm \sqrt{-4(-4-4)}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{2}}{2}$$

$$x = 2(-1 \pm \sqrt{2})$$

$$x_1 = 2(-1 + \sqrt{2})$$

$$x_2 = 2(-1 - \sqrt{2})$$

Here one root is positive, and the other root is negative.
So, the roots cannot be both positive.

9. If the zeroes of the quadratic polynomial $ax^2 + bx + c, c \neq 0$ are equal, then

- (A) c and a have opposite signs
- (B) c and b have opposite signs
- (C) c and a have the same sign
- (D) c and b have the same sign

Solution:

(C) c and a have the same sign

For equal roots $b^2 - 4ac = 0$

As, b^2 is always positive so $4ac$ must be positive or we can say product of a and c must be positive i.e., a and c must have same sign either positive or negative.

10. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

- (A) Has no linear term and the constant term is negative.
- (B) Has no linear term and the constant term is positive.
- (C) Can have a linear term but the constant term is negative.
- (D) Can have a linear term but the constant term is positive.

Solution:

(A) Has no linear term and the constant term is negative.

Taking,

$$f(x) = x^2 + ax + b$$

Let, α, β are the roots of it.

Then,

$$\beta = -\alpha$$

(Given)

$$\alpha + \beta = \frac{-b}{a}$$

and

$$\alpha \cdot \beta = \frac{c}{a}$$

Putting $\beta = -\alpha$ in equation $\alpha + \beta = \frac{-b}{a}$.

$$\alpha - \alpha = \frac{-b}{1}$$

$$0 = -b$$

Also,

$$\alpha(-\alpha) = \frac{b}{1}$$

$$-\alpha^2 = b$$

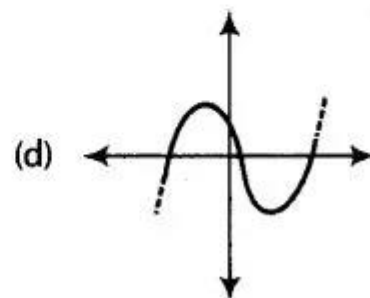
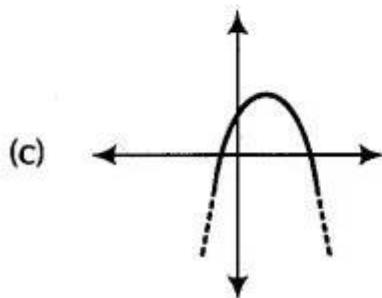
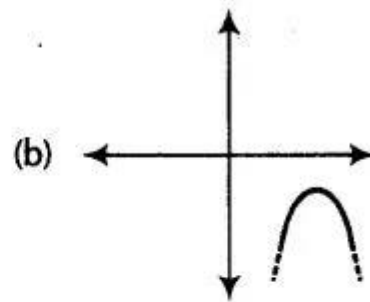
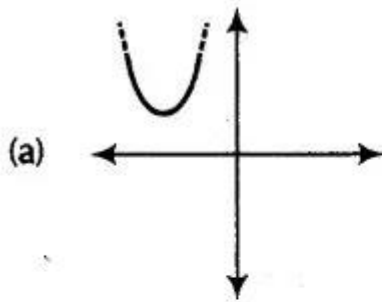
So,

$$a = 0,$$

$b < 0$ or b is negative

Therefore, $f(x) = x^2 + b$ shows that it has no linear term and constant term is negative.

11. Which of the following is not the graph of a quadratic polynomial?



Solution:

(D)

Graph 'D' intersect at three points on X-axis so the roots of polynomial of graph is three, so it is cubic polynomial. Other graph are of quadratic polynomial.

Exercise No: 2.2

Short Answer Questions with Reasoning:

Question 1:

Answer the following and justify:

- i. Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5?
- ii. What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s, p \neq 0$?
- iii. If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?
- iv. If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?
- v. Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

Solution:

(i)

No, $x^2 - 1$ cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5.

Explanation:

When a polynomial with degree 6 is divided by degree 5 polynomial, the quotient will be of degree 1.

Assuming that $(x^2 - 1)$ divides the degree 6 polynomial and the quotient obtained is degree 5 polynomial.

As $a = bq + r$, so,

$$\begin{aligned}(\text{Degree 6 polynomial}) &= (x^2 - 1)(\text{degree 5 polynomial}) + r(x) \\ &= (\text{degree 7 polynomial}) + r(x) \\ &\quad [\text{As, } (x^2 \text{ term} \times x^5 \text{ term} = x^7 \text{ term})] \\ &= (\text{degree 7 polynomial})\end{aligned}$$

So, this contradicts our assumption.

Hence, $x^2 - 1$ cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5.

(ii)

Solution:

Degree of the polynomial $px^3 + qx^2 + rx + s = 3$

Degree of the polynomial $ax^2 + bx + c = 2$

Here, Degree of $px^3 + qx^2 + rx + s$ is greater than degree of the $ax^2 + bx + c$

Hence, the quotient would be zero,

Therefore, the remainder would be the dividend $= ax^2 + bx + c$.

(iii)

Solution:

We have, $p(x) = g(x) \times q(x) + r(x)$

As given in the question,

$q(x) = 0$

When $q(x) = 0$,

$r(x)$ is also $= 0$

Here, when we divide $p(x)$ by $g(x)$,

Then, $p(x)$ should be $= 0$

Therefore, the relation between the degrees of $p(x)$ and $g(x)$ is the degree $p(x) < \text{degree } g(x)$.

(iv)

Solution:

To divide $p(x)$ by $g(x)$

We have,

Degree of $p(x) > \text{degree of } g(x)$

or

Degree of $p(x) = \text{degree of } g(x)$

Hence, the relation between the degrees of $p(x)$ and $g(x)$ is degree of $p(x) > \text{degree of } g(x)$

(v)

Solution:

A Quadratic Equation has equal roots when:

$$b^2 - 4ac = 0$$

Given,

$$x^2 + kx + k = 0$$

$$a = 1,$$

$$b = k,$$

$$c = k$$

Putting values in the equation we get,

$$k^2 - 4(1)(k) = 0$$

$$k^2 - 4k = 0$$

$$k(k - 4) = 0$$

$$k = 0,$$

$$k = 4$$

Here, it is given that k is greater than 1. So, the value of k is 4 if the equation has common roots.

If $k = 4$, then the equation $(x^2 + kx + k)$ will have equal roots.

Question 2:

Are the following statements 'True' or 'False'? Justify your answers.

- i. If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a , b and c all have the same sign.
- ii. If the graph of a polynomial intersects the x -axis at only one point, it cannot be a quadratic polynomial.
- iii. If the graph of a polynomial intersects the x -axis at exactly two points, it need not be a quadratic polynomial.
- iv. If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.
- v. If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.
- vi. If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a , b and c is non-negative.
- vii. The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeros is $\frac{1}{2}$.

Solution:

(i)

False

Taking α and β as the roots of the quadratic polynomial. If α and β are positive then

$$\alpha + \beta = \frac{-b}{a}. \text{ For making sum of roots positive either } b \text{ or } a \text{ must be negative.}$$

(ii)

False

The statement is false, because when two zeroes of a quadratic polynomial are equal, then two intersecting points coincide to become one point.

(iii)

True

If a polynomial of degree more than two has two real zeroes and other zeroes are not real or are imaginary, then graph of the polynomial will intersect at two points on x-axis.

(iv)

True

Taking,

$$\beta = 0,$$

$$\gamma = 0$$

$$f(x) = (x - \alpha)(x - \beta)(x - \gamma)$$

$$= (x - \alpha)x \cdot x$$

$$f(x) = x^3 - \alpha x^2$$

So, it has no linear (coefficient of x) and constant terms.

(v)

True

α , β and γ are all negative for cubic polynomial $ax^3 + bx^2 + cx + d$.

$$\alpha + \beta + \gamma = \frac{-b}{a} \quad \dots (i)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad \dots (ii)$$

$$\alpha\beta\gamma = \frac{-d}{a} \quad \dots (iii)$$

As α , β , γ are all negative so,

$$\alpha + \beta + \gamma = -x$$

(Any negative number)

$$\frac{-b}{a} = -x$$

[From (i)]

$$\frac{-b}{a} = x$$

As, a, b have same sign and product of any two zeroes will be positive.

$$\text{So, } \alpha\beta + \beta\gamma + \gamma\alpha = +y$$

(Any positive number)

$$\frac{c}{a} = +y$$

[From (ii)]

c and a have same sign.

$$\alpha\beta\gamma = -z$$

(Any negative number)

$$\frac{-d}{a} = -z$$

[From (iii)]

$$\frac{-d}{a} = z$$

Here, d and a will have same sign.

So, sign of b, c, d are same as of a.

Signs of a, b, c, d will be same either positive or negative.

(vi)

False: As all zeroes of cubic polynomial are positive

Let $f(x) = x^3 + ax^2 - bx + c$

$\alpha + \beta + \gamma = \text{positive,}$
(say $+x$)

$$\frac{-b}{a} = x$$

a and b has opposite signs ... (i)

$\alpha\beta + \beta\gamma + \gamma\alpha = +y$

$$\frac{c}{a} = y$$

So, signs of a and c are same. ... (ii)

Now, $\alpha\beta\gamma = \text{positive} = +z$

$$\frac{-d}{a} = z$$

a and d have opposite signs.

Therefore, we can conclude that,

From (i) if a is positive, then b is negative.

From (ii) if a is positive, then c is also positive.

From (iii) if a is positive, then d is negative.

So, if zeroes α, β, γ of cubic polynomial are positive then out of a, b, c at least one is negative.

(vii)

False.

$$f(x) = kx^2 + x + k$$

$$a = k,$$

$$b = 1,$$

$$c = k$$

Condition of equal roots,

$$b^2 - 4ac = 0$$

$$(1)^2 - 4(k)(k) = 0$$

$$4k^2 = 1$$

$$k^2 = 1/4$$

$$k = \pm \frac{1}{2}$$

So, the values of k are $\pm \frac{1}{2}$ so that the given equation has equals roots.

Exercise 2.3

Short Answer Questions:

Find the zeroes of the following polynomials by factorization method.

1. $4x^2 - 3x - 1$
2. $3x^2 + 4x - 4$
3. $5t^2 + 12t + 7$
4. $t^3 - 2t^2 - 15t$
5. $2x^2 + \frac{7}{2}x + \frac{3}{4}$
6. $4x^2 + 5\sqrt{2}x - 3$
7. $2s^3 - (1 + 2\sqrt{2})s + \sqrt{2}$
8. $v^2 + 4\sqrt{3}v - 15$
9. $y^2 + \frac{3}{2}\sqrt{5}y - 5$
10. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Solution:

1.

$$4x^2 - 3x - 1$$

By splitting the middle term,
 $4x^2 - 4x + 1x - 1$

Now, taking out the common factors,
 $4x(x-1) + 1(x-1)$
 $(4x+1)(x-1)$

The zeroes are,

$$4x+1=0$$

$$4x = -1$$

$$x = \frac{-1}{4}$$

Also,

$$(x-1) = 0$$

$$x = 1$$

Therefore, zeroes are $\frac{-1}{4}$ and 1 .

2. $3x^2 + 4x - 4$

Solution:

$$3x^2 + 4x - 4$$

By splitting the middle term, we get,

$$3x^2 + 6x - 2x - 4$$

$$3x(x+2) - 2(x+2)$$

$$(x+2)(3x-2)$$

Either,

$$x+2 = 0$$

$$x = -2$$

$$3x-2=0$$

$$3x = 2$$

$$x = \frac{2}{3}$$

Therefore, zeroes are $\frac{2}{3}$ and -2.

3. $5t^2 + 12t + 7$

Solution:

$$5t^2 + 12t + 7$$

By splitting the middle term, we get,

$$5t^2 + 5t + 7t + 7$$

$$5t(t+1) + 7(t+1)$$

$$(t+1)(5t+7)$$

So, the zeroes are,

$$t+1 = 0$$

$$y = -1$$

$$5t+7=0$$

$$5t = -7$$

$$t = -\frac{7}{5}$$

So, the zeroes are $\frac{7}{5}$ and -1

4. $t^3 - 2t^2 - 15t$

Solution:

$$t^3 - 2t^2 - 15t$$

$$t(t^2 - 2t - 15)$$

Splitting the middle term of the equation $t^2 - 2t - 15$, we get,

$$t(t^2 - 5t + 3t - 15)$$

$$t(t(t-5) + 3(t-5))$$

$$t(t+3)(t-5)$$

The zeroes are,

$$t = 0$$

$$t+3=0$$

$$t = -3$$

$$t - 5 = 0$$

$$t = 5$$

So, zeroes are 0, 5 and -3.

$$5. \quad 2x^2 + \frac{7}{2}x + \frac{3}{4}$$

Solution:

$$2x^2 + \frac{7}{2}x + \frac{3}{4}$$

We can write this equation as,

$$8x^2 + 14x + 3$$

Now, splitting the middle term, we get,

$$8x^2 + 12x + 2x + 3$$

$$4x(2x+3) + 1(2x+3)$$

$$(4x+1)(2x+3)$$

The zeroes are,

$$4x+1=0$$

$$x = \frac{-1}{4}$$

$$2x+3=0$$

$$x = \frac{-3}{2}$$

Therefore, zeroes are $\frac{-1}{4}$ and $\frac{-3}{2}$.

$$6. \quad 4x^2 + 5\sqrt{2}x - 3$$

Solution:

By splitting middle term, we get,

$$4x^2 + 5\sqrt{2}x - 3$$

$$4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3$$

$$2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3)$$

$$(2\sqrt{2}x - 1)(\sqrt{2}x + 3)$$

Therefore,

$$x = \frac{1}{2\sqrt{2}}$$

or,

$$x = \frac{-3}{\sqrt{2}}$$

$$7. 2s^3 - (1 + 2\sqrt{2})s + \sqrt{2}$$

Solution:

By splitting middle term, we get,

$$2s^3 - (1 + 2\sqrt{2})s + \sqrt{2} = 0$$

$$2s^3 - 1s - 2\sqrt{2}s + \sqrt{2} = 0$$

$$s(2s - 1) - \sqrt{2}(2s - 1) = 0$$

$$(2s - 1)(s - \sqrt{2}) = 0$$

$$s = \frac{1}{2}$$

$$s = \sqrt{2}$$

8.

Solution:

By splitting middle term, we get,

$$v^2 + 4\sqrt{3}v - 15$$

$$v^2 + 5\sqrt{3}v - \sqrt{3}v - 15$$

$$v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3})$$

$$(v + 5\sqrt{3})(v - \sqrt{3})$$

$$v = -5\sqrt{3} \text{ or } v = \sqrt{3}$$

9. $y^2 + \frac{3}{2}\sqrt{5}y - 5$

Solution:

By splitting middle term, we get,

$$y^2 + \frac{3}{2}\sqrt{5}y - 5 = 0$$

$$2y^2 + 3\sqrt{5}y - 10 = 0$$

$$2y^2 + 4\sqrt{5}y - 1\sqrt{5} - 10 = 0$$

$$2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5}) = 0$$

$$(y + 2\sqrt{5})(2y - \sqrt{5}) = 0$$

$$y = -2\sqrt{5} \text{ or } y = \frac{\sqrt{5}}{2}$$

10. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Solution:

By splitting middle term, we get,

$$7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$

We can write this equation as,

$$21y^2 - 11y - 2 = 0$$

$$21y^2 - 14y + 3y - 2 = 0$$

$$7y(3y - 2) + 1(3y - 2) = 0$$

$$(7y + 1)(3y - 2) = 0$$

$$y = \frac{-1}{7} \text{ or } y = \frac{2}{3}$$

Exercise 2.4

Long Answer Questions:

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorization.

i. $\frac{-8}{3}, \frac{4}{3}$

ii. $\frac{21}{8}, \frac{5}{16}$

iii. $-2\sqrt{3}, -9$

iv. $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

Solution:

(i)

$$\text{Sum of the zeroes} = \frac{-8}{3}$$

$$\text{Product of the zeroes} = \frac{4}{3}$$

$$P(x) = x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

So,

$$P(x) = x^2 - x \left(\frac{-8}{3} \right) + \frac{4}{3}$$

$$P(x) = 3x^2 + 8x + 4$$

Splitting the middle term, we get,

$$3x^2 - 8x + 4 = 0$$

$$3x^2 + 6x + 2x + 4 = 0$$

$$3x(x + 2) + 2(x + 2) = 0$$

$$(x + 2)(3x + 2) = 0$$

$$x + 2 = 0$$

Or

$$3x + 2 = 0$$

$$x = -2, \frac{-2}{3}$$

(ii)

$$\text{Sum of the zeroes} = \frac{21}{8}$$

$$\text{Product of the zeroes} = \frac{5}{16}$$

$$P(x) = x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$P(x) = x^2 - x\left(\frac{21}{8}\right) + \frac{5}{16}$$

$$P(x) = 16x^2 - 42x + 5$$

Splitting the middle term,

$$16x^2 - 42x + 5 = 0$$

$$16x^2 - (2x + 40x) + 5 = 0$$

$$16x^2 - 2x - 40x + 5 = 0$$

$$2x(8x - 1) - 5(8x - 1) = 0$$

$$(8x - 1)(2x - 5) = 0$$

$$x = \frac{1}{8}, \frac{5}{2}$$

(iii)

$$\text{Sum of the zeroes} = -2\sqrt{3}$$

$$\text{Product of the zeroes} = -9$$

$$P(x) = x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$P(x) = x^2 - 2\sqrt{3}x - 9$$

Splitting the middle term,

$$x^2 - 2\sqrt{3}x - 9 = 0$$

$$x^2 - (-\sqrt{3}x + 3\sqrt{3}x) - 9 = 0$$

$$x^2 + \sqrt{3}x - 3\sqrt{3}x - 9 = 0$$

$$x(x + \sqrt{3}) - 3\sqrt{3}(x + \sqrt{3}) = 0$$

$$(x + \sqrt{3})(x - 3\sqrt{3}) = 0$$

$$x = -\sqrt{3}, 3\sqrt{3}$$

(iv)

$$\text{Sum of the zeroes} = \frac{-3}{2\sqrt{5}}$$

$$\text{Product of the zeroes} = -\frac{1}{2}$$

$$P(x) = x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$$

$$P(x) = x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x - \frac{1}{2}$$

$$P(x) = 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

Splitting the middle term,

$$2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$$

$$2\sqrt{5}x^2 + (5x - 2x) - \sqrt{5} = 0$$

$$2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$$

$$\sqrt{5}x(2x + \sqrt{5}) - (2x + \sqrt{5}) = 0$$

$$(2x + \sqrt{5})(\sqrt{5}x - 1) = 0$$

$$x = \frac{-\sqrt{5}}{2} \text{ and } x = \frac{1}{\sqrt{5}}$$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a, a + b, a + 2b$ for some real numbers a and b , find the values of a and b as well as the zeroes of the given polynomial.

Solution:

We have, $a, a+b, a+2b$ are roots of given polynomial $x^3 - 6x^2 + 3x + 10$.

Sum of the roots = $a + 2b + a + a + b$

$$= - \frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$$

$$3a + 3b = -\left(\frac{-6}{1}\right)$$

$$= 6$$

$$3(a+b) = 6$$

$$a+b = 2$$

$$b = 2 - a$$

....(i)

Product of roots = $(a+2b)(a+b)a$

$$= - \frac{\text{constant}}{\text{coefficient of } x^3}$$

$$(a+b+b)(a+b)a = - \frac{10}{1}$$

Putting the value of $a + b = 2$, we get,

$$(2+b)(2) a = -10$$

$$(2+b) 2a = -10$$

$$(2+2-a) 2a = -10$$

$$(4-a)2a = -10$$

$$4a-a^2 = -5$$

$$a^2-4a-5 = 0$$

$$a^2-5a+a-5 = 0$$

$$(a-5)(a+1) = 0$$

$$a-5 = 0 \quad \text{or} \quad a+1 = 0$$

$$a = 5 \text{ and } a = -1$$

Putting values of a in equation (i),

When $a = 5$,

$$5+b = 2$$

$$b = -3$$

When $a = -1$,

$$-1+b = 2$$

$$b = 3$$

3. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

Solution:

According to question, $\sqrt{2}$ is one of the zero of the cubic polynomial.

So,

$(x - \sqrt{2})$ is one of the factor of the given polynomial,

$$p(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$$

So, by dividing $p(x)$ by $x - \sqrt{2}$

$$\begin{array}{r}
 6x^2 + 7\sqrt{2}x + 4 \\
 x - \sqrt{2} \overline{) 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}} \\
 \underline{6x^3 - 6\sqrt{2}x^2} \phantom{- 10x - 4\sqrt{2}} \\
 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 \underline{7\sqrt{2}x^2 - 14x} \phantom{- 4\sqrt{2}} \\
 -4x - 4\sqrt{2} \\
 \underline{-4x + 4\sqrt{2}} \\
 0
 \end{array}$$

Therefore,

$$6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} = (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4)$$

Splitting the middle term, we have,

$$\begin{aligned}
 &(x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4) \\
 &(x - \sqrt{2})[2x(3x + 2\sqrt{2}) + \sqrt{2}(3x + 2\sqrt{2})] \\
 &(x - \sqrt{2})(2x + \sqrt{2})(3x + 2\sqrt{2})
 \end{aligned}$$

For finding the zeroes of $p(x)$,

Take $p(x) = 0$

$$(x - \sqrt{2})(2x + \sqrt{2})(3x + 2\sqrt{2}) = 0$$

$$x = \sqrt{2},$$

$$x = \frac{-\sqrt{2}}{2},$$

$$x = \frac{-2\sqrt{2}}{3}$$

On simplifying,

$$x = \sqrt{2},$$

$$x = \frac{-1}{\sqrt{2}},$$

$$x = \frac{-2\sqrt{2}}{3}$$

So, the other two zeroes of $p(x)$ are $\frac{-1}{\sqrt{2}}$ and $\frac{-2\sqrt{2}}{3}$.

4. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

Solution:

By factor theorem and Euclid's division algorithm, we get
 $f(x) = g(x) \times q(x) + r(x)$

Taking,

$$f(x) = 2x^4 + x^3 - 14x^2 + 5x + 6 \quad \dots(i)$$

$$\text{and } g(x) = x^2 + 2x + k$$

$$\begin{array}{r}
 x^2 + 2x + k \overline{) 2x^4 + x^3 - 14x^2 + 5x + 6} \quad \quad \quad x^2 - 3x - 8 - 2k \\
 \underline{2x^4 + 4x^3 - 2kx^2} \\
 -3x^3 - 14x^2 - 2kx^2 + 5x + 6 \\
 \underline{-3x^3 - 6x^2 - 3kx} \\
 -8x^2 - 2kx^2 + 5x + 3kx + 6 \\
 \underline{-8x^2 - 16x - 8k} \\
 -2kx^2 + 21x + 3kx + 8k + 6 \\
 \underline{-2kx^2 - 4kx - 2k^2} \\
 21x + 7kx + 2k^2 + 8k + 6
 \end{array}$$

$$\text{But, } r(x) = 0$$

$$(21 + 7k)x + 2k^2 + 8k + 6 = 0x + 0$$

So,

$$21 + 7k = 0$$

$$k = \frac{-21}{7}$$

And

$$2k^2 + 8k + 6 = 0$$

Splitting the middle term, we get,

$$2k^2 + 6k + 2k + 6 = 0$$

$$2k(k + 3) + 2(k + 3) = 0$$

$$(k + 3)(2k + 2) = 0$$

$$k + 3 = 0 \text{ or } 2k + 2 = 0$$

Therefore, $k = -3, -1$.

Common solution is $k = -3$.

$$q(x) = 2x^2 - 3x - 8 - 2(-3)$$

$$= 2x^2 - 3x - 8 + 6$$

$$q(x) = 2x^2 - 3x - 2$$

$$f(x) = g(x)q(x) + 0$$

$$= (x^2 + 2x - 3)(2x^2 - 3x - 2)$$

$$= (2x^2 - 4x + 1x - 2)(x^2 + 3x - 1x - 3)$$

$$= [2x(x - 2) + 1(x - 2)][x(x + 3) - 1(x + 3)]$$

$$f(x) = (x - 2)(2x + 1)(x + 3)(x - 1)$$

For zeroes of $f(x)$, $f(x) = 0$

$$(x - 1)(x - 2)(x + 3)(2x + 1) = 0$$

$$(x - 1) = 0,$$

$$(x - 2) = 0,$$

$$(x + 3) = 0$$

and

$$2x + 1 = 0$$

$$x = 1,$$

$$x = 2,$$

$$x = -3 \text{ and } x = -\frac{1}{2}$$

Hence, zeroes of $f(x)$ are 1, 2, -3 and $-\frac{1}{2}$. And, the zeros of $x^2 + 2x - 3$ is 1, -3.

5. Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

Solution:

Let $f(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$ and $g(x) = x - \sqrt{5}$.
 $g(x)$ is a factor of $f(x)$ so $f(x) = q(x) \cdot x - \sqrt{5}$.

On dividing $f(x)$ by $g(x)$ we get,

$$\begin{array}{r}
 x^2 - 2\sqrt{5}x + 3 \\
 x - \sqrt{5} \overline{) x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}} \\
 \underline{x - \sqrt{5}x^2} \phantom{+ 13x - 3\sqrt{5}} \\
 - + \phantom{+ 13x - 3\sqrt{5}} \\
 \underline{- 2\sqrt{5}x^2 + 13x - 3\sqrt{5}} \\
 - 2\sqrt{5}x^2 + 10x \phantom{- 3\sqrt{5}} \\
 \phantom{- 2\sqrt{5}x^2} + - \phantom{- 3\sqrt{5}} \\
 \hline
 \phantom{- 2\sqrt{5}x^2} 3x - 3\sqrt{5} \\
 \phantom{- 2\sqrt{5}x^2} 3x - 3\sqrt{5} \\
 \phantom{- 2\sqrt{5}x^2} - + \phantom{- 3\sqrt{5}} \\
 \hline
 \phantom{- 2\sqrt{5}x^2} 0
 \end{array}$$

$$f(x) = q(x) \cdot g(x)$$

$$f(x) = (x^2 - 2\sqrt{5}x + 3)(x - \sqrt{5})$$

$$f(x) = (x - \sqrt{5} - \sqrt{2})(x - \sqrt{5} + \sqrt{2})(x - \sqrt{5})$$

So,

$$f(x) = 0$$

Therefore,

$$(x - \sqrt{5}) = 0$$

$$(x - \sqrt{5} - \sqrt{2}) = 0$$

$$(x - \sqrt{5} + \sqrt{2}) = 0$$

Therefore, zeroes of polynomial are:

$$x = \sqrt{5}$$

$$x = (\sqrt{5} + \sqrt{2})$$

$$x = \sqrt{5} - \sqrt{2}$$

6. For which values of a and b , are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?

Solution:

We have, factor theorem and Euclid's division lemma to solve this question,

Using factor theorem if $q(x)$ is a factor of $p(x)$, then $r(x)$ must be zero.

$$p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$$

$$q(x) = x^3 + 2x^2 + a$$

$$\begin{array}{r}
 x^3 + 2x^2 + a \overline{) x^5 - x^4 - 4x^3 + 3x^2 + 3x + b} \\
 \underline{x^5 + 2x^4 + ax^2} \\
 -3x^4 - 4x^3 - ax^2 + 3x^2 + 3x + b \\
 \underline{-3x^4 - 6x^3 - 3ax} \\
 +2x^3 - ax^2 + 3x^2 + 3x + 3ax + b \\
 \underline{-2x^3 + 4x^2 + 2a} \\
 +ax^2 - x^2 + 3x + 3ax - 2a + b
 \end{array}$$

So, by factor theorem remainder must be zero so,

$$r(x) = 0$$

$$-(a+1)x^2 + (3a+3)x + (b-2a) = 0x^2 + 0x + 0$$

Comparing the coefficients of x^2 , x and constant on both sides, we get,

$$-(a+1) = 0, 3a+3 = 0 \text{ and } b-2a = 0.$$

So,

$$a = -1 \text{ and}$$

$$b - 2(-1) = 0$$

$$b = -2$$

For $a = -1$ and $b = -2$, zeroes of $q(x)$ will be zeroes of $p(x)$.

For zeroes of $p(x)$,

$$p(x) = 0$$

$$(x^3 + 2x^2 + a)(x^2 - 3x + 2) = 0$$

$$[\because a = -1]$$

$$[x^3 + 2x^2 - 1][x^2 - 2x - 1x + 2] = 0$$

$$(x^3 + 2x^2 - 1)[x(x-2) - 1(x-2)] = 0$$

$$(x^3 + 2x^2 - 1)(x-2)(x-1) = 0$$

So, $x = 2$ and 1 are not the zeroes of $q(x)$.