2 Annuities

2.1 ANNUITY

An **annuity** is a sequence of equal payments made at equal intervals of time. Examples of annuities are weekly wages, monthly home mortgage payments, payments to a recurring deposit, quarterly stock dividend, annual premiums on a life insurance policy etc.

The time period between successive payments is called **payment period** or **payment interval**. It may be weekly, monthly, quarterly, annually etc. or any fixed period of time. The time from the beginning of first interval to the end of the last interval is called the **term** or **duration** of the annuity. The size of each payment of an annuity is called the **periodic payment** of the annuity. The person who receives the payment is called **annuitant**. The sum of all payments made in a year is called *annual rent.* Thus, if ₹ 2000 are being paid every quarterly, the annual rent is ₹ 8000.

The **amount** or **future value** of an annuity is the total amount due at the end of the term of the annuity. It is principal plus interest. Thus, the amount is the sum total of each instalment kept on compound interest till the end of the term. Thus, if ₹ 100 are deposited every month for seven years, and at the end of seven years, the depositor gets ₹ 10000, then the amount of the annuity is ₹ 10000, periodic payment is ₹ 100, term or duration is 7 years, and the payment period or payment interval is one month.

Present value of an annuity is the current value of a sequence of equal periodic payments made over a certain period of time. Thus, if you borrow a car loan of ₹ 2 lacs, and return it in 24 equal monthly instalments of ₹ 10000 each, over a period of 2 years, then the present value of this annuity is ₹ 2 lacs.

Annuities are of three types :

(*i*) In *annuities certain,* the number of payments is fixed *i.e.* the payments begin and end on fixed dates. An example is that you borrow some loan from a bank and repay it, say, in 10 equal annual instalments, the first intalment being paid 1 year after date of borrowing.

(*ii*) A *contingent annuity* is one where the term depends upon some event whose occurrence is not fixed. An example is periodic payments of life insurance premiums which stop when the person dies.

(*iii*) A *perpetual annuity* or *perpetuity* is an annuity whose term does not end *i.e.* it extends till infinity. Thus there is no last payment; they go on forever. An example is freehold property, where you can earn rent in perpetuity.

Annuities are further classified into three categories by payment dates :

(*i*) An *ordinary annuity* or *immediate annuity* is where payments are made at the *end* of each payment period *i.e.* first payment is made at the end of the first payment interval, and so on. Examples are repayment of car loan, house mortgage etc.

(*ii*) An *annuity due* is where payments are made at the *beginning* of each period. Examples are life insurance premium payments, recurring deposit payments, house rent etc.

(*iii*) If the payments start after a specified number of periods, we get a *deferred annuity*. Typical examples are pension plans floated by various insurance companies. When an annuity is left unpaid for a number of years (or payment intervals), it is said to *unpaid* or *fore-borne* for that amount of period; the total amount for these intervals, alongwith interest, is called *amount of deferred annuity*.

2.2 AMOUNT AND PRESENT VALUE OF ORDINARY ANNUITIES

When we speak of annuities certain, we usually speak of ordinary (or immediate) annuity, where payment is at the end of each payment interval. The following notations will be used throughout this chapter :

A = amount of each instalment

V = present value of annuity

M = (future) amount of annuity

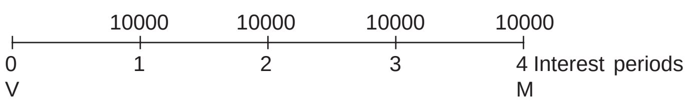
r = rate of interest (on *one* rupee per payment interval)

n = number of instalments (in case of annuity certain)

Note that if payments are semi-annual and rate of interest is 6% p.a., then $r = 0{\cdot}03$.

It is always useful to draw a line diagram to solve problems related to annuity.

Consider an ordinary annuity of ₹ 10000 per year for four years with money worth 5%. (Consider a situation where you buy a computer and pay four annual instalments of ₹ 10000 each).



Here A = ₹ 10000, $r = 0{\cdot}05$, $n = 4$.

To find present value, V, of all 4 instalments together, let us consider present value of each of them separately. Amount of ₹ 10000 paid one year hence is equivalent to ₹ $\frac{10000}{1 + 0{\cdot}05}$ paid now; amount of ₹ 10000 paid two years hence is equivalent to ₹ $\frac{10000}{(1 + 0{\cdot}05)^2}$ paid now, and so on.

$$\therefore \quad V = ₹\,\frac{10000}{1{\cdot}05} + ₹\,\frac{10000}{(1{\cdot}05)^2} + ₹\,\frac{10000}{(1{\cdot}05)^3} + ₹\,\frac{10000}{(1{\cdot}05)^4}$$

$$= ₹\,\frac{10000}{1{\cdot}05}\left[1 + \frac{1}{1{\cdot}05} + \frac{1}{(1{\cdot}05)^2} + \frac{1}{(1{\cdot}05)^3}\right]$$

$$= ₹\,\frac{10000}{1{\cdot}05}\left[\frac{1 - \left(\frac{1}{1{\cdot}05}\right)^4}{1 - \frac{1}{1{\cdot}05}}\right]$$

$$= ₹\,\frac{10000}{0{\cdot}05}\,[1 - 0{\cdot}8227] = ₹\,35460$$

To find the future amount M, we note that ₹ 10000 paid at the end of first year is equivalent to paying ₹ 10000 $(1{\cdot}05)^3$ at the end of fourth year, and so on.

$$\therefore \quad M = ₹\,10000\,(1{\cdot}05)^3 + ₹\,10000\,(1{\cdot}05)^2 + ₹\,10000\,(1{\cdot}05) + ₹\,10000$$

$$= ₹\ 10000\ [1 + 1{\cdot}05 + (1{\cdot}05)^2 + (1{\cdot}05)^3] = ₹\ 10000 \,.\, \frac{(1{\cdot}05)^4 - 1}{1{\cdot}05 - 1}$$

$$= ₹\ \frac{10000}{0{\cdot}05}\ [1{\cdot}2155 - 1] = ₹\ 43100$$

Thus, these four instalments of ₹ 10000 each are equivalent to ₹ 35460 paid now or to ₹ 43100 paid four years later. We can cross check : ₹ 35460 paid now would amount to ₹ 35460 $(1{\cdot}05)^4$ = ₹ 43100 approximately.

Now let us derive a generalised formula.

$$\text{Present value, } V = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \ldots + \frac{A}{(1+r)^n}$$

$$= \frac{A}{(1+r)^n}\left[(1+r)^{n-1} + (1+r)^{n-2} + \ldots + 1\right]$$

$$= \frac{A}{(1+r)^n}\left[1 + (1+r) + \ldots + (1+r)^{n-1}\right]$$

$$= \frac{A}{(1+r)^n}\left[\frac{(1+r)^n - 1}{(1+r) - 1}\right]$$

$$= \frac{A}{r}\left[1 - \left(\frac{1}{1+r}\right)^n\right] = \frac{A}{r}[1 - (1+r)^{-n}]$$

$$\boxed{V = \frac{A}{r}[1 - (1+r)^{-n}]}$$

Note that r is interest *per payment period* and n is number of *interest periods*; if payment is not annual, values of r and n should be stated correctly.

Similarly, future amount M of annuity is

$$M = A(1 + r)^{n-1} + A(1 + r)^{n-2} + \ldots + A(1 + r) + A$$

$$= A[1 + (1 + r) + \ldots + (1 + r)^{n-1}]$$

$$= A\left[\frac{(1+r)^n - 1}{(1+r) - 1}\right] = \frac{A}{r}[(1+r)^n - 1]$$

$$\boxed{M = \frac{A}{r}[(1+r)^n - 1]}$$

Observe that $M = V(1 + r)^n$, which makes sense.

2.3 AMOUNT OF ANNUITY LEFT UNPAID

If an annuity is left unpaid for n interest periods, then the equivalent amount which may be paid at the end of nth interest period is nothing but the amount of ordinary annuity certain.

$$\therefore \qquad M = \frac{A}{r}\ [(1 + r)^n - 1]$$

Unpaid annuity is also called **foreborne** annuity for that period.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the present value and amount of an ordinary annuity of 8 quarterly payments of ₹ 500 each, the rate of interest being 8% per annum compounded quarterly.*

Solution. Here periodic instalment, A = ₹ 500,

number of periods $n = 8$,

and rate of interest $r = 8\%$ p.a. = 0·02 (per quarter)

$\therefore$ Present value, $V = \frac{A}{r}[1-(1+r)^{-n}]$ = ₹ $\frac{500}{0\cdot02}[1-(1\cdot02)^{-8}]$

Now let $x = (1\cdot02)^{-8} \Rightarrow \log x = -8 \log 1\cdot02 = -8(0\cdot0086)$

$\Rightarrow \log x = -0\cdot0688 = \bar{1}\cdot9312 \Rightarrow x = 0\cdot8535$

$\therefore \quad V = \frac{500}{0\cdot02}[1 - 0\cdot8535] = 500 \times 50 \times 0\cdot1465$ = ₹ 3662·50

Now, amount of annuity, $M = \frac{A}{r}[(1+r)^n - 1]$ = ₹ $\frac{500}{0\cdot02}[(1\cdot02)^8 - 1]$

Let $x = (1\cdot02)^8 \Rightarrow \log x = 8 \log 1\cdot02 = 8(0\cdot0086) = 0\cdot0688$

$\Rightarrow \quad x = 1\cdot171$

$\therefore \quad M = \frac{500}{0\cdot02}[1\cdot171 - 1] = 500 \times 50 \times 0\cdot171$ = ₹ 4275

Thus, the present value of annuity is ₹ 3662·50 and amount is ₹ 4275.

Note. These days students are allowed to use calculators in council examination, but not in most of the competitive exams. Answers in annuities can be different if you use log tables or calculators. In the examples and exercises here, we will use calculator. In the example above, answers using calculator will be V = ₹ 3662·74, M = ₹ 4291·48.

Example 2. *Find the amount of an annuity of ₹ 2000 payable at the end of every month for 5 years if money is worth 6% per annum compounded monthly.* (*I.S.C. 2004*)

Solution. Here, monthly instalment = A = ₹ 2000,

number of periods = $n = 5 \times 12 = 60$

and rate of interst r = 6% p.a. = $\frac{6}{12}\%$ per month

$= \frac{6}{12} \times \frac{1}{100}$ per month per rupee

= 0·005 per period per rupee.

Amount of annuity = $M = \frac{A}{r}[(1+r)^n - 1]$

= ₹ $\frac{2000}{0\cdot005}[(1+0\cdot005)^{60} - 1]$

= ₹ $\frac{2000}{0\cdot005}[(1\cdot005)^{60} - 1]$

= ₹ 139540·06.

Example 3. *What amount should be set aside at the end of each year to amount to ₹ 10 lacs at the end of 15 years at 6% per annum compound interest ?*

Solution. Let A be the amount set aside at the end of each year.

Here number of periods, $n = 15$, rate of interest per period = 0·06,

amount of annuity, M = ₹ 1000000.

Using $M = \frac{A}{r}[(1+r)^n - 1]$, we get

₹ $1000000 = \frac{A}{0\cdot06}[(1\cdot06)^{15} - 1]$

$$\Rightarrow \quad A = ₹ \frac{60000}{(1{\cdot}06)^{15} - 1} = ₹\ 42962{\cdot}76$$

$\therefore$ A = ₹ 42960 approx.

Example 4. *A man borrowed some money and returned it in 3 equal quarterly instalments of ₹ 4630·50 each. What sum did he borrow if the rate of interest was 20% per annum compounded quarterly ? Find also the interest charged.*

Solution. We have to find present value of an ordinary annuity certain. Here, periodic instalment A = ₹ 4630·50, number of periods, $n = 3$, rate of interest = 20% p.a. *i.e.* 5% per quarter *i.e.* 0·05 per period

$$\therefore \text{ Present value } V = \frac{A}{r}[1-(1+r)^{-n}] = ₹ \frac{4630{\cdot}50}{0{\cdot}05}[1-(1{\cdot}05)^{-3}]$$

$$= ₹\ 12610$$

Thus, the sum borrowed was ₹ 12610

Now, total money repaid = ₹ (3 × 4630·50) = ₹ 13891·50

$\therefore$ Interest paid = ₹ 13891·50 – ₹ 12610 = ₹ 1281·50

Example 5. *An iPod is purchased on instalment basis, such that ₹ 8000 is to be paid on the signing of the contract and four yearly instalments of ₹ 3000 each, payable at the end of the first, second, third and fourth years. If compound interest is charged at 5% per annum, what would be the cash price of the iPod ? (Take 1·05)$^{-4}$ = 0·8227).* (I.S.C. 2009)

Solution. Here, cash down payment = ₹ 8000,

yearly instalment = ₹ 3000,

number of periods = n = 4 and

rate of interest = 5% p.a. = 0·05 per period per rupee.

$$\text{Present value of annuity} = V = \frac{A}{r}[1 - (1 + r)^{-n}]$$

$$= ₹ \frac{3000}{0{\cdot}05}[1 - (1 + 0{\cdot}05)^{-4}]$$

$$= ₹ \frac{3000}{0{\cdot}05}[1 - (1{\cdot}05)^{-4}]$$

$$= ₹ \frac{3000}{0{\cdot}05}(1 - 0{\cdot}8227)$$

$$= ₹ \frac{3000 \times 0{\cdot}1773}{0{\cdot}05} = ₹\ 10638.$$

$\therefore$ Cash price of the iPod = ₹ 8000 + ₹ 10638

= ₹ 18638.

Example 6. *Mr. Aggarwal buys a house at ₹ 3000000 for which he agrees to make payments at the end of each year for 10 years. If the money is worth 10% p.a., find the amount of each instalment. [Take (1·1)$^{-10}$ = 0·3855]* (I.S.C. 2008)

Solution. Here, present value V = ₹ 3000000, n = 10, r = 10% = 0·1

We wish to find the amount of each instalment, A.

Using $V = \frac{A}{r}[1 - (1 + r)^{-n}]$, we get

$$₹\ 3000000 = \frac{A}{0{\cdot}1}[1 - (1{\cdot}1)^{-10}]$$

$$\Rightarrow \quad A = ₹ \frac{3000000 \times 0{\cdot}1}{1-(1{\cdot}1)^{-10}} = ₹ \frac{300000}{1-0{\cdot}3855}$$

$$= ₹ \frac{300000}{0{\cdot}6145} = ₹\ 488201{\cdot}79.$$

Hence the value of each instalment is ₹ 488202 approximately.

Example 7. *Amit borrows ₹ 60000 at 6% effective and promises to repay the loan in 20 equal instalments beginning at the end of first year. Find the value of each instalment.*

Solution. Here, present value V = ₹ 60000, $n = 20$,

$r = 6\% = 0{\cdot}06.$

We wish to find the amount of each instalment, A.

Using $V = \frac{A}{r}[1 - (1 + r)^{-n}]$, we get

$$₹\ 60000 = \frac{A}{0{\cdot}06}[1 - (1{\cdot}06)^{-20}]$$

$$\Rightarrow \quad A = ₹\ \frac{60000 \times 0{\cdot}06}{1 - (1{\cdot}06)^{-20}} = ₹\ 5231{\cdot}07$$

Hence, the value of each instalment is ₹ 5231 approximately.

Example 8. *A person takes a loan on compound interest and returns it in 2 equal annual instalments. If the rate of interest is 16%. p.a. and the yearly instalment is ₹ 1682, find the principal and the interest charged with each instalment.*

Solution. Here, A = ₹ 1682, $r = 0{\cdot}16$, $n = 2$

So, principal $V = \frac{A}{r}[1 - (1 + r)^{-n}]$

$$= \frac{1682}{0{\cdot}16}[1 - (1{\cdot}16)^{-2}]$$

$$= 2700$$

Hence, principal is ₹ 2700

Now, the first instalment of ₹ 1682 includes interest on ₹ 2700 for 1 year

i.e. ₹ (2700 × 0·16 × 1) = ₹ 432, and principal repayment of ₹ (1682 – 432) *i.e.* ₹ 1250.

Hence, principal remaining after first payment = ₹ (2700 – 1250)

= ₹ 1450

Now, second instalment of ₹ 1682 includes interest on the remaining principal

i.e. ₹ (1450 × 0·16 × 1) = ₹ 232, and principal repayment of ₹ (1682 – 232) = ₹ 1450.

Example 9. *Sanjay borrows a loan of ₹ 400950 on condition to repay it with compound interest at 6% p.a. by annual instalments of ₹ 150000 each. In how many years will the debt be paid off ?*

Solution. Here, present value, V = ₹ 400950,

each instalment A = ₹ 150000,

$r = 6\%$ p.a. $= 0{\cdot}06$

We wish to calculate the number of instalments, n.

Using $V = \frac{A}{r}[1 - (1 + r)^{-n}]$

$$₹\ 400950 = ₹\ \frac{150000}{0{\cdot}06}[1 - (1{\cdot}06)^{-n}]$$

$$\Rightarrow \quad 1 - (1{\cdot}06)^{-n} = \frac{400950 \times 0{\cdot}06}{150000} = 0{\cdot}1604$$

$$(1{\cdot}06)^{-n} = 1 - 0{\cdot}1604 = 0{\cdot}8396$$

Taking logs, $-n \log 1{\cdot}06 = \log 0{\cdot}8396$

$\Rightarrow \ -n \times 0{\cdot}0253 = -0{\cdot}759$

$\Rightarrow \ n = 3.$

Hence, it will take 3 years to pay the debt off.

Example 10. *You have taken a loan and have to repay it in 10 annual instalments of ₹ 1000 each. The rate of interest is 10% p.a. If you want to pay off the loan in annual instalments of ₹ 2000 each, how many instalments will be required to be paid ? What would be the amount of last instalment ?*

Solution. Amount of loan borrowed is the present value of annuity of ₹ 1000. Here A = 1000, $n = 10$, $r = 0{\cdot}1$

$$\therefore \text{ Loan borrowed, } V = \frac{A}{r}[1-(1+r)^{-n}]$$

$$= ₹\ \frac{1000}{0{\cdot}1}[1-(1{\cdot}1)^{-10}]$$

$$= ₹\ 6145 \text{ (approx.)}$$

Now if n instalments of ₹ 2000 will pay off this loan, then

$$₹\ 6145 = ₹\ \frac{2000}{0{\cdot}1}[1-(1{\cdot}1)^{-n}]$$

$$\Rightarrow \quad 1-(1{\cdot}1)^{-n} = \frac{6145 \times 0{\cdot}1}{2000} = 0{\cdot}3072 \quad \Rightarrow (1{\cdot}1)^{-n} = 0{\cdot}6928$$

$$\Rightarrow \quad -n \log 1{\cdot}1 = \log 0{\cdot}6928 \quad \Rightarrow -n\,(0{\cdot}0414) = -0{\cdot}1594$$

$$\Rightarrow \quad n = \frac{0{\cdot}1594}{0{\cdot}0414} = 3{\cdot}85$$

Thus, four instalments will be required, first three of ₹ 2000 each and the fourth less than ₹ 2000.

Now present value of first 3 instalments of ₹ 2000 each

$$= ₹\ \frac{2000}{0{\cdot}1}[1-(1{\cdot}1)^{-3}] = ₹\ 4974$$

∴ Present value of fourth instalment = ₹ (6145 – 4974) = ₹ 1171

Hence, amount of fourth instalment

$$= ₹\ 1171 \times (1{\cdot}1)^4 = ₹\ 1714 \text{ (approx.)}$$

Example 11. *A man borrows ₹ 37500 and agrees to repay in semi-annual instalments of ₹ 2250 each, the first due in 6 months. How many payments must he make if rate of interest is 6% compounded semi-annually ?*

Solution. Here we have an ordinary annuity certain of present value V = ₹ 37500, periodic payment A = ₹ 2250, rate of interest per period, $r = 0{\cdot}03$ and we have to find the number of payments, n.

$$V = \frac{A}{r}[1-(1+r)^{-n}] \Rightarrow ₹\ 37500 = ₹\ \frac{2250}{0{\cdot}03}[1-(1{\cdot}03)^{-n}]$$

$$\Rightarrow 1-(1{\cdot}03)^{-n} = \frac{37500 \times 0{\cdot}03}{2250} = 0{\cdot}5$$

$$\Rightarrow \quad (1{\cdot}03)^{-n} = 0{\cdot}5 \quad \Rightarrow \quad -n \log 1{\cdot}03 = \log 0{\cdot}5$$

$$\Rightarrow -n\,(0{\cdot}0128) = -0{\cdot}3010$$

$$\Rightarrow \quad n = \frac{-0{\cdot}3010}{-0{\cdot}0128} = 23{\cdot}51$$

Thus, the money may be repaid in 23 instalments with 23rd instalment slightly more than ₹ 2250, or the money may be repaid in 24 instalments, the 24th instalment slightly less than ₹ 2250. Following the procedure of previous example, the amount of 24th instalment

$$= ₹\ \left\{37500 - \frac{2250}{0{\cdot}03}[1-(1{\cdot}03)^{-23}]\right\} \times (1{\cdot}03)^{24}$$

$$= ₹\ 1020{\cdot}22$$

Thus, there will be 23 instalments of ₹ 2250 each and 24th instalment of ₹ 1020·22.

Example 12. *Anshul buys a flat for ₹ 800000 on the following conditions : 25% cash down payment and balance in 10 equal semi-annual instalments, the first to be paid six months after the date of purchase. Calculate the amount of each instalment, if the rate of interest is 10% per annum compounded half-yearly. Also calculate the total amount of interest paid by Anshul.*

Solution. Cash down payment = 25% of ₹ 800000 = ₹ 200000

Balance = ₹ 800000 – ₹ 200000 = ₹ 600000

This is to be paid in 10 equal semi-annual instalments, say A.

Here $V = ₹\ 600000,\ n = 10,\ r = 0{\cdot}05$

$$V = \frac{A}{r}[1 - (1 + r)^{-n}]$$

$$\Rightarrow\quad ₹\ 600000 = \frac{A}{0{\cdot}05}[1 - (1{\cdot}05)^{-10}]$$

$$\Rightarrow\quad A = ₹\ \frac{600000 \times 0{\cdot}05}{1 - (1{\cdot}05)^{-10}} = ₹\ 77702{\cdot}75$$

Hence, the amount of each instalment is ₹ 77703 approximately

Now to calculate the interest paid, we note that in lieu of ₹ 600000 cash payment, 10 instalments of ₹ 77703 each were paid.

Hence, total interest paid = ₹ (77703 × 10 – 600000) = ₹ 177030.

Example 13. *Mr. Mehta purchased a house, paying ₹ 50000 down and promising to pay ₹ 2000 every 3 months for the next 10 years. The interest is 6% p.a. compounded quarterly.*

(i) What is the cash value of the house ? Round off your answer to nearest ₹ 100.

(ii) If Mr. Mehta misses first 6 payments, how much should he pay at the time of 7th payment to bring himself upto date ?

(iii) If at the end of 5th year, he wants to finish his liability by a single payment, how much should he pay ?

Solution. (*i*) We have an ordinary annuity certain, where each payment A = ₹ 2000, number of payments = 40, rate of interest per period = 6% p.a. = 1·5% per quarter = 0·015 per quarter

Present value of annuity, $V = \frac{A}{r}[1 - (1 + r)^{-n}]$

$$\Rightarrow\quad V = ₹\ \frac{2000}{0{\cdot}015}[1 - (1{\cdot}015)^{-40}]$$

$$= ₹\ 59831{\cdot}69$$

Hence, V = ₹ 59832 approximately.

Thus, total cash value of house = ₹ 50000 + ₹ 59832

= ₹ 109832 approx

= ₹ 109800 (correct to nearest ₹ 100)

(*ii*)

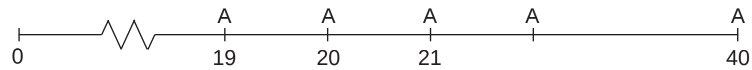
At the time of seventh payment, equivalent of first 6 missed payments has also to be paid. Thus, total payment required at end of 7th period is the amount of annuity of 7 terms ; hence, amount required to be paid

$$= \frac{A}{r}[(1 + r)^n - 1] = ₹\ \frac{2000}{0{\cdot}015}[(1{\cdot}015)^7 - 1]$$

$$= ₹\ 14645{\cdot}99$$

i.e. ₹ 14646 approximately.

(*iii*)



If at the end of 5th year *i.e.* at the time of 20th payment, he wants to finish off the liability, then lump sum payment required

$$= ₹\ 2000 + ₹\ \frac{2000}{0{\cdot}015}\ [1 - (1{\cdot}015)^{-20}]$$

$$= ₹\ 2000 + ₹\ 34337{\cdot}28$$

$$= ₹\ 36337 \text{ approximately.}$$

EXERCISE 2.1

1. Mr. X purchased an annuity of ₹ 2500 per year for 15 years from an insurance company which reckons the interest at 3% compounded annually. If the first payment is due in one year, what did the annuity cost X ?

2. Find the amount of an ordinary annuity if payment of ₹ 600·00 is made at the end of every quarter for 10 years at the rate of 4% per year compounded quarterly. (*I.S.C. 2005*)

3. A man borrowed some money and paid back in 3 equal annual instalments of ₹ 2160 each. What sum did he borrow if the rate of interest charged was 20% p.a. compounded annually ? Find also the total interest charged. (*I.S.C. 2007*)

4. Anshul has been depositing ₹ 1000 at the end of every year out of his pocket money in a savings account which pays $3\frac{1}{2}$% effective. What is the amount in his credit just after the tenth deposit ?

5. Find the amount and the present value of an ordinary annuity of ₹ 150 a month for 6 years 3 months at 6% compounded monthly.

6. Calculate the amount of ordinary annuity of ₹ 7000 at the rate of 10% per annum compounded annually for 10 years.

7. Find the present value of an annuity of ₹ 1200 payable at the end of each 6 months for 3 years when the interest is earned at 8% per year compounded semi-annually.

8. Find the future amount of an ordinary annuity of 12 monthly payments of ₹ 1000 that earn an interest at 12% per year compounded monthly.

9. The price of a tape recorder is ₹ 1661. A person purchased it by making a cash payment of ₹ 400 and agrees to pay the balance with due interest in 3 half-yearly equal instalments. If the dealer charged interest at the rate of 10% per annum compounded half-yearly, find the value of the instalment. (*I.S.C. 2011*)

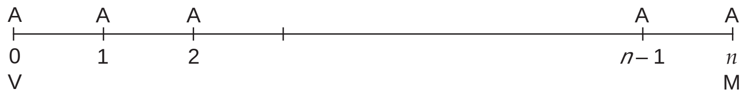
10. Mr A buys a television for ₹ 6000 down and ₹ 500 per month for the next 12 months. If interest is charged at 9% compounded monthly, find the equivalent cash value.

11. A washing machine was on sale for ₹ 13500 cash or ₹ 6000 as initial payment and the rest paid in equal monthly instalments over a period of 10 months at 20% interest per annum compound interest. Find the equal monthly instalments to be paid, giving your answer to nearest rupee, if not exact.

12. At 6 month intervals Mr. Gupta deposited ₹ 5000 in a savings account paying 6% compounded semi-annually. The first deposit was made when his son was 6 months old and the last deposit was made when the son was 21 years old. The money remained in account and was presented to the son on his 25th birthday. How much did he receive ?

2.4 AMOUNT AND PRESENT VALUE OF ANNUITY DUE

The previous section was about ordinary or immediate annuity, where payment is at end of each payment period. In annuity due, payment is at *beginning* of each payment period, for example, paying monthly house rent in advance each month.



Note that first payment is made at beginning of first period *i.e.* time zero; last payment is made the beginning of nth period *i.e.* time $(n - 1)$.

Here, present value

$$V = A + \frac{A}{1+r} + \frac{A}{(1+r)^2} + \ldots + \frac{A}{(1+r)^{n-1}} = \ldots = \frac{A}{r}(1+r)[1-(1+r)^{-n}]$$

and future amount

$$M = A\,(1+r)^n + A(1+r)^{n-1} + \ldots + A(1+r) = \ldots = \frac{A}{r}\,(1+r)\,[(1+r)^n - 1]$$

Note that this is easily obtained from formulae of ordinary annuity by multiplying by factor $(1 + r)$; that can be easily understood if you visualise that all instalments are shifted from end of period to beginning of period, and so they amount to $(1 + r)$ times more.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the amount and present value of an annuity due of ₹ 500 per quarter for 8 years and 9 months at 6% compounded quarterly.*

Solution. Here, rate of interest $r = 1{\cdot}5\%$ per interest period $= 0{\cdot}015$, number of interest periods, $n = 4 \times 8 + 3 = 35$, and each instalment, A = ₹ 500

Present value of annuity due,

$$V = \frac{A}{r}(1+r)\,[1-(1+r)^{-n}]$$

$$= ₹\ \frac{500}{0{\cdot}015}(1{\cdot}015)\,[1-(1{\cdot}015)^{-35}]$$

$$= ₹\ 13740{\cdot}86$$

Amount of annuity due,

$$M = \frac{A}{r}(1+r)\,[(1+r)^n - 1]$$

$$= ₹\ \frac{500}{0{\cdot}015}(1{\cdot}015)\,[(1{\cdot}015)^{35} - 1]$$

$$= ₹\ 23137{\cdot}98$$

Example 2. *What equal payments made at the beginning of each year for 10 years will pay for a piece of land priced at ₹ 400000, if money is worth 7% per annum compounded annually ?*

Solution. Here we have an annuity due with

rate of interest, $r = 7\%$ p.a. $= 0{\cdot}07$,

number of interest periods, $n = 10$,

present value, V = ₹ 400000

We wish to calculate the amount of each instalment, A

Now, for annuity due, present value

$$V = \frac{A}{r}(1+r)\,[1-(1+r)^{-n}]$$

5. A rich man dies, leaving his widow with ₹ 2 lacs in a bank earning 5% interest. If the widow spends ₹ 18000 every year, show that she will be ruined before the end of the 17th year.

6. A person deposits ₹ 15000 at the end of each year in an account that pays 15% interest compounded annually. Find the amount in his account at the end of 5 years.

Round off your answer to nearest hundred rupees.

7. A firm anticipates a capital expenditure of ₹ 50000 for a new equipment in 5 years. How much should be deposited quarterly in a sinking fund carrying 12% per annum compounded quarterly to provide for the purchase ?

8. A person sets up a sinking fund in order to have ₹ 100000 after 10 years for his children's college education. How much amount should be set aside after every 6 months into an account paying 5% p.a. compounded half-yearly ?

9. A person borrows ₹ 50000 at 8% p.a. interest compounded semi-annually and agrees to pay both the principal and interest at 10 equal instalments at the end of every six months. Find the amount of these instalments.

10. A company borrows a loan of ₹ 400950 on the condition to repay it with compound interest at 6% p.a. by annual instalments of ₹ 150000 each. In how many years will the debt be paid off ?

11. Find the present value of a sequence of annual payments of ₹ 6000 each, the first being made at the end of 5 years and the last at the end of 12 years, if money is 6% effective.

12. A man wishes to buy a government house valued at rupees one lac. He will have to pay 20% initially and the balance in 20 equal annual instalments. If the rate of interest is 12% p.a., calculate the amount of each instalment.

13. Beginning on 1st October 2010 and continuing for 4 more years, ₹ 20000 will be needed (every year) to retire certain school bonds. What equal annual deposits in a fund paying 5% effective beginning on 1st October 2000 and continuing for 14 more years are necessary to retire the bonds as they fall due ?

14. A man wants to have ₹ 12000 in his account after 10 years. He deposits annual payments in an account that pays 4% rate of interest compounded annualy. How much should he deposit each year?

ANSWERS

EXERCISE 2.1

1. ₹ 29844·84. **2.** ₹ 29331·82. **3.** ₹ 4550; ₹ 1930.

4. ₹ 11731·39. **5.** ₹ 136089·98; ₹ 9362·05. **6.** ₹ 111561·97.

7. ₹ 6290·56. **8.** ₹ 12682·50. **9.** ₹ 463·05.

10. ₹ 11717·46. **11.** ₹ 820·45. **12.** ₹ 519522·66.

13. ₹ 15600·41. **14.** ₹ 2615·54. **15.** ₹ 1128·25.

16. ₹ 18780. **17.** ₹ 3397·23. **18.** ₹ 926·10

19. ₹ 1450. **20.** ₹ 12979·36.

21. ₹ 292423·76; ₹ 100000; ₹ 192423·76.

22. (*i*) ₹ 196·97 (*ii*) ₹ 63529·30. **23.** ₹ 45242·49.

24. Comparing present values ₹ 53349·26, ₹ 54509·23, it is better to purchase second annuity.

25. ₹ 21537·23 **26.** 14·2 years approximately.

27. Never (As the annual instalment of ₹ 200 does not cover the interest on ₹ 2500 at 10% p.a. *i.e.* ₹ 250).

28. 27 years 8 months. **29.** 11 years.